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Option pricing using jump diffusion model: a case of stock markets of selected east African countries

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OPTION PRICING USING JUMP DIFFUSION MODEL: A CASE OF STOCK MARKETS OF SELECTED EAST AFRICAN COUNTRIES

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A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Master's in Mathematical and Computer Sciences and Engineering of the Nelson Mandela African Institution of Science and Technology

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ABSTRACT

The stock price is characterized by several features which can only be captured by the best model. To investigate this the Merton's jump-diffusion model was developed and applied to the selected stocks of three East African community countries' stock markets. The daily closing stock prices of the Nairobi Securities Exchange, the Dar es Salaam Stock Exchange and Uganda Securities Exchange over a period of five (5) years from 1st July, 2013 to 1st July, 2018 were analyzed. The objective of this analysis was to investigate how best the developed model do price options when the stock price features of three East African stock markets are incorporated into the model. The Merton's jump-diffusion model was employed as a stochastic differential equation. While the Maximum Likelihood Estimation method was used to estimate the optimal model parameters and implemented with MATLAB. For comparison purpose, the researcher estimated the parameters of the Black- Scholes model. The empirical results show that the Merton Jump Diffusion gives realistic option price values for the selected stocks due to the incorporation of the compound Poison process. On the other hand, the selected stocks from all three markets exhibit several jumps as it was evidenced from non-zero values of jump intensities (lambda). Also, the log-returns density of Merton reveals the presence of volatility and leptokurtic features due to the presence of both negative and positive skewness and excessive kurtosis values.

Keywords: Kurtosis, Options, Leptokurtic.

DECLARATION

I, Kimaro Novat do hereby declare to the Senate of Nelson Mandela African Institution of Science and Technology that this dissertation is my original work and that it has neither been submitted nor being concurrently submitted for degree award in any other institution.

…………………………… **…………………………**

Kimaro Novat **Date**

Name and signature of the candidate

The above declaration is confirmed by:

………………………… **…………………………**

Dr. Wilson M. Charles **Date**

Name and Signature of Supervisor 1

………………………….. **……………………….**

Prof. Verdiana G. Masanja **Date**

Name and Signature of Supervisor 2

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CERTIFICATION

We, the undersigned certify that we have read and hereby recommend for acceptance by the Nelson Mandela African Institution of Science and Technology, a dissertation entitled *Option Pricing Using a Jump Diffusion Model: A Case of Stock Markets of East African Countries,* in partial fulfillment of the requirements for the degree of Master's in Mathematical and Computer Sciences and Engineering of the Nelson Mandela African Institution of Science and Technology.

………………………… **…………………………**

Dr. Wilson M. Charles **Date**

Name and Signature of Supervisor 1

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Prof. Verdiana G. Masanja **Date**

Name and Signature of Supervisor 2

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DEDICATION

This dissertation is dedicated to my wife Renatha and my children Brilliant and Brightness.

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LIST OF ABBREVIATIONS AND SYMBOLS

ARJI-GARCH	Autoregressive Jump Intensity-Generalized Autoregressive Conditional Heteroscedastic.
BATU	British American Tobacco Uganda.
BOBU	Bank of Baroda Uganda.
BS	Black-Scholes Model.
C&G	Car & General (K) Ltd Ord 5.00.
DSE	Dar es Salaam Stock Exchange.
KCB	Kenya Commercial Bank Group Plc. Ord 1.00.
KQ	Kenya Airways Ltd Ord 5.00.
LIMT	Limuru Tea Co. Plc. Ord 20.00AIMS.
MJD	Merton's Jump Diffusion Model.
NIC	National Insurance Corporation.
NMB	National Microfinace Bank Plc.
NSE	Nairobi Securities Exchange.
SBU	Stanbic Bank Uganda.
SCOM	Safaricom Plc. Ord 0.05.
SDE	Stochastic Differential Equation.
SWIS	Swissport Tanzania Plc.
TBL	Tanzania Breweries Limited.
TCC	Tanzania Cigarette Company.
UMEM	UMEME Limited.
URT	United Republic of Tanzania.
USE	Ugandan Securities Exchange.

VODA	Vodacom Tanzania Limited.
$B(t)$	Standard Brownian Motion.
$N(t)$	Poisson Process.
$S(t)$	The Stock Price at Time t .
α	Drift Coefficient, Mean Log Return.
δ	Standard Deviation of the Logarithm of the Jump Size. Distribution.
λ	Jump Intensity/Amplitude.
μ	Mean of the Logarithm of the Jump Size Distribution. .
σ	Volatility, Standard Deviation Of Daily Log Return.

CHAPTER ONE

INTRODUCTION

1.1 Background of the problem

Recently, several empirical studies have pointed out that financial returns exhibit volatility with a stochastic pattern and fatter tails than the standard normal model, which is not suitable for capturing the asset price dynamics (Scoot, 1997 ; Costabile, Leccadito, Massabó & Russo, 2014). Models such as the Black-Scholes model have been used in option valuation as a basis for nearly four decades (Yi, 2010). The Black-Scholes method is based on assumption that the asset's price, in this regard that is the underlying asset of the option follows a geometrical Brownian motion. Nevertheless a geometrical Brownian motion cannot accommodate most of features of a stock price (Burger & Kliaras, 2013). Despite, the usefulness of the BS model as the basic model for pricing option, it fails to capture stock price features such as the leptokurtic and empirical abnormity called volatility smile (Kou, 2008).

However, several models have been proposed to accommodate such properties, particularly the leptokurtic and volatility smile (Toivanen, 2008; Lin, Chao & Miao, 2017). These include the Model based on exponential Lévy processes (Cont &Tankov, 2004) such as Merton Jump diffusion model (Merton, 1976), the Kou's jump-diffusion model (Kou, 2008) and Variance Gamma Model (Matsuda, 2004).

Additionally, the financial derivatives in particular the options are traded both on stock exchanges markets and over the counter. The main two kinds of options styles are European options and American options, European options can only be exercised at the expiration time but American options can be exercised at any time up to expiration. Furthermore options are categorized into Call options and Put options. Call options give the buyer the right but not the obligation to buy a particular asset, and the seller has to provide it. A put option gives the buyer the right but not the obligation to sell the asset (Hull, 2015).

Moreover, price of options is determined by the forces of supply and demand in the markets but likewise it is done by mathematical models. Since the market is characterized with imbalances in supply and demand then the mathematical models are fair good in options valuations (Burger & Kliaras, 2013).

As discussed before the Black-Scholes model (BS) which is based on the Brownian motion and normal distribution is not suitable for option valuation because of failure to accommodate the leptokurtic features and the empirical abnormity called volatility smile. Thus, the jumps model with added compound poison process remidies these weaknesses in the Black-Scholes approach.

This study concentrated on extending the Black-Scholes model by adding a Compound Poison Process and using the daily closing prices data from stock exchange markets of Dar es Salaam Stock Exchange (DSE), Nairobi Securities Exchange (NSE) and Ugandan Securities Exchange (USE) to compute options and test whether the model captures most of the stock price features or not.

1.2 Statement of the problem

The government of Tanzania ordered all telecommunications companies in the country to sell at least 25% stake at DSE market to increase domestic ownership (URT, 2016). The first telecommunication company to implement this was Vodacom-Tanzania. Initially, it offered 560 million ordinary shares at 850/- per share. The response was that 60% of investors were Tanzanians and the remaining 40% were non-Tanzanians (Daily News Tanzania, 2017). Also, Tanzania mining companies registered at DSE following the mining regulation of 2017 (TanzaniaInvest.com, 2018) and many other companies registered to make a total of 28 (DSE, 2018) from 4 companies since its establishment in 1998 (Ziorklui, 2001). In Kenya and Uganda, the registered companies at Nairobi Securities Exchange (NSE) and Ugandan Securities Exchange (USE) are 64 and 18 respectively (African Markets, 2018).

Though many companies are registering and trading at the stock markets to raise their capital investment, stock trading is associated with high risk. To mitigate the risk exposure which arises from the unpredictability of stock prices the companies can hedge by investing in financial derivatives such as options which give the holder the right but not the obligation to buy or sell an asset at or before a future date for the predetermined price (DSE, 2018; Hull, 2015).

Moreover, to be able to exercise the financial derivatives (options) needs to have the best model which can predict in advance when and how to exercise the options and the one which accommodates most of the stock features. In many works in literature, the available models have been applied to other stock markets rather than East African stock markets. Example the study on the Japanese stock market using Kou Jump model (Maekawa, Lee, Morimoto & Kawai, 2008), the pricing options in jump-diffusion using Mellin transform at an American stock index of S&P 500 (Frontczak, 2013) and the study on Egypt, Nigeria and South Africa stock markets using Autoregressive Jump Intensity-Generalized Autoregressive Conditional Heteroscedastic (ARJI-EGARCH) model (Kuttu, 2017) just to mention a few. Therefore, this study intends to do option modeling using the Merton's jump-diffusion model (Merton, 1976) to investigate if the model captures most of the stock features basing on the data from East African stock markets.

1.3 Rationale of the study

This study provides useful information on the valuation of options at stock markets. The stock markets are characterized with fluctuation of prices due to many factors such as economic, political, social and technological changes. Due to these factors, determining the prices of underlying becomes problematic. Therefore due to these challenges in the stock markets, the model developed in this study will have great contribution in curbing these challenges of market failure.

1.4 Objectives of the study

1.4.1 General objective

To develop the jump-diffusion model and using data from East African countries' stock markets to investigate the best option pricing model.

