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Modeling and optimization of clean water distribution networks

Edward, Lucas

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MODELING AND OPTIMIZATION OF CLEAN WATER DISTRIBUTION NETWORKS

Lucas Edward

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Master's in Mathematical and Computer Science and Engineering of the Nelson Mandela African Institution of Science and Technology

Arusha, Tanzania

October, 2021

ABSTRACT

In this study, a model has been developed to find the minimum cost in distributing clean water. Linear Programming (LP) technique was used to formulate the model for Dodoma city. The developed model consists of both hydraulic and water treatment parameters. The model was then tested with real data collected from Ihumwa water network of Dodoma city and other treatment cost data from the literature to test the workability of the model. Hydraulic parameters such as head loss of the pipes, flow velocity and pipe pressure are calculated using water flow software. The resulted model was solved using lingo software by testing different intermediate values of pressure and velocity to obtain the minimum cost of distributing clean water. As a result, the values 650 N/m^2 and 700 N/m^2 as a maximum and minimum pressure and 0.5m/s and 2m/s as minimum and maximum velocity give the minimum cost of distributing clean water. Consequently, the objective value of resulted optimization model shows that the original cost of distributing clean water was reduced by 3.48%.

DECLARATION

I, Lucas Edward do hereby declare to the Nelson Mandela Institution of Science and Technology that this dissertation is my own original work and that it has neither been submitted nor being concurrently submitted for degree award in any other university.

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Lucas Edward (Candidate)

4 November 2021

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Date

The above declaration is confirmed

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Prof. Verdiana Grace Masanja (Supervisor 1)

4 November 2021

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Date

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Dr. Mashaka James Mkandawile (Supervisor 2)

4 November 2021

....

Date

CERTIFICATION

The undersigned certify that they have read the dissertation titled "Modeling and optimization of clean water distribution networks" and recommend for acceptance in fulfillment for the requirements for the award of Master's in Mathematical and Computer Science and Engineering of the Nelson Mandela African Institution of Science and Technology.

The above declaration is confirmed

....

Prof. Verdiana Grace Masanja (Supervisor 1)

4 November 2021

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Date

....

Dr. Mashaka James Mkandawile (Supervisor 2)

4 November 2021

....

Date

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DEDICATION

I would like to dedicate this work to my family (My mother, brothers and sister) and friends.

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LIST OF ABBREVIATIONS AND SYMBOLS

ACO	Ant Colony Optimization
BIP	Binary Integer Programming
BIP	Binary Integer Programming
CANDA-GA	Cellular Automata for Network Design Algorithm Combined with Genetic Algorithm
CCLM COSMO-	Climate Limited Area Modeling
DP	Dynamic Programming
DUWASA	Dodoma Urban Water and Sanitation Authority
EWURA	Energy and Water Utilities Regulatory Authority
GA	Genetic Algorithm
GIS	Geographical Information System
INGOs	International Nongovernmental Organizations
IWA	International Water Association
LP	Linear Programming
MCP	Minimum Connector Problem
MICS	Multiple Indicator Cluster Survey
MST	Minimum Spanning Trees
NBS	National Bureau of Statistics
NLP	Non Linear Programming
OC	Operation Energy Cost
PRV	Pressure Reducing Valves
PSO	Particle Swarm Optimization
PSO	Particles Swarm Optimization
PVC	Pipe Polyvinyl Chloride pipe
RTO	Real-Time Optimisation
WDS	Water Distribution System
WSSA	Water Supply and Sanitation Authority

CHAPTER ONE

INTRODUCTION

1.1 Background of the Problem

A network of pipes, reservoirs, pumps, valves and other hydraulic components make up the water distribution system. The goal is to provide quality water to consumers under specific pressure levels and a variety of requirements.

Water distribution systems are an essential component of modern societies' economic infrastructure. Previous research on the design optimization of water distribution systems, on the other hand, generally involved few decision variables and, as a result, small solution spaces; piecemeal-solution methods based on pre-processing and search space reduction; and/or combinations of techniques working in concert (Tanyimboh & Seyoum, 2020).

Increased population, the development of new technologies, increased user awareness and the environmental sustainability needs of clean water distribution must be increasingly effective. These goals encourage the scientific community to propose innovative approaches to addressing new challenges through the use of quantitative predictive tools (Boano *et al*., 2015).

The UN Water Development Report of Water (2018) shows that many people will be affected by drinking water shortages by 2050. This is due to increased demand for water, reduced water resources and increasing water pollution driven by spectacular population and economic increase.

