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Aminiel, Joram

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Mathematical Modeling on the Spread of Awareness Information to Infant Vaccination

Joram Aminiel[1,*], Damian Kajunguri[1], Emmanuel A. Mpolya[2]

[1]Department of Applied Mathematics and Computational Science, Nelson Mandela African Institution of Science and Technology, Arusha, Tanzania
[2]Department of Life Science and BioEngineering, Nelson Mandela African Institution of Science and Technology, Arusha, Tanzania

Abstract In this paper, we examine the importance of spreading awareness information about infant vaccination in a population. A mathematical model for the spread of infant vaccination awareness information is proposed and analyzed quantitatively using the stability theory of the differential equations. The basic reproduction number R_0 is obtained and its sensitivity analysis is carried out. The awareness free equilibrium is also proved to be locally and globally stable. Consideration is taken when R_0 is greater than unity, which indicates that infant vaccination awareness information will invade the population and cause immunization to succeed. It is also proved that the maximum awareness equilibrium is locally stable if R_0 is greater than unity. Numerical results show that word-of-mouth has a more impact on infant vaccination as compared to mass media, but better results are obtained by a combination of both word-of-mouth and mass media. For a successful infant vaccination programme, there is a need to emphasize both forms of awarenes.

Keywords Awareness information, Infant vaccination

1. Introduction

Vaccination is a powerful tool in the public-health control arsenal, and allows for the mass prevention of infection rather than treating the symptoms of infection [19]. Barriers to immunization are grouped as system of barriers (eg, those involving the organization of the health care system and economics), health care provider barriers (eg, inadequate clinician knowledge about vaccines and contraindications to their use), and parent or patient barriers (eg, fear of immunization related adverse events). These barriers affect immunization rates and increase the burden of preventable disease in our society [20].

There can be many reasons for fear of or opposition to vaccination. Some people have religious or philosophic objections.

Some see mandatory vaccination as interference by the government into what they believe should be a personal choice.

Others are concerned about the safety or efficacy of vaccines, or may believe that vaccine-preventable diseases do not pose a serious health risk [9].

Sufficient references should be provided to sources containing more information about childhood vaccination, especially about the effectiveness of vaccines and vaccine components and the risks, such as possible side effects and benefits of vaccination. This may satisfy parents' information needs and enable them to make a sufficiently informed choice whether or not to vaccinate their child [8].

The spread of vaccination awareness information in a population can potentially alter individuals' decisions and hence alter the effectiveness of immunization program. Lack of correct awareness information on vaccinations, result into disease outbreak. The information from public campaigns and mass media reporting can change peoples' behavior and perception on vaccination hence improve human immunizations. Mathematical models have been helpful decision-making tools for vaccination strategies against infectious diseases, in particular for those sheltered by the expanded program on vaccination [3].

Various studies have been done to study the importance of awareness in vaccinations [1, 6, 7, 13, 14, 15]. Most of the studies are based on individual awareness decisions on whether to accept vaccination or not. In this study, we intended to find out the importance of awareness information to parents and guardians for their infants using a mathematical model.

2. Model Formulation

In this section, we formulate a mathematical model for the spread of infant vaccination awareness information. We incorporate two stages; awareness stage and a

* Corresponding author:
joram_ngowi@yahoo.com (Joram Aminiel)
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decision-making stage in the model. Basically, we modify the Bass model for diffusion of awareness information for a new product presented in [4] by introducing the concept of new vaccination. We extend the model by adding I class (a class of individuals who have awareness information on infant vaccination but did not adopt it) and two forms of awareness; mass media and word-of-mouth. The population is divided into three classes: N, the number of individuals who are not aware with infant vaccination; I, the number of individuals who have awareness information on infant vaccination but did not adopt it; and A, the number of individuals who have adopted the vaccination awareness.

We assume a recruitment rate, β, for the number of individuals who are not aware with infant vaccination (N) which is through birth and immigration. A proportion p of unaware individuals in class N is assumed to become aware and adopt infant vaccination and progress to class A through mass media at a rate δ and the remaining proportion become aware but do not adopt and progresses to class I at the same rate. A proportion q in class N is assumed to progress to class A through the word-of-mouth at a rate θ and the remaining proportion progresses to class I at the same rate. Individuals in class I can progress to the adopters class, A at a rate α. Adopters can discontinue from the infant awareness vaccination class, A and move to class I at a rate ρ. Individuals in the unadopters class, I forget the information and return to the class N at a rate γ. A schematic representation of the model for the spread of infant vaccination awareness information is shown in Figure 1.

Table 1. Parameters and variables used in the model formulation and their descriptions

Parameter/Variable	Description
M	Total number of people interacting with vaccination awareness information.
N	Number of individuals who are not aware with vaccination.
I	Number of individuals who have awareness information on vaccination but did not adopt it.
A	Number of individuals who have adopted the vaccination awareness.
p	Proportion of unaware individuals that become aware through mass media.
q	Proportion of unaware individuals that become aware through word-of-mouth.
β	Recruitment rate.
δ	Vaccine awareness rate through media.
θ	Vaccine awareness rate through word- of-mouth.
γ	Rate at which individuals in awareness class forget vaccine awareness information.
ρ	Discontinuance rate of adopters of vaccine awareness information.
μ	Natural mortality rate.
α	Progression rate from unadopters to adopters class.