1.4.2 Specific objectives

The specific objectives of this study are:

(i) To develop the jump-diffusion model which captures most of the stock features.

(ii) To compare the Jump diffusion model with the Black-Scholes model based on data gathered from the NSE, DSE, and USE.

(iii) To establish whether the log return distribution of the developed model incorporates the leptokurtic and asymmetric features.

1.5 Research questions

The study intends to address the following questions;

(i) What are the assumptions and procedures in the development of a jump-diffusion model?

(ii) What are the differences and similarities between the jump-diffusion model and the BS model based on data from East African stock markets?

(iii) Are leptokurtic and asymmetric features in the log return distribution of a jump-diffusion model incorporated?

1.6 Significance of the study

The results can help the stock exchange markets to address the challenges of market failure in determining the best option. Also, the results will help people to understand the importance of holding their money in financial assets rather than cash and other forms and last but not least the study will provide the platform for further research.

1.7 Delineation of the study

The study focused on option pricing using jump diffusion model. Only three stock markets of the selected East African countries were considered. The markets were Nairobi Securities Exchange (NSE) of Kenya, Dar es Salaam Stock Exchange (DSE) of Tanzania and Ugandan Securities Exchange (USE) of Uganda. The five stocks from each market were chosen randomly to make a total of fifteen stocks at all three stock markets. The study also, concentrated on martingale approach in option valuation.

CHAPTER TWO

LITERATURE REVIEW

2.1 Definition of terms

2.1.1 Option

An option is a financial security that gives the holder the right but not an obligation to buy or sell a specified quantity of a specified asset at a specified price on or before a specified date (Hull, 2015).

2.1.2 Types of option

(i) A call option gives the holder the right but not the obligation to buy the underlying asset by a certain date for a certain price.

(ii) A put option gives the holder the right but not the obligation to sell the underlying asset by a certain date for a certain price.

2.1.3 Option styles

(i) A European Call (Put) Option gives the right but not the obligation to purchase (sell) a stock at a specific time called maturity T for a specific amount K called the strike price.

(ii) An American Call (Put**)** Option gives the right but not the obligation to purchase (sell) stock for a specific strike price K, at any time up to maturity T.

2.2 Assumptions of Merton jump diffusion model

Merton (1975) (see pages 1, 2, 4 and 5) as cited by Burger and Kliaras (2013), provided the following assumptions which must be made in the process of developing the MJD model regarding the market situation.

(i) Frictionless markets, this means there are no transaction costs of differential taxes.

(ii) No dividend payments. The risk-free interest rate is available and constant over time.

(iii) No restrictions regarding the value of transaction and price development of the asset.

(iv) Short trading is not prohibited.

(v) Stocks are randomly divisible.

(vi) All information is available to all market participants.

(vii) No arbitrage possibilities.

(viii) The option is a European style option.

2.3 Literature review of empirical studies

Kuttu (2017), used ARJI-EGARCH model to examine the time-varying conditional discrete jump dynamics in thinly-traded adjusted equity returns of Egypt, Nigeria and South Africa. The results show that conditional discrete jump sensitivity is determined in all considered three markets and only South Africa is more probable to show asymmetric conditional jump volatility. These three markets are the largest stock markets in Africa. The study concentrated only on determining discrete jump dynamics but did not extend to options valuation and examine other stock features like leptokurtic.

Mwaniki (2015), on the study using log-ARCH –Levy type model to study daily asset return, the empirical analyses of the Standard and Poor (S & P 500) index and NSE 20 index shows that in both markets features such as volatility and leptokurtic (features of financial time series data) were captured. This study focused much on the correlation and autoregression between these financial features (volatility and leptokurtic) rather than showing how they can be captured in financial derivatives markets.

Mayanja, Mataramvura and Charles (2013) on their study of a mathematical approach that dwelled much on stocks portfolio selection. They established that Uganda brokers use the qualitative approach and speculation because models available have not been customized to suit their situation. However, this study emphasis on the use of models based on optimization in stocks portfolio selection rather than option pricing.

Namugaya, Patrick and Charles (2014) used Generalized Autoregressive Conditional Heteroscedastic (GARCH) models to study stock return volatility on USE. The study established that, the USE returns are non-normal and heteroscedasticity was existing. Also the USE return series exhibited volatility clustering and leptokurtosis as evidenced from the high kurtosis values. Likewise the study did not show how the jumps in the log return can be determined.

Urama and Ezepue (2018) on their work of “Stochastic Ito-Calculus and Numerical Approximation for Asset Price Forecasting in the Nigerian Stock Market” emphasis on the need of a mathematical model for modeling financial derivatives. Their work based much on the estimation of parameters of the BS model using the Euler-Maruyama method rather than possible extension to incorporate the compound jump process.

Kou (2008), cited by (Yi, 2010) the distribution of the standard and poor (S & P 500) index daily log return was plotted and it was found that the jump was significantly higher and the tails were fatter than a normal distribution. The Black-Scholes model could not be suitable for modeling the option price. This is because the jump component could not be captured as the Black Scholes model base on the geometric brown motion. It is proposed that an extra jump part must be added to the model to give a response to the under reaction and the overreaction to the external news.

Burger and Kliaras (2013) on the study which involved comparing the jump-diffusion model and Black Scholes model concluded that the stock price exhibit the jumps and the standard BS model is not suitable for option pricing. They suggested extra assumptions to be made regarding the BS model in order to a good result when valuing options. For instance, the assumption on jump intensity and how jumps are determined could be anticipated. They further emphasized that the jump-diffusion models like that of double exponential which does not rely on normal distribution of the stock returns but a distribution which has got a higher peak and two heavier tails fit stock data better (Kou, 2002).

Lin, Chao and Miao (2017) emphases that the suitable model for modeling the price of the underlying asset must include the jump component and according to their study on “analysis of jump-diffusion option pricing model with serially correlated jump size”, establish that the jump size is serially correlated with the return of the underlying stock price. Friesen, Weller and Dunham (2009) as cited by (Lin *et al.*, 2017) conveyed that positive jump size correlation was present in the prices of stocks constituting the Standard and Poor (S & P 100) index.

Xu and Jia (2019), their study on the calibration of parameters of jump-diffusion models establishes that the jump-diffusion models generate several volatility smiles and skews when the parameters of the jump-diffusion process are chosen properly. It was investigated that the jump component is very important especially when the derivative (option) is nearby expiration date. This shows that a good model for options pricing should incorporate the jump component.

Therefore, these literatures provide information on how the stock price features can be determined using various models. Some literatures suggest the need to add the jump component when valuing options which is the focus of this study. Since the capital markets especially the markets for financial derivatives are characterized with fluctuations such as sudden upsurges and crashes (jumps). To model these fluctuations of stocks this study came up with Merton's Jump Diffusion model (MJD) model which accommodates most of stock price features.

2.4 Research framework

The relationship of the variables in the jump model which will be the basis in this study is given by the equation.

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dB(t) + \eta dN(t) \tag{2.1}$$

where $S(t)$ is the stock price at time t, $B(t)$ is the Brownian motion, $N(t)$ is a Poisson process with an intensity λ, μ is the drift coefficient , σ is volatility and η is an impulse function which causes a jump of $S(t)$ to $S(t+1)$ (Kou, 2008; Xu & Jia, 2019) .

The stock price, $S(t)$ is generally described by a continuous diffusion part and a discontinuous jump part. The typical fluctuation in the stock price is accounted for by the continuous diffusion part (determine by Brownian motion). The discontinuous jump part is responsible for the extreme (rare) events and is determined by η.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Study design

The cross-sectional design was used, whereby the DSE, NSE and USE were examined simultaneously.

3.2 Data types and collection methods

Secondary data from DSE, NSE and USE of the daily closing stock prices for the period of 5 years from 1 July, 2013 to 1 July, 2018 were used. The only five (5) actively trading companies from each stock market were randomly chosen making a total of fifteen (15) companies from all the three stock markets. The trading days per year in all the three markets were assumed to be 252 days.

3.3 Model development and estimation

3.3.1 Development of jump diffusion model

Merton Jump Diffusion model is the new version which extend BS model in the way that it enables to incorporate the stock price features such as skewness and kurtosis of the log stock price density $P\left(\ln\left(\frac{S(t)}{S(0)}\right)\right)$ by adding a compound Poisson jump process (Matsuda, 2004). This compound Poison jump process leads to additional of new more parameters λ, μ, and δ to the basic model called BS model.

Merton Jump Diffusion model is one of an exponential Lévy model of the form.

$$S(t) = S(0)e^{\mathcal{L}(t)} \quad (3.1)$$

For which, $(S(t); 0 \leq t \leq T)$ is treated as the exponential of a Lévy process. The Lévy process is chosen because it consists of two parts which are a continuous diffusion process (Brownian motion with drift) and a discontinuous jump process determined by compound Poisson process as:

$$\mathcal{L}(t) = \left(\alpha - \frac{\sigma^2}{2} - \lambda k\right)t + \sigma B_t + \sum_{m=1}^{N(t)} Y_m \quad (3.2)$$

where $(B(t); 0 \leq t \leq T)$ is a standard Brownian motion. The term $\left(\alpha - \frac{\sigma^2}{2} - \lambda k\right)t + \sigma B(t)$

is a continuous diffusion process or Brownian motion with drift process and on the other hand $\sum_{m=1}^{N(t)} Y_m$ is a compound Poisson jump process (Matsuda, 2004). According to Merton (1976), the jump part enables to model sudden and unexpected price jumps of the underlying asset. The compound Poisson jump process encompasses two sources of randomness.