According to the report of Energy and Water Utilities Regulatory Authority (EWURA, 2018) on the performance of water utilities for the year 2017/18, the use of water supply system is analyzed in terms of a number of interconnections per kilometer of the water distribution system. The report indicates that the regional number of water links per kilometer of the water supply network improved from 52 to 54 in 2015/16 and 2016/17. Also from the report, the WSSAs of Arusha, Mwanza, DAWASCO, Morogoro and Dodoma registered the highest connection density compared to other Regional WSSAs. The connection density for each water utility was Arusha 98.5, Mwanza 97.3, DAWASCO 96.4, Morogoro 80.5 and Dodoma 80.5.

Flow pressure in a water pipe is experienced when the flow front is moving due to pressure differences. Water flows from high pressure toward a lower pressure in a system. The water distribution system is made up of a series of pumps, pipes, valves, and node sets that include a reservoir and pipe connections. The flow pressure in the network is determined by a set of fixed points, many of which are non-linear. The experimentally determined pressure-flow rate relationship is associated with non-linear conditions.

Collins *et al*. (1978) assert that the problem of traditional pipe network estimate is based on calculating a series of flows and pressures in a water supply pipe network given known sources and withdrawals. New water distribution networks (WDNs) are difficult to manage due to increased population growth, changing customer needs, outdated infrastructural facilities, high operating costs, and a scarcity of water resources.

According to Caballero and Ravagnani (2019), water distribution networks (WDNs) are found in a variety of industrial processes and urban areas. Water distribution networks are made up of reservoirs, pipes, nodes, loops, and pumps and their design can be formulated as an optimization problem. The main goal of a formulated optimization problem is to reduce the network's cost, which is determined by pipe diameters and flow directions.

However, several models for optimizing water distribution networks have been proposed, such as the model-based in Carini *et al*. (2018), where the nonlinear programming (NL) constrained method is used. The research takes into account hydraulic parameters such as pressure and velocity, which are also taken into account in this study with the addition of a treatment term and its treatment constraints. The first term of the formulated model in this study has nonlinearity properties due to its pressure and velocity constraints, therefore it should account be multiplied by zero unit variables to make it linear together with its constraints i.e. Pressure and velocity.

To validate the model's applicability, this study has used hydraulic and cost data from the Dodoma Water and Sanitation Authority (DUWASA) as well as water treatment data from the literature review. Because the government is relocating to its capital city from Dar es Salaam to Dodoma, hydraulic data from the Dodoma water supply is being considered for this study. Clean water distribution systems are planned or are being planned for communities as the population grows and new residential areas are built. As a result, the findings of this study will be useful. The model will simulate the most cost-effective systems while taking into account the city's growing population and the need for clean water.

1.1.1 Water Distribution Networks

A water distribution network (WDNs) is a system containing pipes, reservoirs, pumps, and valves of different types, which are connected to each other to provide water to consumers (Mansouri *et al*., 2015).

A WDN is a network containing connected pipes and other appurtenances to supply water for a given demand and pressure requirements. One way to model the structure WDN is to use a mathematical graph with node-edge representation.

Node/Vertices is the component in a network that represents a specific location such as reservoir, tank, and consumer junction. Nodes in a network can represent both sources and demand centers.

Edges/links/arc in water distribution network edge represents pipes and expresses connectivity between nodes.

A mathematical graph of nodes and edges is represented as in Equation (1)

$$G = G(V, E) \tag{1}$$

where V is the set nodes and E is set edges.

To show the topological structure of the incidence matrix is introduced as in Equation (2)

$$A_{ij} = \begin{cases} 1 \\ -1 \\ 0 \end{cases} \tag{2}$$

where A_{ij} represent the connectivity relationship between node i and edge j, for 1 Node i is the initial node of edge j, for 0 Node i is the uncounted node of edge j and for -1 Node i is the terminal node of edge j.