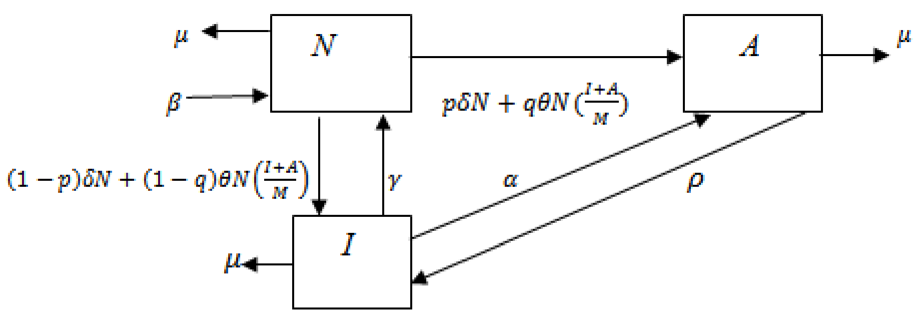


Figure 1. Compartmental diagram for the infant vaccination awareness information in a population

The model can be described by a system of equations given by

$$\frac{dN}{dt} = \beta - \delta N - \theta N\left(\frac{I+A}{M}\right) + \gamma I - \mu N,$$
$$\frac{dI}{dt} = (1-p)\delta N + (1-q)\theta N\left(\frac{I+A}{M}\right) - (\alpha+\gamma+\mu)I + \rho A, \tag{1}$$
$$\frac{dA}{dt} = p\delta N + q\theta N\left(\frac{I+A}{M}\right) - (\rho+\mu)A + \alpha I,$$

where: $M = N + I + A, N > 0, I > 0, A > 0$. All variables and parameters in the model (1) are considered to be positive, and the model lies in the region $\Omega = \{(N, I, A) \in R_+^3 : M \le \beta/\mu\}$.

2.1. Basic Properties

2.1.1. Invariant Region

Lemma 1. *All feasible regions Ω defined by*
$\Omega = \{(N(t), I(t), A(t)) \in R_+^3 : N(t) + I(t) + A(t) \ge 0\}$, *with the initial conditions* $N(0) \ge 0,\ I(0) \ge 0, A(0) \ge 0$ *are positively invariant for system (1).*

Proof. Adding the system of three equations of (1), we have

$$\frac{dM}{dt} = \beta - \mu M. \quad (2)$$

Solving equation (2) using $e^{\mu t}$ as an integrating factor we obtain $M(t) = \frac{\beta}{\mu} + ce^{-\mu t}$, where c is a constant of integration. As $t \to \infty$, $M \to \frac{\beta}{\mu}$.

It implies that the region $\Omega = \{(N(t), I(t), A(t)) \in R_+^3 : N(t) + I(t) + A(t) \geq 0\}$, is a positively invariant set for (1). So we consider dynamics of system (1) on the set Ω in this paper.

2.1.2. Positivity of Solutions

For the system(1), to ensure that the solutions of the system with positive initial conditions remain positive for all $t \geq 0$. It is necessary to prove that all the state variables are nonnegative, so we have the following lemma.

Lemma 2. *If* $N(0) > 0, I(0) > 0, and\ A(0) > 0$, then the solutions $N(t), I(t), and\ A(t)$ *of system (1) are positive for all* $t \geq 0$.

Proof. Under the given initial conditions, it is easy to prove that the solutions of system (1) are positive; if not, we assume a contradiction that there exists a first time t_1 such that

$$N(0) > 0, \quad N(t_1) = 0, \quad N'(t_1) \leq 0,$$
$$I(t) > 0, \quad A(t) > 0, \quad 0 \leq t < t_1.$$

From the first equation of system (1), we have $N'(t_1) = \beta + \gamma I > 0$, which is a contradiction meaning that $N(t) > 0$, $t \geq 0$.

Similarly, it can be shown that the variables I and A remain positive for all $t \geq 0$. Thus, the solutions $N(t), I(t), and\ A(t)$ of the system (1) remain positive for all $t \geq 0$.

3. Model Analysis

The model system (1) is analyzed qualitatively to get insights into its dynamical features which give better understanding of the impact of awareness on infant vaccination.

3.1. Awareness Free Equilibrium Point, E_0

Awareness free equilibrium point is the point at which there is no awareness about infant vaccination in the entire population, i.e when $A = I = 0$.

Considering system (1), when there is no awareness, then $\beta - \delta N - \mu N = 0$, $N = \frac{\beta}{\delta+\mu}$.

Then the awareness free equilibrium point E_0, is given by

$$E_0 = \left(\frac{\beta}{\delta+\mu}, 0, 0\right) \quad (3)$$

3.2. The Basic Reproduction Number, R_0

Diekmann *et al.*, [16], define the basic reproduction number denoted by R_0, as the average number of secondary infections caused by an infectious individual during the entire period of infectiousness. In this study, a secondary infection will be treated as a secondary awareness acquired by an individual during the period of getting vaccination awareness information. The basic reproduction number will be an important quantity in this study as it sets the threshold in the study of awareness information on infant vaccination for predicting the increase or decrease of number of awared people. Thus, whether the number of awared people increase or decrease in a population depends on the value of the reproduction number. For this study, if $R_0 < 1$, it means that every awared individual on infant vaccination will cause less than one secondary awared individual which cause the decrease of the number of awared people; but when $R_0 > 1$, every awared individual will cause more than one secondary awared individuals and hence the awareness information on infant vaccination will invade the population.