(i) Poisson process $dN(t)$ with intensity λ which cause the asset price to jump randomly.
(ii) Random jump size i.e. how much it jumps. It is assumed that $(Dx_i) \sim i.i.d. Normal(\mu, \delta^2)$ where Dx_i is log stock price jump size.

$$f(Dx_i) = \frac{1}{\sqrt{2\pi\delta}} exp\left\{-\frac{(Dx_i-\mu)^2}{2\delta^2}\right\} \quad (3.3)$$

The assumption is that, the two sources of randomness are independent of each other. The Lévy measure $\ell(Dx_i)$ of a compound Poisson process is given as a product of the intensity and the jump size density as:

$$\ell(Dx_i) = \lambda f(Dx_i). \quad (3.4)$$

Considering equation (3.1) then, the log-return $\ln\left(\frac{S(t)}{S(0)}\right)$ is now modeled as a Lévy process such that

$$\ln\left(\frac{S(t)}{S(0)}\right) = \mathcal{L}(t) = \left(\alpha - \frac{\sigma^2}{2} - \lambda k\right) t + \sigma B_t + \sum_{m=1}^{N(t)} Y_m \quad (3.5)$$

3.3.2 Model derivation

For the case of MJD model the changes in the asset price is determined by two components which are a continuous diffusion component that is modeled by a Brownian motion with drift process and a discontinuous component that is modeled by a compound Poisson process (Matsuda, 2004). It should be noted that, the asset's price jumps are assumed to occur independently and identically. The probability when the asset price jumps in a very small time interval dt can easily be stated using a Poisson process $dN(t)$ as:

$$P\{an\ asset\ price\ jumps\ once\ in\ dt\} = Pr\{dN(t) = 1\} \cong \lambda dt.$$

$$P\{an\ asset\ price\ jumps\ more\ than\ once\ in\ dt\} = Pr\{dN(t) \geq 2\} \cong 0.$$

$$P\{an\ asset\ price\ does\ not\ jump\ in\ dt\} = Pr\{dN(t) = 0\} \cong 1 - \lambda dt.$$

Where the parameter $\lambda \in \mathbb{R}^+$ is the intensity of the jump process which is independent of time t.

For a small dt the asset price is expected to change (jump) from $S(t)$ to $y_t S(t)$, where y_t is absolute price jump size. Therefore the percentage change in the asset price triggered by jump (i.e. the relative price jump size) is given as:

$$\frac{dS(t)}{S(t)} = \frac{y_t S(t) - S(t)}{S(t)} = y_t - 1 \tag{3.6}$$

Where (Merton, 1976), assumes that y_t is a nonnegative random variable drawn from a lognormal distribution, i.e. $\ln(y_t) \sim i.i.d.\, N(\mu, \delta^2)$. The implication of this is that:

$$E(y_t) = e^{\mu + \frac{1}{2}\delta^2} \tag{3.7}$$

$$E[(y_t - E(y_t)^2] = var[y_t] = e^{\mu + \frac{1}{2}\delta^2}\left(e^{\delta^2} - 1\right). \tag{3.8}$$

This is because if $\ln x \sim N(a, b)$ then

$$x \sim Lognormal\left(e^{a + \frac{1}{2}b^2}, e^{a + \frac{1}{2}b^2}\left(e^{b^2} - 1\right)\right) \tag{3.9}$$

Incorporating the above properties into the MJD dynamics of the asset price, we obtain an SDE of the form;

$$\frac{dS(t)}{S(t)} = (\alpha - \lambda k)dt + \sigma dB(t) + (y_t - 1)dN(t). \quad (3.10)$$

It is assumed that the processes $B(t), N(t)$ and y_t are independent of each other. Hence, $y_t - 1$ is log-normally distributed with a mean

$$E[y_t - 1] = e^{\mu + \frac{1}{2}\delta^2} - 1 = k \quad (3.11)$$

and the variance.

$$E[(y_t - 1 - E(y_t - 1)^2] = e^{2\mu + \delta^2}\left(e^{\delta^2} - 1\right) \quad (3.12)$$

It is assumed that, the log-return jump size $\ln(y_t) \equiv Y_t$ is a normal random variable such that

$$\ln\left(\frac{y_t S(t)}{S(t)}\right) = Y_t = \ln(y_t) \sim i.i.d.\, N(\mu, \delta^2) \quad (3.13)$$

If we consider the jump part $dN(t)$, the expected relative price change $E\left[\frac{dS(t)}{S(t)}\right]$ in the time interval dt is $\lambda k dt$ since$E[(y_t - 1)dN(t)] = E[dN(t)]E[y_t - 1] = \lambda k dt$ which is the predictable part of the jump. To ensure that the jump part is unpredictable αdt is adjusted by $-\lambda k dt$ in the drift term as;

$$E\left[\frac{dS(t)}{S(t)}\right] = E[(\alpha - \lambda k)dt] + E[\sigma dB(t)] + E[(y_t - 1)dN(t)]$$

$$= (\alpha - k\lambda)dt + 0 + k\lambda dt$$

$$= \alpha dt \quad (3.14)$$

When $dN(t) = 0$ means that, there is no jump in the asset price in a small time interval dt, then equation (3.10) becomes a SDE for Brownian motion;

$$\frac{dS(t)}{S(t)} = (\alpha - \lambda k)dt + \sigma dB(t) \quad (3.15)$$

However, when the asset price jumps in small time interval dt $(i.e, dN(t) = i)$, then the JDM for the relative price jump of $(y_t - 1)$ is given by;

$$\frac{dS(t)}{S(t)} = (\alpha - \lambda k)dt + \sigma dB(t) + (y_t - 1)dN(t) \quad (3.16)$$

3.3.3 Solution to Merton's jump diffusion model

Consider SDE in equation (3.15) and multiply by $S(t)$ on both sides then

$$dS(t) = (\alpha - \lambda k)S(t)dt + \sigma S(t)dB(t) + (y_t - 1)S(t)dN(t) \tag{3.17}$$

From It$\hat{o}$ formula for the jump-diffusion process (Cont &Tankov, 2004) given as:

$$df(X(t), t) = \frac{\partial f(X(t),t)}{\partial t}dt + b_t \frac{\partial f(X(t),t)}{\partial x}dt + \frac{{\sigma_t}^2}{2}\frac{\partial^2 f(X(t),t)}{\partial x^2}dt + \sigma_t \frac{\partial f(X(t),t)}{\partial x}dB(t) + [f(X(t) + \Delta X(t)) - f(X(t))] \tag{3.18}$$

Where, b_t and σ_t correspond to the drift term and volatility term of a jump-diffusion process respectively, also the function is considered as $f \in C^{1,2}([0,T] \times \mathbb{R})$

$$\text{Let } f(S(t), t) = \ln(S(t)) \tag{3.19}$$

$$\frac{\partial f}{\partial t} = 0 \tag{3.20}$$

$$\frac{\partial f}{\partial S} = \frac{1}{S(t)} \tag{3.21}$$

$$\frac{\partial^2 f}{\partial S^2} = -\frac{1}{S(t)^2} \tag{3.22}$$

$$b_t = (\alpha - \lambda k)S(t) \tag{3.23}$$

$$\sigma_t = \sigma S(t) \tag{3.24}$$

Substituting results from equation (3.20) to (3.24) into equation (3.18) then,

$$d\ln(S(t)) = \frac{1}{S(t)}(\alpha - \lambda k)S(t)dt - \frac{(\sigma S(t))^2}{2} \times \frac{1}{S(t)^2}dt + \sigma S(t) \times \frac{1}{S(t)}dB(t) + [\ln(y_t S(t)) - \ln(S(t))] \tag{3.25}$$

$$d\ln(S(t)) = (\alpha - \lambda k)\,dt - \frac{\sigma^2}{2}dt + \sigma dB(t) + [\ln(y_t S(t)) - \ln(S(t))] \tag{3.26}$$

$$d\ln(S(t)) = \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)dt + \sigma dB(t) + \ln(y_t) \tag{3.27}$$

Integrating equation (3.27) over the time interval $0 \le s \le t$, we get

$$\int_0^t d\ln(S(s)) = \int_0^t \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)ds + \int_0^t \sigma dB(s) + \sum_{m=1}^{N(t)} \ln y_m \tag{3.28}$$

$$\ln(S(t))|_0^t = \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t\Big|_0^t + \sigma B(t)|_0^t + \sum_{m=1}^{N(t)} \ln y_m \tag{3.29}$$

Substituting the limits then;

$$\ln\left(\frac{S(t)}{S(0)}\right) = \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t) + \sum_{m=1}^{N(t)} \ln y_m \tag{3.30}$$

By the assumption in equation (3.13) that log price (return) jump size $\ln y_m \equiv Y_m$, then

$$\ln\left(\frac{S(t)}{S(0)}\right) = \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t) + \sum_{m=1}^{N(t)} Y_m \tag{3.31}$$

$$S(t) = S(0)exp\left\{\left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t) + \sum_{m=1}^{N(t)} Y_m\right\} \tag{3.32}$$

From equation (3.1) then, stock price process $(S(t): 0 \leq t \leq T)$ is modeled as:

$$S(t) = S(0)e^{\mathcal{L}(t)} \tag{3.33}$$

In which $S(t)$ is a Lévy process of finite jumps such that;

$$\ln\left(\frac{S(t)}{S(0)}\right) = \mathcal{L}(t) = \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t) + \sum_{m=1}^{N(t)} Y_m \tag{3.34}$$