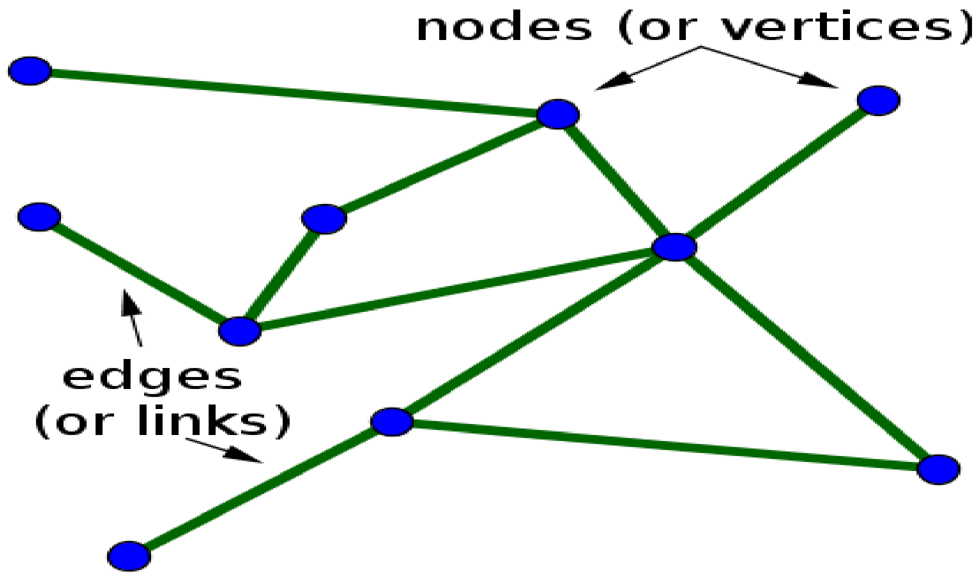

Figure 1: **Simple Network**

1.2 Statement of the Problem

The water supply system consists of a network of hydraulic elements that are linked together to distribute predetermined water flow rates from sources to required centers. The use of a network structure as geometrical support for model optimization has proven particularly advantageous. Nodes in a network can represent both sources and demand centers, as well as control elements. Arcs are the pipes that connect nodes. Aside from the obvious benefits, the graphic structure enables the use of a highly efficient data structure for optimization (Fanni *et al*., 2000).

For many years, the optimal design of urban water supply networks has become a key problem in the water sector. The most cost-effective solution for transferring water from the reservoir to customers is sought by the optimized distribution network layout (Heydari *et al.*, 2020).

A municipal clean water distribution network can be designed using a variety of pipe sizes, reservoirs, and pumps. Almost all water systems consume a significant amount of energy, resulting in high costs, based on the topography of the sources and targets. As energy prices rise, researchers' goal of reducing power consumption and costs becomes a budgetary one.

Energy efficiency and sustainable energy concepts have also gained a great deal of importance in recent years and will continue to be significant as energy needs increase. This phenomenon forces researchers to try to minimize energy expenditure (Gencer & Merzi, 2016).

According to Marchionni *et al*. (2015), an appropriate decision in wastewater and water services necessitates a detailed model to achieve the best performance at a reasonable risk level while accounting for the costs of capital asset construction, operation, repairs, and disposal over their lifespan.

Furthermore, to design water distribution modeling and optimization, one must have the best model that will optimize the cost of clean water distribution systems in the specified area or data of a particular area. In many literatures, provided models for optimization of water distribution system have been used to optimize the cost are based on hydraulic parameters, such as a model in the research paper of Maiolo *et al*. (2017), that also uses a nonlinear optimization (NP) method to improve drinking water supply systems in regards to the effects of climate change. Bao-Feng and Du Xue (2014) use particles swamp optimization (PSO) method to optimize tree pipe networks layout. The mixed-integer Linear Programming (MLP) is used by Veintimilla-Reyes *et al*. (2019) to improve water distribution and reservoir location.

Although the present study has been conducted on various cost optimizations for water distribution networks, previous studies do not consider the cost of water purification/water treatment parameters and hydraulic parameters in the same model while formulating water cost optimization models.

In the study of Surco *et al*. (2018a), the use of a modified particle swarm optimization method was utilized to determine the minimum cost of distributing water in a given network. The study focused to minimize the cost of water distribution based on pipe diameter, thus there is one term in the objective function.

The purpose of this study is to develop an optimization model for a clean water distribution network using the (Samani & Zanganeh, 2010) model, for the hydraulic parameter used i.e. Pressure, velocity and flow rate and model by Boah *et al*. (2016) for the case of water treatment parameters. With a slight modification of the parameters for these two models, a new model is being developed to optimize the cost of a clean water distribution network.

1.3 Rationale of the Study

Clean water distribution in WDS is a serious challenge for many water supply authorities (WSA) worldwide. The vast majority of the growing demand for water will occur in countries with developing or emerging economies. An urban clean water distribution network can be designed using a variety of pipe sizes, valves, reservoirs, and pumps. Almost all water systems consume a significant amount of energy, resulting in high operating costs and maintenance costs, based on the topography of the sources and targets. This challenge raises the need for research into clean water distribution networks in the WDS.

Optimization models have played a great role in understanding water distribution networks and making the right decisions on water distribution network parameters. The optimization models developed in previous studies are either for hydraulic parameters or water treatment parameters. This study focuses on the development of a cost optimization model that involves both hydraulic parameters and water treatment parameters.