For this study, a large number of R_0 may indicate the possibility of having more awared people about infant vaccination.

We use the method presented in [17] to derive the expression for the basic reproduction number, R_0.

Let $\mathcal{F}_i(x)$ be the rate of appearance of new awareness information in compartment i. The information transmission model consists of the equations, $x_i' = f_i(x) = \mathcal{F}(x)_i - \mathcal{V}_i(x)$, where $\mathcal{V}_i(x) = \mathcal{V}_i(x)_i - \mathcal{V}_i^+(x)$. We then compute the matrices F and V which are $m \times m$ matrices, where m represent the awareness classes, defined by $F = \frac{\partial \mathcal{F}_i(x_0)}{\partial x_j}$, and $V = \frac{\partial \mathcal{V}_i(x_0)}{\partial x_j}$, with $1 \leq i, j \leq m$. F is nonnegative and V is non-singular m - matrix.

We then compute FV^{-1}, defined as the next generation matrix. The basic reproduction number, R_0 is then defined by $R_0 = \rho(FV^{-1})$, where $\rho(B)$ is the spectral radius of matrix B, (or the maximum modulus of the eigenvalues of B).

From system (1) we define $\mathcal{F}$ and $\mathcal{V}$ as

$$\mathcal{F} = \begin{pmatrix} (1-p)\delta N + (1-q)\theta N\left(\frac{I+A}{M}\right) \\ p\delta N + q\theta N\left(\frac{I+A}{M}\right) \end{pmatrix},$$

$$\mathcal{V} = \begin{pmatrix} -(\alpha+\gamma+\mu)I + \rho A \\ -(\rho+\mu)A + \alpha I \end{pmatrix}.$$

The awareness compartments are I and A, giving $m = 2$.

Differentiating $\mathcal{F}$ and $\mathcal{V}$ with respect to I and A gives

$$F = \begin{pmatrix} (1-q)\theta & (1-q)\theta \\ q\theta & q\theta \end{pmatrix},$$

and

$$V = \begin{pmatrix} (\rho+\gamma+\mu) & -\rho \\ -\alpha & \rho+\mu \end{pmatrix}.$$

The eigenvalue of equation FV^{-1} can be computed by the characteristic equation: $|FV^{-1} - \lambda I| = 0$.

This gives the basic reproduction number,

$$R_0 = \frac{q\gamma\theta + \theta L + \sqrt{\theta^2\left(D^2 + 4(-1+q)q(-\alpha\rho + LH)\right)},}{2(-\alpha\rho + LH)}$$

where $D = q\gamma + \mu + \rho, L = \mu + \rho, and$

$$H = \gamma + \mu + \rho.$$

Note: R_0 does not depends on awareness rate through media, δ.

3.3. Sensitivity Analysis of Model Parameters

According to Chitnis *at al.*, [2], sensitivity analysis is the method used to determine the robustness of model predictions to parameter values. We use sensitivity analysis to discover parameters that have a high impact on R_0. Sensitivity indices allow us to measure the relative change in a state variable when a parameter changes. The normalized forward sensitivity index of a variable to a parameter is the ratio of the relative change in the variable to the relative change in the parameter. When the variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivatives.

The normalized forward sensitivity index Υ, of a variable, u, that depends differentiably on a parameter, p, is defined as: $\Upsilon_p^u := \frac{\partial u}{\partial p} \times \frac{p}{u}$. As we have an expression for R_0, we derive an analytical expressions for the sensitivity index to R_0 for each of the six different parameters described in **Table 1** that appears in R_0. For example; using the set of values of estimated parameters in **Table 3,** the sensitivity indices for R_0 with respect to θ and ρ are given by $\Upsilon_\theta^{R_0} := \frac{\partial R_0}{\partial \theta} \times \frac{\theta}{R_0} = +1.0135$ and

$\Upsilon_\rho^{R_0} := \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0} =$ -0.5902 respectively. Other indices $\Upsilon_q^{R_0}$, $\Upsilon_\gamma^{R_0}$, $\Upsilon_\mu^{R_0}$, and $\Upsilon_\alpha^{R_0}$ are obtained following the same method and are shown in table 2.

Table 2. Sensitivity Indices of model parameters to R_0

Parameter	Index
α	-1.1374
θ	$+1.0135$
q	$+0.9989$
γ	$+0.6550$
ρ	-0.5902
μ	-0.0640

Interpretations of sensitivity indices.

Table 2 shows that the parameters θ, q, and γ increase the value of R_0 as they have positive indices, implying that they maximize the awareness on infant vaccination. The parameters α, μ, and ρ decrease the value of R_0 implying that they minimize the awareness on vaccination as they have negative indices. But individually, the most sensitive parameter is the rate of un-adoptor to adoptors class (α), and the least sensitive parameter is the motality rate (μ).