It should be noted that for the case of Black-Scholes:

$$\ln\left(\frac{S(t)}{S(0)}\right) \sim N\left[\left(\alpha - \frac{\sigma^2}{2}\right)t, \sigma^2 t\right] \tag{3.35}$$

However, due to the presence of the term $\sum_{m=1}^{N(t)} Y_m$ makes log return not normal for the case of MJD model. The assumption on the log return jump size $Y_m \sim N(\mu, \delta^2)$ enables to obtain the probability density of the log return $x_t = \ln\left(\frac{S(t)}{S(0)}\right)$ as a converging series of the form:

$$P(x_t) = \sum_{i=0}^{\infty} P(N(t) = i)P(x_t|N(t) = i) \tag{3.36}$$

$$P(x_t) = \sum_{i=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^i}{i!} \phi\left(x_t; \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + i\mu, \sigma^2 t + i\delta^2\right). \tag{3.37}$$

Where $\phi\left(x_t; \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + i\mu, \sigma^2 t + i\delta^2\right)$

$$= \frac{1}{\sqrt{2\pi(\sigma^2 t + i\delta^2)}} exp\left\{-\frac{\left(x_t - \left(\left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + i\mu\right)\right)^2}{2(\sigma^2 t + i\delta^2)}\right\} \tag{3.38}$$

The term $P(N(t) = i) = \frac{e^{-\lambda t}(\lambda t)^i}{i!}$ represents the probability that the asset price jumps i times in the time interval of length and $P(x_t|N(t) = i) = \phi\left(x_t; \left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + i\mu, \sigma^2 t + i\delta^2\right)$ is the Black-Scholes normal density of log return under the assumption that the asset price jumps i times in the interval of t.

3.3.4 Maximum likelihood estimation method

To estimate the five parameters α, σ, λ, μ and, δ the maximum likelihood estimation (MLE) method (Tang, 2018) was used. Meanwhile, there are no analytic expressions of the optimal parameter values, the MATLAB code fminsearch was used to estimate optimal parameters (Honoré , 1998).

3.4 Option pricing: Martingale approach

If we let $\{B(t); 0 \leq t \leq T\}$ which is a standard Brownian motion process be on a space $(\Omega, \mathcal{F}, \mathbb{P})$. The stock price process under actual probability measure $\mathbb{P}$, is given by equation (3.39) in the form of:

$$S(t) = S(0)exp\left\{\left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t) + \sum_{m=1}^{N(t)} Y_m\right\} \tag{3.39}$$

It should be noted that, MJD model is one of an incomplete model with the reason that there are many equivalent martingale risk-neutral measures $\mathbb{Q} \sim \mathbb{P}$ under which $\{e^{-rt}S(t); 0 \leq t \leq T\}$ i.e., the discounted asset price process becomes a martingale (Matsuda, 2004). Merton comes up with an equivalent martingale risk-neutral measure $\mathbb{Q}_M \sim \mathbb{P}$ by changing $B(t)$ while keeping the other parts unchanged. The equation (3.39) under $\mathbb{Q}_M$ becomes:

$$S(t) = S(0)exp\left\{\left(r - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma B(t)^{\mathbb{Q}_M} + \sum_{m=1}^{N(t)} Y_m\right\} \tag{3.40}$$

Note that $B(t)^{\mathbb{Q}_M}$ is on space $(\Omega, \mathcal{F}, \mathbb{Q}_M)$ and the process $\{e^{-rt}S(t); 0 \leq t \leq T\}$ is a martingale under $\mathbb{Q}_M$. Then a European option price $V^{merton}(t, S(t))$ with payoff function $H(S(T))$ is computed as:

$$V^{merton}(t, S(t)) = e^{-r(T-t)} E^{\mathbb{Q}_M}\left[H(S(T))|\mathcal{F}_t\right]. \tag{3.41}$$

The standard assumption is $\mathcal{F}_t = S(t)$ thus;

$$V^{merton}(t,S(t)) = e^{-r(T-t)}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\lambda k-\frac{\sigma^2}{2}\right)(T-t)+\sigma B(T-t)^{\mathbb{Q}_M}+\sum_{m=1}^{N(T-t)}Y_m\right\}\right)\middle|S(t)\right]$$ (3.42)

$$V^{merton}(t,S(t)) = e^{-r(T-t)}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\lambda k-\frac{\sigma^2}{2}\right)(T-t)+\sigma B(T-t)^{\mathbb{Q}_M}+\sum_{m=1}^{N(T-t)}Y_m\right\}\right)\right]$$. (3.43)

Using the index i for the number of jumps:

$$N(T-t) = 0,1,2\ \ldots \equiv i.$$ (3.44)

And the compound Poisson process is distributed as:

$$\sum_{m=1}^{N(T-t)}Y_m \sim Normal(i\mu, i\delta^2).$$ (3.45)

Consequently, $V^{merton}(t,S(t))$ can be expressed as:

$$V^{merton}(t,S(t))$$
$$= e^{-r(T-t)}\sum_{i=0}^{\infty}\mathbb{Q}_M(N(T-t)=i)E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\lambda k-\frac{\sigma^2}{2}\right)(T-t)+\sigma B(T-t)^{\mathbb{Q}_M}+\sum_{m=1}^{i}Y_m\right\}\right)\right].$$ (3.46)

Using equation (3.47).

$$\tau = T-t$$ (3.47)

$$V^{merton}(t,S(t))$$
$$= e^{-r\tau}\sum_{i=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+\sigma B(\tau)^{\mathbb{Q}_M}+\sum_{m=1}^{i}Y_m\right\}\right)\right].$$ (3.48)

It should be noted from equation (3.48) that:

$$\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+\sigma B(\tau)^{\mathbb{Q}_M}+\sum_{m=1}^{i} Y_m$$

$$\sim Normal\left(\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+i\mu,\sigma^2\tau+i\delta^2\right). \qquad (3.49)$$

Rewriting equation (3.50) without changing its distribution then:

$$\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+i\mu+\sqrt{\frac{\sigma^2\tau+i\delta^2}{\tau}}B(\tau)^{\mathbb{Q}_M}$$

$$\sim Normal\left(\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+i\mu,\sigma^2\tau+i\delta^2\right). \qquad (3.50)$$

Since a normal density is uniquely determined by mean and variance, now we can rewrite equation (3.36) as:

$$V^{merton}(t,S(t))$$

$$= e^{-r\tau}\sum_{i=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\frac{\sigma^2}{2}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+i\mu+\sqrt{\frac{\sigma^2\tau+i\delta^2}{\tau}}B(\tau)^{\mathbb{Q}_M}\right\}\right)\right]. \qquad (3.51)$$

Adding $\left(\frac{i\delta^2}{2\tau}-\frac{i\delta^2}{2\tau}\right)=0$ inside the exponential function in equation (3.38) then:

$$V^{merton}(t,S(t))$$

$$= e^{-r\tau}\sum_{i=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\left(r-\frac{\sigma^2}{2}+\left(\frac{i\delta^2}{2\tau}-\frac{i\delta^2}{2\tau}\right)-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\right)\tau+i\mu+\sqrt{\frac{\sigma^2\tau+i\delta^2}{\tau}}B(\tau)^{\mathbb{Q}_M}\right\}\right)\right]. \qquad (3.52)$$

$$= e^{-r\tau}\sum_{i=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}E^{\mathbb{Q}_M}\left[H\left(S(t)exp\left\{\langle r-\frac{1}{2}\left(\sigma^2+\frac{i\delta^2}{\tau}\right)+\frac{i\delta^2}{2\tau}-\lambda\left(e^{\mu+\frac{1}{2}\delta^2}-1\right)\rangle\tau+i\mu+\sqrt{\sigma^2+\frac{i\delta^2}{\tau}}B(\tau)^{\mathbb{Q}_M}\right\}\right)\right]. \qquad (3.53)$$

Let $\sigma_i^{\ 2} = \sigma^2 + \frac{i\delta^2}{\tau}$ and rearrange:

$$= e^{-r\tau} \sum_{i=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} E^{\mathbb{Q}_M} \left[H \left(S(t) exp \left\{ \left(r - \frac{1}{2}\sigma_i^{\ 2} + \frac{i\delta^2}{2\tau} - \lambda \left(e^{\mu + \frac{1}{2}\delta^2} - 1 \right) \right) \tau + i\mu + \sigma_i B(\tau)^{\mathbb{Q}_M} \right\} \right) \right]. \tag{3.54}$$

$$= e^{-r\tau} \sum_{i=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} E^{\mathbb{Q}_M} \left[H \left(S(t) exp \left\{ i\mu + \frac{i\delta^2}{2} - \lambda \left(e^{\mu + \frac{1}{2}\delta^2} - 1 \right) \tau \right\} exp \left\{ \left(r - \frac{1}{2}\sigma_i^{\ 2} \right) \tau + \sigma_i B(\tau)^{\mathbb{Q}_M} \right\} \right) \right]. \tag{3.55}$$

BS price can be expressed as:

$$V^{BS}(\tau = T - t, S(t), \sigma) = e^{-r\tau} E^{\mathbb{Q}_{BS}} \left[H \left(S(t) exp \left\{ \left(r - \frac{1}{2}\sigma^2 \right) \tau + \sigma B(\tau)^{\mathbb{Q}_{BS}} \right\} \right) \right]. \tag{3.56}$$