3.4. Local Stability Analysis of Awareness Free Equilibrium point, E_0

To determine the local stability of awareness free equilibrium, the variation Jacobian matrix at equilibrium point, $\boldsymbol{J_{E_0}}$ of the model system (1) is obtained as

$$\boldsymbol{J_{E_0}} = \begin{bmatrix} -\delta-\mu & -\theta+\gamma & -\theta \\ (1-p)\delta & (1-q)\theta-(\alpha+\gamma+\mu) & (1-q)\theta+\rho \\ p\delta & q\theta+\alpha & q\theta-(\rho+\mu) \end{bmatrix} \quad (4)$$

The stability of the awareness free equilibrium point can be clarified by studying the behaviour of $\boldsymbol{J_{E_0}}$ in which for local stability of awareness free equilibrium we seek for all its eigenvalues to have negative real parts. The characteristic function of the matrix (4) with λ being the eigenvalues of $\boldsymbol{J_{E_0}}$, is obtained and by using mathematica software, we have the following eigenvalues;

$$\lambda_1 = -\mu,$$

$$\lambda_2 = -\frac{1}{2}(\alpha+\gamma+\delta+2\mu+\rho-\theta) - k.$$

$$\lambda_3 = -\frac{1}{2}(\alpha+\gamma+\delta+2\mu+\rho-\theta) - k,$$

where

$k = \sqrt{w^2 - 2\delta z + 2\theta h + 2\rho g - 4\gamma j + \delta^2 + \theta^2}$, and

$w = \alpha + \gamma$, $g = \alpha - \gamma$, $z = \alpha - \gamma + \theta + \rho$,

$h = \alpha - \gamma + \rho$, $j = p\delta - q\theta$.

Considering real parts;

$$\lambda_1 = -\mu,$$

$$\lambda_2 = -\frac{1}{2}(\alpha+\gamma+\delta+2\mu+\rho-\theta),$$

$$\lambda_3 = -\frac{1}{2}(\alpha+\gamma+\delta+2\mu+\rho-\theta).$$

The awareness free equilibrium of system (1) is l.a.s given that $+\gamma+\delta+2\mu+\rho > \theta$. Thus for $R_0 < 1$ the awareness free equilibrium point is locally asymptotically stable.

3.5. Global Stability of Awareness Free Equilibrium

In this section, we analyze the global stability of awareness free equilibrium. Here we use the method developed in [23]. We rewrite the model system (1) as

$$\frac{dX}{dt} = F(X, Z),$$

$$\frac{dZ}{dt} = G(X, Z), \hat{G}(X, 0) = 0,$$

where $X \in R^m$ denotes (its components) the number of unawared individuals and $Z \in R^n$ denote (its components) the number of individuals who have vaccine awareness information. The awareness free equilibrium is now denoted by $E_0 = (X^0, 0)$. The following conditions, (H_1) and (H_2) must be met to guarantee a local asymptotic stability:

(H_1) For $\frac{dX}{dt} = F(X, 0)$, then X^0 is globally asymptotically stable (g.a.s),

(H_2) $G(X, Z) = KZ - \hat{G}(X, Z)$, where $\hat{G}(X, Z) \geq 0$ for $(X, Z) \in \Omega$,

where $K = D_Z G(X^0, 0)$ is an M –matrix (the off- diagonal elements of K are non-negative) and Ω is the region where the model makes sence. Then the following lemma holds:

Lemma 1.

The fixed point $E_0 = (X^0, 0)$, is globally asymptotic stable (g.a.s) equilibrium of system (1) provided $R_0 < 1$ is (l.a.s) and that the assumptions (H_1) and (H_2) are satisfied.

Proof:

Consider (H_1).

Considering the model system (1), we have

$$\mathrm{F(X,Z)} = \begin{bmatrix} \beta - \delta N - \theta N\left(\frac{I+A}{M}\right) + \gamma I - \mu N \\ (1-p)\delta N + (1-q)\theta N\left(\frac{I+A}{M}\right) - (\alpha+\gamma+\mu)I + \rho A \\ p\delta N + q\theta N\left(\frac{I+A}{M}\right) - (\rho+\mu)A + \alpha I \end{bmatrix},$$

$$\mathrm{G(X,Z)} = \begin{bmatrix} (1-p)\delta N + (1-q)\theta N\left(\frac{I+A}{M}\right) - (\alpha+\gamma+\mu)I + \rho A \\ p\delta N + q\theta N\left(\frac{I+A}{M}\right) - (\rho+\mu)A + \alpha I \end{bmatrix}$$

Then $X = (N), Z = (I, A)$.

Now,

$$\mathrm{F(X,0)} = \begin{bmatrix} \beta - \delta N - \mu N \\ 0 \\ 0 \end{bmatrix},$$

It is clear that $E_0 = (\frac{\beta}{\delta+\mu}, 0,0)$ is a g.a.s of $\frac{dX}{dt} = \mathrm{F(X,0)}$. Hence condition ($H_1$) is satisfied.

Now consider (H_2).

$$\hat{G}(X,Z) = \begin{bmatrix} (1-q)\theta(I+A)\left(1-\frac{N}{M}\right) - (1-p)\delta N \\ q\theta(I+A)\left(1-\frac{N}{M}\right) - p\delta N \end{bmatrix}.$$

Since $q \le 1, p \le 1$ *and* $0 \le (I+A) \le N \le M$, it is clear that $\hat{G}(X,Z) \ge 0$.

Then we have

$$X = (N),\ \ Z = (I, A),$$

$$K = \begin{bmatrix} (1-q)\theta - (\alpha+\gamma+\mu) & (1-q)\theta + \rho \\ q\theta + \alpha & q\theta - (\rho+\mu) \end{bmatrix},\ Z = \begin{bmatrix} I \\ A \end{bmatrix},$$

and $\hat{G}(X,Z) = \begin{bmatrix} (1-q)\theta(I+A)\left(1-\frac{N}{M}\right) - (1-p)\delta N \\ q\theta(I+A)\left(1-\frac{N}{M}\right) - p\delta N \end{bmatrix}.$

Then on substituting the above values we have

$$KZ - \hat{G}(X,Z) = \begin{bmatrix} (1-p)\delta N + (1-q)\theta N\left(\frac{I+A}{M}\right) - (\alpha+\gamma+\mu)I + \rho A \\ p\delta N + q\theta N\left(\frac{I+A}{M}\right) - (\rho+\mu)A + \alpha I \end{bmatrix}$$
$$= G(X,Z).$$

Hence (H_2) satisfied.

Then $E_0 = (\frac{\beta}{\delta+\mu}, 0,0)$ is globally asymptotic stable to our model system (1).

Existence of Maximum Awareeness Equilibrium ($M_a{}^*$).

The maximum awareness equilibrium of the system (1) is given by $M_a{}^* = (N^*, I^*, A^*)$ and it is obtained by setting the right hand side of equations equal to zero. In this paper, maximum awareness equilibrium works as for endemic equilibrium in disease model.

$$\beta - \delta N^* - \theta N^*\left(\frac{I^*+A^*}{M}\right) + \gamma I^* - \mu N^* = 0,$$
$$(1-p)\delta N^* + (1-q)\theta N^*\left(\frac{I^*+A^*}{M}\right) - (\alpha+\gamma+\mu)I^* + \rho A^* = 0,$$
$$p\delta N^* + q\theta N^*\left(\frac{I^*+A^*}{M}\right) - (\rho+\mu)A^* + \alpha I^* = 0. \tag{5}$$

For the existence and uniquiness of maximum awareness equilibrium $M_a{}^* = (N^*, I^*, A^*)$, the conditions $N^* > 0$, or $I^* > 0, or\ A^* > 0$, must be satisfied. It was not possible to get analytical solutions of system (5), so we resort to simulations to obtain insight in the dynamics of the model.

3.5.1. Local Stability of Maximum Awareeness Equilibrium ($\boldsymbol{M_a}^*$)

To analyse the stability of maximum awareness equilibrium, the additive compound matrix approach is used using the idea of [5, 25].

If $R_0 > 1$, then the equation model (5) has a unique maximum awareness equilibrium given by

$M_a{}^* = (N^*, I^*, A^*)$ in Ω, with

$$N^* = \frac{M\beta + MI^*\gamma}{M\delta + \theta(I^* + A^*)},$$

$$I^* = \frac{M(w - Mk\delta\gamma) - Mz\theta(\beta + \gamma A^*) +, - [(Mk\delta\gamma + Mz\theta(\beta + \gamma A^*) - Mwh) - 4\Big(M\theta(Mz\gamma - h)\big(M^2k\delta\beta + M\theta(z\beta A - \rho A)\big)\Big)]^{\frac{1}{2}}}{2M\theta(z\gamma - h)},$$

$$A^* = \frac{-(M\theta(p\delta N - (G+\alpha)I^* + q\theta M(\beta + I^*\gamma) - GMw)) +, -((M\theta(p\delta N - GI^* - \alpha I^*) + q\theta M(\beta + I^*\gamma) - GMw)^2 + 4GM\theta(w + N\theta I^*)p\delta NM + Mq\theta(M\beta + MI^*\gamma + \beta I^* + \gamma I^{*2}))^{\frac{1}{2}} - (w + I^*\theta)\alpha I^* M}{(GM\theta)^2}.$$

where $k = 1 - p, z = 1 - q, h = \alpha + \gamma + \mu,$

$$G = \rho + \mu, and\ w = M\delta + M\mu.$$

Local stability of the maximum awareness equilibrium is determined by the variational matrix $J_{(M_a{}^*)}$ of the nonlinear system

$$\begin{bmatrix} -\delta - \frac{\theta(I^*+A^*)}{M^*} - \mu & \frac{\theta N^*}{M^*} + \gamma & \frac{-\theta N^*}{M^*} \\ (1-p)(1-q)\delta\theta\frac{(I^*+A^*)}{M^*} & (1-q)\frac{\theta N^*}{M^*} - (\alpha+\gamma+\mu) & (1-q)\frac{\theta N^*}{M^*} + \rho \\ p\delta + \frac{q\theta(I^*+A^*)}{M^*} & \frac{q\theta N^*}{M^*} + \alpha & \frac{q\theta N^*}{M^*} - (\rho+\mu) \end{bmatrix} \quad (6)$$

Lemma 1: Let $J_{(M_a{}^*)}$ be the variational matrix corresponding to $M_a{}^*$. If tr ($J_{(M_a{}^*)}$), det ($J_{(M_a{}^*)}$), and ($J^{[2]}{}_{(M_a{}^*)}$), are all negative, then all eigenvalues of $J_{(M_a{}^*)}$ have negative real parts.