Lastly, when BS price is conditioned on the number of jumps i as weighted average, then MJD pricing formula can be found as:

$$V^{merton}(t, S(t))$$

$$= \sum_{i=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} V^{BS} \left(\tau, S(i) \equiv S(t) exp \left\{ i\mu + \frac{i\delta^2}{2} - \lambda \left(e^{\mu + \frac{1}{2}\delta^2} - 1 \right) \tau \right\}; \sigma_i \equiv \sqrt{\sigma^2 + \frac{i\delta^2}{\tau}} \right). \tag{3.57}$$

Alternatively:

$$V^{merton}(t, S(t)) = e^{-r\tau} \sum_{i=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!} E^{\mathbb{Q}_M} \left[H \left(S(t) exp \left\{ r - \lambda \left(e^{\mu + \frac{1}{2}\delta^2} - 1 \right) + \frac{i\mu + \frac{i\delta^2}{2}}{\tau} - \frac{1}{2}\sigma_i^{\ 2} \right\} \tau + \sigma_i B(\tau)^{\mathbb{Q}_M} \right) \right]. \tag{3.58}$$

$$= \sum_{i=0}^{\infty} \frac{e^{-\bar{\lambda}\tau}(\bar{\lambda}\tau)^i}{i!} V^{BS}(\tau = T - t, S(t), \sigma_i, r_i). \tag{3.59}$$

Where;

$$\bar{\lambda} = \lambda(1 + k) = \lambda e^{\mu + \frac{1}{2}\delta^2} \tag{3.60}$$

$$\sigma_i = \sqrt{\sigma^2 + \frac{i\delta^2}{\tau}} \tag{3.61}$$

$$r_i = r - \lambda\left(e^{\mu+\frac{1}{2}\delta^2} - 1\right) + \frac{i\mu+\frac{i\delta^2}{2}}{\tau} = r - \lambda k + \frac{i\ln(1+k)}{\tau} \tag{3.62}$$

$$V^{BS}(\tau, S(t), \sigma_i, r_i) = S\phi(d_1) - Ke^{-r\tau}\phi(d_2). \tag{3.63}$$

Where, $\phi(x)$ is cumulative normal distribution and

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma_i^2}{2}\right)\tau}{\sigma_i\sqrt{\tau}} \tag{3.64}$$

$$d_2 = d_1 - \sigma_i\sqrt{\tau} \tag{3.65}$$

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Parameter estimates for BS-model

The following tables summarizes the parameter estimates for BS-model and were obtained using MATLAB. The code for parameter estimates is included in the appendix.

Table 1: Estimation results of BS model for USE

STOCK	$\hat{\alpha}$	$\hat{\sigma}$
BATU	0.5315	0.2884
BOBU	0.0787	0.3102
NIC	-0.0202	0.4961
UMEM	0.0219	0.2457
SBU	0.1551	0.5247

Table 1 shows that the instantaneous returns for all stocks except National Insurance Corporation (NIC) are positive. The British American Tobacco Uganda (BATU) has high log return of about 53.2%. The returns for National Insurance Corporation (NIC) and UMEME LIMITED (UMEM) are nearly equal about 2% but with opposite signs. On the other hand, the Stanbic Bank Uganda (SBU) has a high value of diffusion coefficient (standard deviation) of about 52.5% compared to other stocks. Therefore it can be seen that with the absence of sudden events in the market, the stock price of Stanbic Bank Uganda (SBU) is more variant.

Table 2: Estimation results of BS for NSE

STOCK	$\hat{\alpha}$	$\hat{\sigma}$
C&G	0.1496	0.7396
KCB	0.0757	0.2433
KQ	0.1364	0.5184
LIMT	0.5148	0.9731
SCOM	0.3181	0.2316

Table 2 shows that the expected instantaneous returns for all stocks of NSE are positively indicating that they all have increasing returns. The Limuru Tea Co. Plc Ord 20.00AIMS (LIMT) has a high return of about 51.5% among all stocks but it has a high coefficient of diffusion (standard deviation) of about 97.3% which makes it more variant.

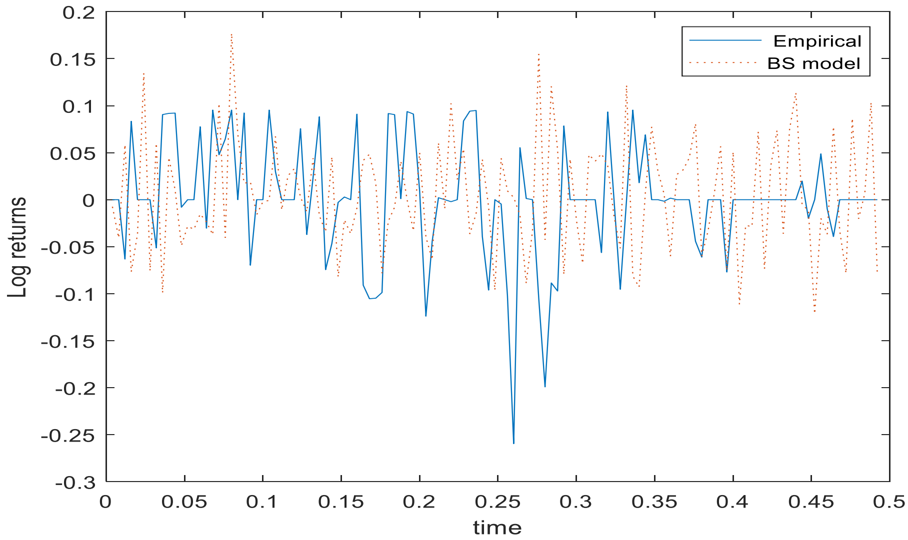


Figure 1: Comparison of empirical and BS model for LIMT

Figure 1 shows the simulated empirical log return for LIMT and BS models. The empirical LIMT in blue shows high sparks compared to the BS model in red.

Table 3: Estimation results of BS for DSE

STOCK	$\hat{\alpha}$	$\hat{\sigma}$
TBL	0.3755	0.1380
SWIS	0.1473	0.2331
NMB	0.1379	0.2411
TCC	0.3167	0.4342
VODA	-0.1153	0.2103

Table 3 shows that the expected instantaneous returns for all stocks of DSE are positive except Vodacom Tanzania (VODA) with a decreasing expected instantaneous rate of return of about 11.5%. The Tanzania Breweries Limited (TBL) has a low diffusion coefficient (standard deviation) value of about 13.8% but its expected rate of return is high about 37.6%.

When I consider only the rate of return under the BS model, then TBL seems to be the best stock to invest among other stocks.

4.2 Parameter estimates for MJD-model

Table 4 summarizes the parameter estimates for MJD-model; they were simulated using MATLAB code which is hereby appended.

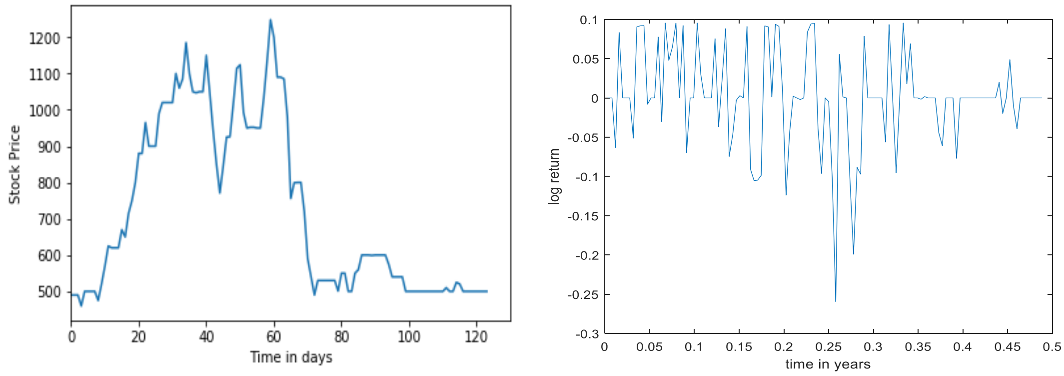
Table 4: Estimation results of MJD model for NSE, USE and DSE

STOCK	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\mu}$	$\hat{\delta}$
KCB	0.0442	0.1260	108.0856	0.0001	0.0193
KQ	-0.6293	0.1692	135.8863	0.0048	0.0355
LIMT	1.3715	0.8509	4.2835	-0.2260	0.000002
SCOM	0.1584	0.0884	236.4803	0.0006	0.0138
C&G	0.1491	0.7388	50.3132	-0.0156	0.1101
TBL	0.1555	0.0333	14.9270	0.0187	0.0213
SWIS	0.0641	0.0560	53.8672	0.0011	0.1284
NMB	0.0853	0.2171	31.4789	0.0039	0.0125
TCC	0.1941	0.1983	9.7657	0.0047	0.4353
VODA	-0.1154	0.2098	1.1156	-0.0251	0.0991
BATU	0.0094	0.0181	16.0868	0.1314	0.1396
BOBU	-0.0275	0.0506	11.0200	0.0054	0.0516
NIC	0.0219	0.0215	71.1078	-0.0054	0.1900
UMEM	-0.0108	0.0725	55.1214	0.0004	0.0774
SBU	0.0106	0.0390	84.6321	0.0002	0.2570

Table 4 shows, the instantaneous returns for Kenya Airways Ltd Ord 5.00 (KQ), VODA, BATU and UMEM are negative indicating that they have a decreasing return. On the other hand, the rest of the stocks are positive indicating that they have an increasing return. The Safaricom Plc Ord 0.05 (SCOM) has the highest jump among all stocks while VODA has the smallest jump (lambda) compared to the rest of stocks. The expected jump sizes for LIMT, Car & General (K) Ltd Ord 5.00 (C&G), VODA and NIC are all negative compared to the rest whose expected jump sizes are positive. The sign of expected jump size $\hat{\mu}$ determines the skewness of the stocks. Although the moments of the log return of stocks will be discussed in the next sections but in a nutshell those whose values of expected jump size are positive have positive skewness and those with negative values have negative skewness. The presence of skewness in the stock price has significant implications as it will be discussed in the next sections.