Using the above lemma, we will study the stability of the maximum awareness equilibrium.

Theorem 2: If $R_0 > 1$, the maximum awareness equilibrium $M_a{}^*$ of the model (6) is locally asymptotically stable in Ω.

Proof:

From jacobian matrix $J_{(M_a{}^*)}$ in (6), we have

$$\text{tr}\,(J_{(M_a{}^*)}) = -(\delta + \pi + \theta\mu) - (\varepsilon + \varphi) - (\rho + \mu) < 0.$$

Det $(J_{(M_a{}^*)}) = -(\alpha + \kappa)(\frac{\tau\theta N(1-p)}{M^*} + \big(-\delta - \pi - \mu)(\tau + \rho)\big) - (\pi(1-p)(1-q)\delta(\gamma + \varphi) - (\varepsilon - \tau)(-\delta - \pi + \mu)(\kappa - \mu - \rho + (p\delta + \pi q)(-\varphi\alpha + q\gamma + \mu + \rho - \varphi 22q - 2 - \rho + \alpha\rho) < 0,$

where $\kappa = \frac{q\theta N^*}{M^*}, \pi = \frac{\theta(I^*+A^*)}{M^*}, \varphi = \frac{\theta N^*}{M^*},$

$$\varepsilon = (\alpha + \gamma + \mu), \tau = \frac{N^*\theta(1-q)}{M^*}.$$

Hence the trace and determinant of the Jacobian matrix $J_{(M_a{}^*)}$ are all negative.

The second additive compound matrix is obtained from the following lemma.

Lemma 2: Let *P* and *Q* be subset of {1,2,3}. The (*P*,*Q*) entry of $N_{ij}(J_{(M_a{}^*)})$ is the coefficient of *C* in the expansion of the determinant of the sub matrix of $J_{(M_a{}^*)}$ +*CI* indexed by row in *P* and column in *Q*.

Proof:

The sub matrix of $J_{(Ma^*)}$+*CI* is given as

$$\begin{bmatrix} -\left(\delta + \frac{\theta(I^*+A^*)}{M^*} + \mu\right) + C & \frac{\theta N^*}{M^*} + \gamma & \frac{-\theta N^*}{M^*} \\ (1-p)(1-q)\delta\theta\frac{(I^*+A^*)}{M^*} & -\left((\alpha+\gamma+\mu)\frac{\theta N^*}{M^*} - (1-q)\right) + C & (1-q)\frac{\theta N^*}{M^*} + \rho \\ p\delta + \frac{q\theta(I^*+A^*)}{M^*} & \frac{q\theta N^*}{M^*} + \alpha & -\left((\rho+\mu) - \frac{q\theta N^*}{M^*}\right) + C \end{bmatrix}.$$

The sub matrix of $J_{(M_a{}^*)}$+*CI* indexed by rows and columns is given by

$$\begin{bmatrix} -\left(\delta+\frac{\theta(I^*+A^*)}{M^*}+\mu\right)+C & \frac{\theta N^*}{M^*}+\gamma \\ (1-p)(1-q)\delta\theta\frac{(I^*+A^*)}{M^*} & -\left((\alpha+\gamma+\mu)\frac{\theta N^*}{M^*}-(1-q)\right)+C \end{bmatrix}.$$

The coefficient of C in the determinant of this matrix is $-(\alpha+\gamma+2\mu)+(1-q)\theta\frac{(I^*+A^*)}{M^*}-\theta\frac{(I^*+A^*)}{M^*}$, and thus the (1,1) entry of N_{ij} is

$-(\alpha+\gamma+2\mu)+(1-q)N\theta\frac{(I^*+A^*)}{M^*}-\theta\frac{(I^*+A^*)}{M^*}$. Other entries were obtained by the same method and the following $J^{[2]}{}_{(M_a{}^*)}$ was obtained as

$$\det\left(J^{[2]}{}_{(M_a{}^*)}\right)=\begin{bmatrix} -(\alpha+\gamma+2\mu)+(1-q)N^*\theta\frac{(I^*+A^*)}{M^*}-\theta\frac{(I^*+A^*)}{M^*} & (1-q)\frac{\theta N^*}{M^*}+\rho & \frac{-\theta N^*}{M^*} \\ \frac{q\theta N^*}{M^*}+\alpha & -\delta-\rho-\frac{(I^*+A^*)\theta}{M^*}+\frac{q\theta N^*}{M^*} & \frac{\theta N^*}{M^*}+\gamma \\ p\delta+\frac{q\theta(I^*+A^*)}{M^*} & (1-p)(1-q)\delta\theta\frac{(I^*+A^*)}{M^*} & -(\rho+2\mu+\gamma+\alpha)+\frac{\theta N^*}{M^*} \end{bmatrix}.$$

$$\det\left(J^{[2]}{}_{(M_a{}^*)}\right)=-(I+A)(1-p)(1-q)\delta\theta[\varphi(\alpha+\kappa)\ +(\alpha+\varphi)(-\omega-\pi-\tau)]+[-\tau(\alpha+\kappa)\ +(-\omega-\pi-\tau)(-\delta-\pi+\kappa-\rho)]$$
$$(-\omega+\varphi-\rho)+(-p\delta-\pi q)\ \left(\varphi(\gamma-\kappa-\delta-\pi\frac{\theta}{M}-\varphi-\rho)\right)<0.$$

Where $\omega=(\alpha+\gamma+2\mu)$.

Therefor $\det\left(J^{[2]}{}_{(M_a{}^*)}\right)<0$.

Thus, acording to lemma 1, the maximum awareness equilibrium $M_a{}^*$ of the model system (1) is locally asymptotically stable in Ω.

Table 3. Parameters estimates of the model

Parameter	Description	Value	Source
p	Proportion of unaware individuals that become aware through mass media.	0.7	Estimated
q	Proportion of unaware individuals that become aware through word-of-mouth.	0.6	Estimated
ρ	Discontinuance rate of adopters of vaccination awareness information	0.0001	Estimated
β	Recruitment rate	0.5	Estimated
θ	Vaccination awareness rate through word- of- mouth	0.4	Estimated
γ	Rate of forgetting vaccination awareness information	0.03	Estimated
δ	Vaccination awareness rate through media	0.2	Estimated
μ	Mortality rate	0.0001	Estimated
α	Progression rate from unadopters to adopters class	0.6	Estimated

3.6. Simulation and Discussion

The main objective of this study is to assess the impact of awareness information to infant vaccination. In order to support the analytical results, graphical representations showing the variations in parameters have been presented in this section. Since, most of the parameters were not readily available; it was found convenient to estimate them just for illustration purposes on how the model would behave in different real situation.

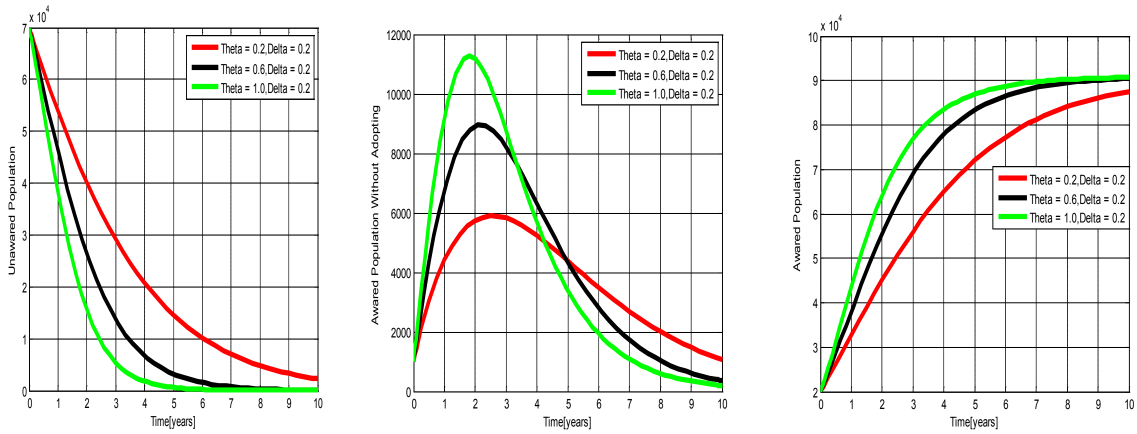
4. Discussion

In simulations the observation shows that when awareness information through word-of-mouth increases with time keeping awareness rate through media constant, unawared population and awared population without adopting infant vaccination decrease while increasing the number of awared population, **figure 2** (a), (b) and (c). The same observations ware obtained when we vary awareness rate through mass media and keep awareness rate through word-of-mouth constant **figure 3** (a), (b) and (c). But when we increase both means of information with the same values, we have beter results. That is, unawared population and awared population without adoting infant vaccination decrease more, and getting more awared population, **figure 4** (a), (b) and (c). **Figure 5** shows the dynamic of the model system that when the time increases, the maximum values approach to their steady-state values, the awareness becomes maximum while minimizing unawared population.

Thus, for the infant vaccination programme to succeed, there must be enough awareness information in both word-of-mouth and mass media before the programme starts.