Moreover, looking at the diffusion coefficients (standard deviation), LIMT has about 85.1% which is the highest among other stocks. BATU has a very small diffusion coefficient of about 1.8%. It should be noted that the diffusion coefficient measures the variability of a stock price. The BS model, is always assumed to be constant.

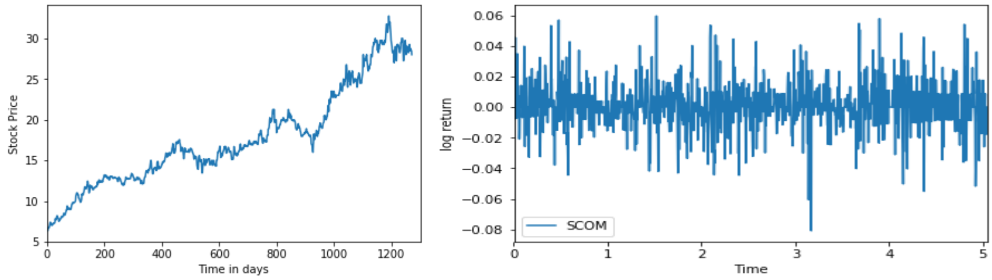
Figure 2 illustrates how the stock of LIMT moves by considering the daily price movement and the daily log return. The Fig. 2 (a) is the daily price movement and Fig. 2 (b) is the daily log return. It is observed that the price movement in both figures is random and as per Table 4 LIMT shows a very small jump intensity value. This implies that its daily ups and downs (jumps) are relatively small.



(a) The LIMT daily price level **(b) The LIMT log return**

Figure 2: The LIMT daily price level and log return (July, 2013 to July, 2018)

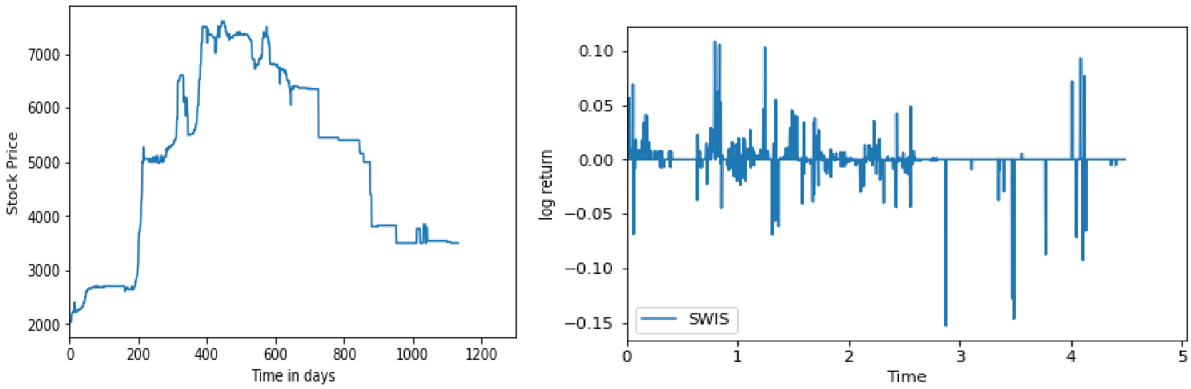
Furthermore, Fig. 3 shows the movement of price for SCOM, whereby Fig. 3 (a) shows the daily price level while Fig. 3 (b) shows the daily log return movement. It is observed that there are very high jumps in both figures contrary to Fig. 2 and the subsequent Figs. 4 and 5. When looking at the log return in Fig. 3 (b) the jumps are very close. A very interesting property is that SCOM is having high jump intensity not only at NSE but also for the rest of the stocks at USE and DSE.



(a) The SCOM daily price level **(b) The SCOM log return**

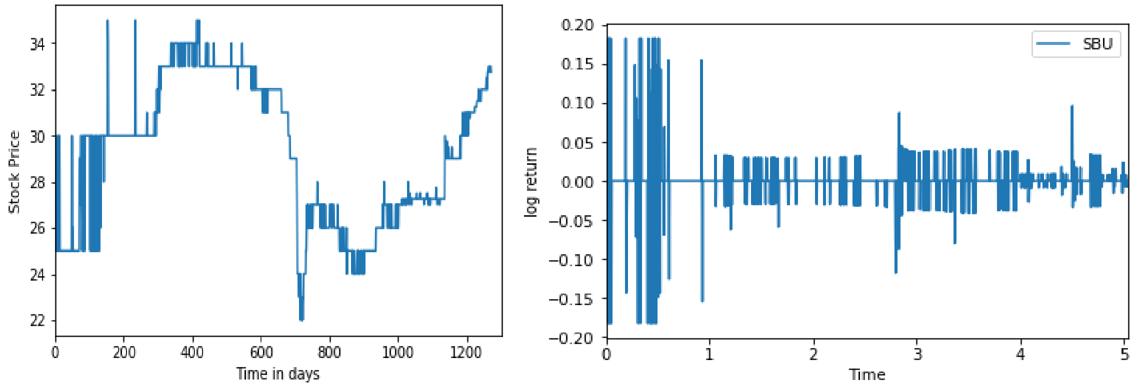
Figure 3: The SCOM daily price level and log return (July, 2013 to July, 2018)

Figure 4 and Fig. 5 illustrate the situation of price movement at DSE and USE, respectively. From Table 4 it is observed that Swissport Tanzania plc (SWIS) at DSE is having a high value of jump intensity of about 53.9 compared to the rest. As illustrated in Fig. 4 and Fig. 5, at USE, the stock of SBU is having a high value of jump amplitude of about 84.6.



(a) **The SWIS daily stock price level** (b) **The SWIS log return**

Figure 4: The SWIS daily price level and log return (July, 2013 to July, 2018)



(a) The SBU daily stock price level **(b) The SBU log return**

Figure 5: The SBU daily price level and log return (July, 2013 to July, 2018)

4.3 Option pricing

In this work, we have considered the European call option which is only exercised at the expiration time. To do this we needed to make some assumptions because none of the considered East African stock market trades financial derivatives (option). Closing prices have been used as the initial price S_0 for each stock. Also we assumed that the riskless interest rate, r, is a constant over the period 1st July, 2013 to 1st July, 2018. In this study the respective country's fixed-rate were used (benchmark interest rate as at 2019) i.e. 7% for Tanzania, 9% for Kenya and 10% for Uganda. The currencies are in TZS for Tanzania, KES for Kenya and UGX for Uganda.

Table 5: European Call option prices from MJD and BS Models for NSE, DSE, and USE

STOCK	S_0	K	MJD	BS
KCB	37.25	30	7.2393	7.2607
KQ	10	8	1.9971	2.0029
LIMT	490	400	88.1440	90.1461
SCOM	6.45	5	1.4482	1.4518
C&G	23	20	3.0197	3.0075
TBL	3220	3000	219.1667	220.8332
SWIS	2040	2000	58.0677	41.7231
NMB	1620	1600	22.0809	23.2003
TCC	6500	6000	539.8199	501.7480
VODA	900	800	99.7858	100.2222
BATU	2540	2500	47.1271	45.6261
BOBU	120	100	19.9603	20.0397
NIC	35	30	5.2145	5.0119
SBU	30	25	5.3715	5.0099
UMEM	344	300	43.9989	44.1190

Table 5 shows the comparison of prices of European Call option for MJD and BS models. The MJD model gives relative small prices compared to the BS model for KCB Group Plc Ord 1.00 (KCB), KQ, LIMT and SCOM for the case of NSE. At DSE, MJD gives small prices for the stocks of TBL, NMB Bank plc. (NMB) and VODA. Moreover MJD model gives small prices for the stocks of the Bank of Baroda Uganda (BOBU) and UMEM at USE. On the other hand, the BS model gives small prices for C&G, SWIS, the Tanzania Cigarette Company (TCC), BATU, NIC and SBU compared to MJD model. The MJD prices for most stocks are small due to the presence of the expected jump amplitude. Therefore, it can be said that incorporation of the compound jump for the case of MJD model has made the prices to be more realistic than for the case of the BS model which ignores the jump component.

4.4 The moments of Merton's log return density

Table 6: The mean, variance, skewness and kurtosis of Merton's log return density for all selected stocks

Stock	Mean	Variance	Skewness	Kurtosis
KCB	-0.0003	0.0002	0.0144	6.5977
KQ	-0.0006	0.0008	0.4306	7.1041
LIMT	-0.0018	0.0037	-0.8574	6.1680
SCOM	0.0008	0.0002	0.1057	5.3227
C&G	-0.0055	0.0046	-0.3614	7.2623
TBL	0.0016	0.00005	5.0553	40.0757
SWIS	0.00030	0.0035	0.0553	16.9359
NMB	-0.00004	0.0002	0.0783	3.2522
TCC	0.0002	0.0075	0.1594	77.2260
VODA	-0.0014	0.0002	-1.0182	32.6454
BATU	0.0084	0.0023	5.5848	43.0367
NIC	-0.0015	0.0102	-0.1603	13.6279
UMEM	-0.0002	0.0013	0.0324	16.2888
BOBU	-0.0007	0.0001	1.3093	61.1292
SBU	-0.00005	0.0222	0.0040	11.9279

Table 6 shows that, with exception of C & G, VODA) and NIC, the LIMT which are negatively skewed the rest of stocks are positively skewed. The presence of skewness indicates that the stock prices have non-symmetric return and exhibit empirical abnormity called volatility smile. The volatility measures the degree of variation of a trading price series over time as it is measured by the standard deviation of log returns. Also in a Table 6, NMB has very small kurtosis value of about 3.2 among all selected stocks at all three markets, likewise TCC has a very high kurtosis value of about 77 compared to other stocks. Kurtosis signifies the presence of the leptokurtic features (fatter tails and high pick than normal curves).

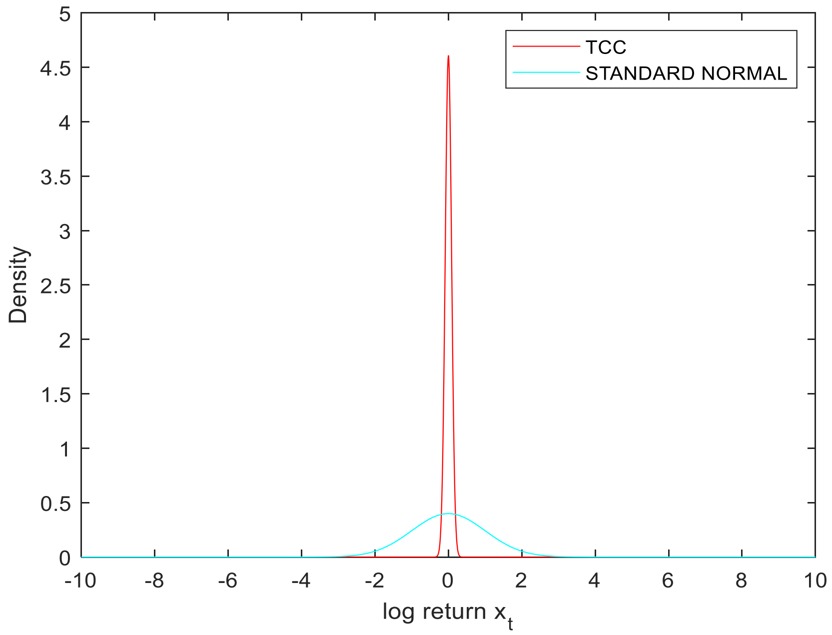


Figure 6: The comparison of kurtosis for empirical TCC and standard normal

Figure 6 shows TCC in red with a high pick than the standard normal distribution which is drawn in cyan color. The kurtosis value for all of the stocks is greater than three compared to the normal distribution which is not supposed to exceed three. This shows that the underlying distribution of the returns are leptokurtic or heavy-tailed as it can be seen in the Fig. 6.

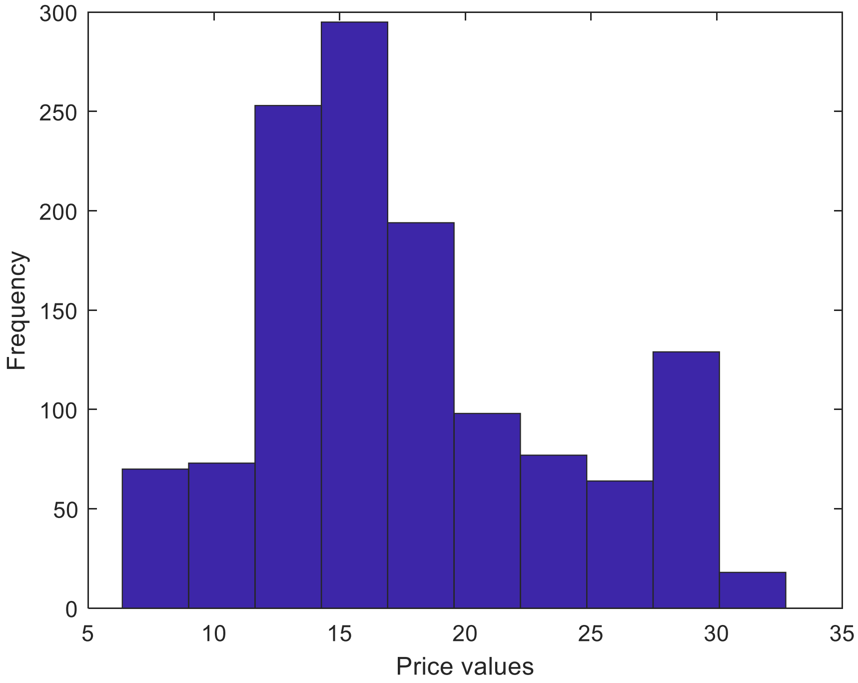


Figure 7: The histogram for SCOM with positive skewness

From Fig. 7, the histogram of SCOM at NSE is having long right tail implying that the stock is positively skewed. There are many bars with small values to the right of the mode of the histogram. The right skewness is due to the positive value of the expected jump size in Table 4.

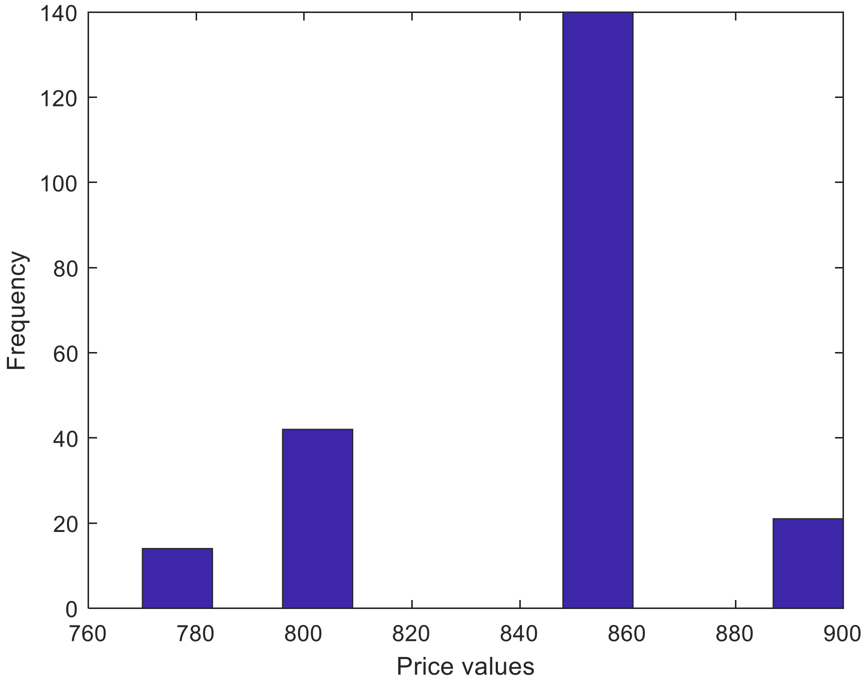


Figure 8: The histogram for VODA with negative skewness

Figure 8 shows, the histogram of VODA at DSE is having long left tail implying that the stock is negatively skewed. There are many bars with small values to the left of the mode of the histogram. The left skewness is due to the negative value of the expected jump size in Table 4.

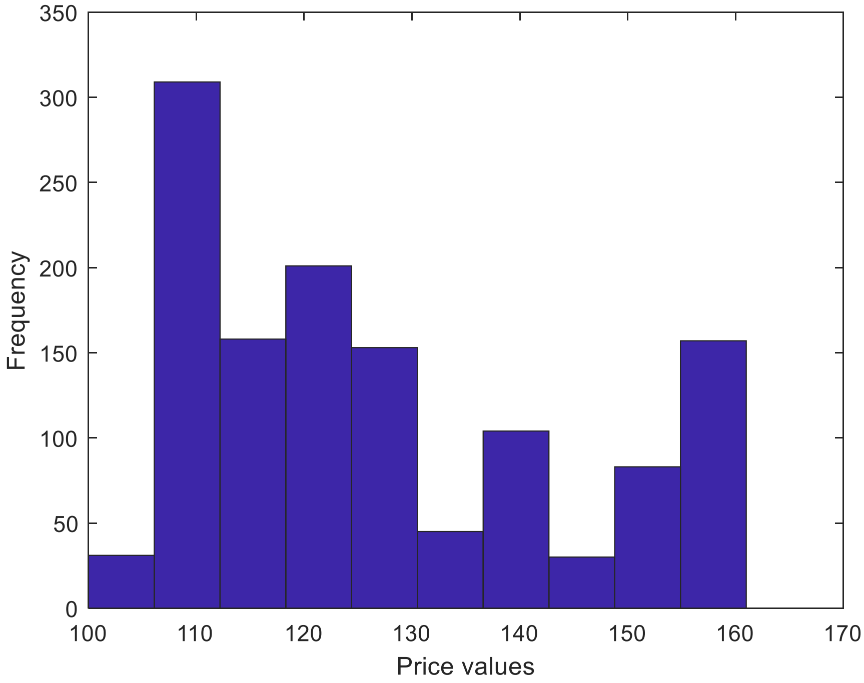


Figure 9: The histogram for BOBU with positive skewness

From Fig. 9, the histogram of BOBU for the case of USE is having long right tail implying that the stock is positively skewed. There are many bars with small values to the right of the mode of the histogram. The right skewness is due to the positive value of the expected jump size in Table 4.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The developed MJD model have been used to calculate European call options using the daily closing stock prices from NSE, DSE and USE. Also, for comparison purpose the researcher computed the call options using the basic model called BS model. The results shows that both models give different option price values but because of the addition of the compound jump Poisson process, the MJD model formulated in this study seems to give more realistic option price values than the BS model.