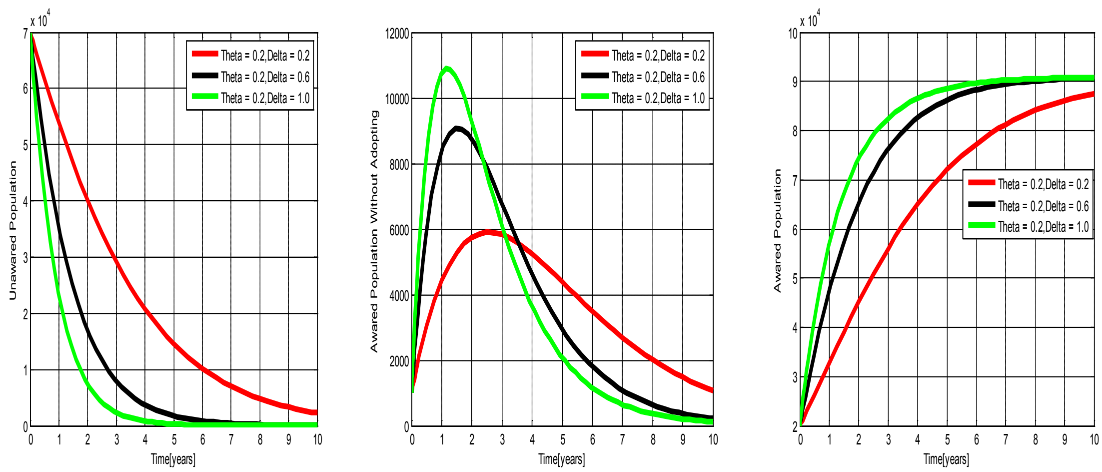
5. Conclusions

In this paper, we formulated a mathematical model which shows the importance of spreading awareness information on infant vaccination to the population. From the model we derived the basic reproduction number, R_0. For this study, we found out that if $R_0<1$, every awared individual on vaccination will cause less than one secondary awared individual and hence cause the number of people who are awared on infant vaccination to decrease, and when $R_0 > 1$, every awared individual on vaccination will cause more than one secondary awared individual and hence the awareness information on vaccination will invade the population. In this study, we were interested with large number of R_0, i.e $R_0 > 1$.



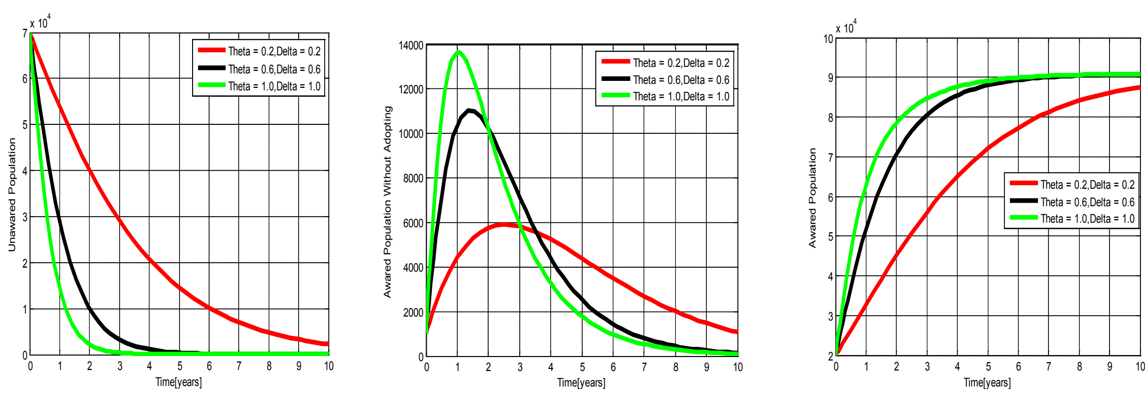
(a) (b) (c)

Figure 2. Variation of awareness information through word-of-mouth with constant value of awareness information through media to (a) unawared population, (b)awared population without adopting, and (c)awared population



(a) (b) (c)

Figure 3. Variation of awareness information through media with constant value of awareness information through word-of-mouth to (a) unawared population, (b) awared population without adopting, and (c) awared population



(a) (b) (c)

Figure 4. Variation of both awareness information through media, and awareness information through word-of-mouth, to (a) unawared population, (b) awared population without adopting, and (c) awared population

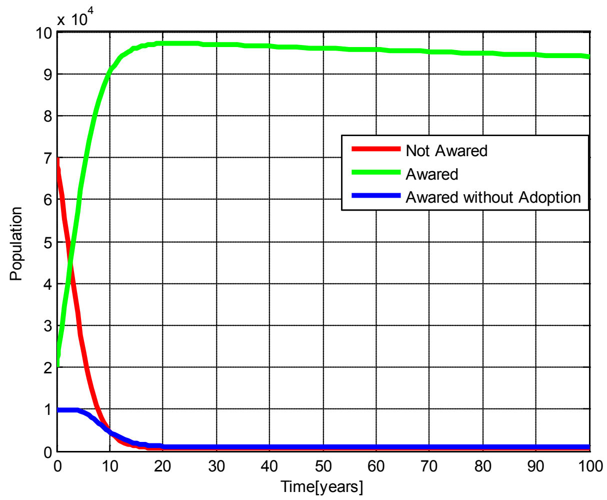


Figure 5. Dynamic of the model system

We found that the maximum awareness equilibrium exists and it is locally and asymptotically stable.

We performed sensitivity analysis on the basic reproduction number from which we noted that the parameter θ (awareness rate through word-of-mouth), is the most sensitive index on maximizing the infant vaccination awareness to the population due to its big positive value.

From numerical simulations we observed that both awareness information through word-of-mouth and mass media are important in reducing unawared people and increase awared people for beter succesion of infant vaccination.

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