Moreover, based on the established empirical results, this study has established that the stocks from all three markets exhibit several jumps as it can be evidenced from non-zero values of jump amplitude (lambda). The graphs of the daily price movement and daily log returns have shown that the stock price moves randomly with ups and downs (jumps). Also, the log-returns density of the MJD model that has been formulated in this study exhibits volatility and leptokurtosis as evidenced by the presence of skewness and kurtosis values. Thus, it is suggested that the Merton's jump-diffusion model with added compound Poisson process fits well the stock prices data of NSE, DSE and USE markets by being more realistic in terms of price valuation and it has accommodated most of the key features of stock prices.

5.2 Recommendations

There are several individual investors, financial institutions and companies that are now trading at the NSE, DSE and USE markets. As indicated in the introduction the number of investors keeps on growing. These investors use brokers to trade and manage their trading activities at these markets. The brokers use the qualitative analyses method of market investigation intelligence and speculation. This is mainly because the models available have not been used effectively at these stock markets. It should be noted that due to the price movement of the stocks being random, the use of speculation is ineffective. Therefore, it is recommended that the NSE, DSE and USE should adapt and use the available models for trading activities especially when dealing with more risk underlying asset such as stocks.

Moreover, currently NSE, DSE and USE trade in Government Bonds, Corporate Bonds, and Ordinary Shares thus they do not trade financial derivatives. It is recommended that to avoid risk resulting from fluctuation of stocks and widening the market, these stock markets should start trading financial derivatives especially options.

Lastly but not least, many people indeed have little understanding of capital markets which leads to the majority of them to invest their money in other assets rather than financial assets. It is recommended that the NSE, DSE and USE markets through various strategies should educate people on how to invest their money in financial assets such as stocks, bonds and derivatives.

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APPENDICES

Appendix 1: Codes for the parameters and figures

```
clc
clear all
% estimation code of parameters of BS model
S=csvread('C:/Users/HP/Improved/BATU.csv',1,0); %CHANGE STOCK
dt=1/252;
R=diff( log (S) ,1); % the return of the emprical stock price
muhat=mean(R)/dt+var(R) /(2*dt)
sigmahat=sqrt (var(R)/dt)
%Code for Comparison of simulated BS-modeled log-returns with the empirical LIMT log-returns.
Ns=1; %number of simulation
dt=1/252;
t=linspace (0, (124)*dt ,124)';
W=cumsum([zeros(1, Ns);sqrt(dt).*normrnd(0 ,1 , length(t)-1,Ns)]) ;
mu=0.51480; sigma=0.9731;S0=490;% estimate the parameter mu and sigma
Ssim=S0*exp((mu-sigma^2/2)*t*ones(1 ,Ns)+sigma*W) ; %stock price of simulation
Rsim=diff(log(Ssim) ,1) ; % return of the simulated stock price
df=csvread('C:/Users/HP/data/NSE_2_Data.csv',1,0);
DF=df(1:124,:); %SKIP ZERO VALUES
S=DF(:,4); %Taking specific column values at a particular row(the emprical stock price)
R=diff( log(S) ,1) ; % return of the real stock price figure (1)
x=t(2: end) ;
plot(x ,R, x ,Rsim ,': ')
xlabel ( 'time' )
ylabel ( 'Log returns')
legend ( ' Empirical ', 'BS model ' )
%title label('Comparison of Empirical and BS Model for LIMT')

clc
clear all
% estimation code of MJD model
```

```
S=csvread ('C: /Users/HP/Improved/KCB.csv', 1, 0);
dt=1/252;
%m=0.186;
R=diff(log(S) ,1);
epsilon =0.02; %values to verify jump
jumpindex=find(abs(R)>epsilon );%if true then considered as jump
lambdahat=length(jumpindex)/((length (S)-1)*dt); % jump intensity , the number of jumps in
per? year
Rjumps=R(jumpindex);% the data of ’jumpindex ’
diffusionindex=find (abs(R)<=epsilon ); % without jumps , the diffusion data can be consider
as in BS model
Rdiffusion=R(diffusionindex) ;% the data of ’ diffusionindex ’
sigmahat=std( Rdiffusion )/sqrt (dt);
muhat=((2*mean(Rdiffusion)+(sigmahat^2)*dt))/(2*dt);
%sigma_jhat=sqrt(m)*sigmahat;
sigma_jhat=sqrt(( var(Rjumps)-sigmahat^2*dt));
mu_jhat=mean(Rjumps)-(muhat-sigmahat^2/2)*dt ;
theta0=[muhat sigmahat lambdahat mu_jhat sigma_jhat];% initial value
Logmerton=@(mu, sigma ,lambda , mu_j , sigma_j )-sum( log(logmertonpdf(R, dt ,mu,
sigma , lambda , mu_j , sigma_j))) ;
[theta, fval, maxiter, exitflag]=fminsearch(@( theta)Logmerton( theta(1) ,theta(2)
,theta(3),theta(4),theta(5)),theta0)
% Maximum likelihood method from initial value
thetas=fminsearch(@(theta)Logmerton(theta(1) ,theta(2) , theta(3) , theta(4) , theta(5) ) ,theta
);
disp ([ 'mu' num2str([theta0(1) theta(1) thetas(1)])]);
disp ([ 'sigma' num2str([theta0(2) theta(2) thetas(2)])]);
disp (['lambda' num2str([ theta0(3) theta(3) thetas(3)])]);
disp ([ 'mu_j' num2str([ theta0(4) theta(4) thetas(4)])]);
disp ([ 'sigma_j' num2str([theta0(5) theta(5) thetas(5)])]);
M1=(theta(1)-theta(2)*2/2+theta(3)*theta(4))*dt % Mean
M2=(theta (2)^2+theta(3)*( theta(5)^2+theta(4)^2))*dt  %Variance
M3=(3*theta(5)^2+theta(4)^2)*theta(4)*theta(3)*dt;
```

```
M4=(3*theta(5)^4+6*theta(4)^2*theta(5)^2+theta(4)^4)*theta(3)*dt+(3*theta(3)^(2)*(theta(
5)^2+theta(4)^2)^(2)+6*theta(3)*theta(2)^(2)*(theta(5)^2+theta(4)^2)+3*theta(2)^(4))*(dt^2
);
Beta3=M3/(M2^(1.5))%Skewness coefficient
Beta4=M4/(M2^(2))%Kurtosis coefficient
Beta5=sqrt(M2) %std

function pdflog=logmertonpdf(r , dt ,mu, sigma , lambda , mu_j , sigma_j ) % build a density
function of MJD model
if lambda>0
nterm=100;
term=zeros(length(r) ,nterm);% return a matrix length(r)*100
for i=1:nterm
   poisson=(lambda*dt)^i/prod(1:i)*exp(-lambda*dt) ;
   normal=1/sqrt (2*pi*(sigma^2*dt+sigma_j^2*i))*exp(-(r-((mu-sigma^2/2)*dt+mu_j*i)
).^2/(2*(sigma^2*dt+sigma_j^2*i)));
   term (: ,i) =poisson*normal;
end
pdflog=sum([1/sqrt(2*pi*sigma^2*dt)*exp(-(r-(mu-
sigma^2/2)*dt).^2/(2*sigma^2*dt))*exp(-lambda*dt) term], 2);
else % lambda <=0
   pdflog=1/sqrt(2*pi*sigma^2)*exp(-(r-(mu-sigma^2/2)*dt).^2/(2*(sigma^2*dt)));
end
end




%MJD CALL OPTION CODE
c = 0;
S0 = 30;%stock price at time 0
K =25; %strike price
tau=1/252
r = 0.1; %risk-free interest rate
lambda = 84.6321;
```

```
delta =0.2570;
mu = 0.0002;
sigma =0.0390;
k = exp(mu + 0.5 * delta ^ 2) - 1;
N = 100; %number of terms
for i = 0 : N
    r_i = r - lambda * k + i * log(1 + k)/tau;
    sigma_i = sqrt(sigma ^ 2 + i * delta ^ 2 / tau);
    d1 = (log(S0/K) + (r_i + 0.5 * sigma_i ^ 2) * tau) /(sigma_i * sqrt(tau));
    d2 = d1 - sigma_i * sqrt(tau);
    p1 = S0 * normcdf (d1,0,1) - (K * exp(r_i *tau) * normcdf(d2,0,1));
    p2 = (exp(-lambda * tau)* ((lambda * tau) ^ i))/factorial(i);
    c = c + (p1 * p2);

end
disp (c)
```

```
%BS-option pricing matlab-code
S=30;
K=25;
tau=1/252;
sigma=0.5247;
r=0.1;
D=0;
d1=(log(S./K)+(r+0.5*sigma^2)*tau)/(sigma*sqrt(tau));
d2=d1-sigma.*sqrt(tau);
Call=S.*exp(-D.*tau).*normcdf(d1,0,1)-K.*exp (-r.*tau).*normcdf(d2,0,1)
```