1.4 Research Objectives

1.4.1 General Objective

The general objective of the research is to formulate a linear programming model that proposes the distribution of clean water at a minimum cost in a given network.

1.4.2 Specific Objectives

The following are the study's specific objectives:

(i) To develop a mathematical model that minimizes the cost of clean water distribution networks.

(ii) To analyze the appropriate pressure of the pipe, the flow velocity in the network will optimize the cost of the distribution of clean water.

(iii) Find the minimum cost for the distribution of clean water.

1.5 Research Questions

The following issues are intended to be addressed by the study:

(i) For the formulation of a mathematical model to optimize, clean water distribution networks, what assumptions and methods are suitable?

(ii) What is the appropriate pipe pressure, flow velocity in a network that optimizes the distribution of clean water?

(iii) What is the minimum cost for the distribution of clean water?

1.6 Significance of the Study

This research will determine the suitable pipeline configuration for forming water distribution networks that will reduce the cost of distributing clean water in cities. As a result, if the findings of this study are used by the relevant authorities, they may guide to municipal civil engineers and management in general on designing WDNs that minimize costs. Results of this work will be helpful to municipal administration to figure out how to reduce the cost of clean water distribution. The developed model and computational techniques will add to the body of knowledge in water distribution.

1.7 Delineation of the Study

This study uses data that were collected from the ongoing water projects in Dodoma city under DUWASA. The developed model will simulate the most cost-effective systems while taking into account the city's growing population and the need for clean water. According to the findings of this study, similar strategies for clean water distribution can be implemented in other water authorities that operate under similar conditions.

CHAPTER TWO

LITERATURE REVIEW

2.1 Models of Water Distribution Networks

In the study of Kumar (2014), a transportation programming model was considered to optimize water distribution from a reservoir to farmland, and optimization methods are used to solve the problem. The initial stage was to identify an achievable baseline solution, and the next stage was to identify the optimal solution. In the achievable baseline solution, the North-west corner (NWC) method was applied, and for an optimal solution, the Modified Distribution (MD) Method was applied. Finally, the minimum total cost of water distribution was obtained from the reservoir to farmland which meets all the requirements of the reservoir capacity.

In the study of Ezzeldin and Djebedjian (2020), for the lowest cost design of pipe networks, a new lowest optimization technique known as the Whale Optimization Algorithm (WOA) is used. The algorithm is applied to three pipe networks, and the results are compared to those obtained by optimizing the networks using various optimization techniques. The findings demonstrated that WOA is a promising optimization model because it provides the minimum cost network design when compared to the majority of optimization methods available in the literature.

Awe *et al*. (2019) examines optimization of water distribution systems and introduces different aspects of WDS. It also presents various methods of optimization in detail. Finally, modern and sophisticated optimization methods are sufficiently sophisticated to address complex real-world issues such as the design and operation of the WDS.

In the study of Cassiolato *et al*. (2021), the optimization model known as deterministic Mathematical programming was proposed to determine the minimum cost of looped WDNs. The model took into account pipe lengths and a discrete set of commercially available diameters and the constraints are mass balances in nodes, energy balances in loops and hydraulic Equations. The discrete optimization problem is reformulated by generalized disjunctive programming to a non-linear integer-blended programming problem (MINLP). The problem is solved by General Algebraic Modeling System (GAMS) environment.

To enhance the resiliency of the networks, the optimization of pipe networks is achieved with two goals, such as reducing the capital cost and maximizing the measurement of the resiliency of the arrangement. In the study of Suribabu (2017), two objectives are treated as a single objective and the optimization model is resolved by the differential evolutionary method. The planned objective function tends to be effective under the weighted method of the multi-objective water distribution design model. The numerical results of the design specify the merits of sizing the pipes along the shortest flow path for having improved resilience indexes.

The work of Reca *et al.* (2017), presents a method to increase the efficiency of the heuristic approach useful to the optimal strategy of water distribution systems (WDS). The method is based on minimizing the search space by limiting the diameters which can be used for each pipe network.

In the work of Surco *et al.* (2018b), an optimization model was developed for the rehabilitation and expansion of water distribution systems based on particle swarm optimization (PSO). The Epanet hydraulic simulator was used to calculate speeds and node pressures. The decision variables (diameters) were treated as integer variables and the change in the internal smoothness of the pipe to be used was considered. The problem was formulated using the non-linear mixed integer programming (MINLP) method.

The study of Martin-Candilejo *et al.* (2020), shows a design method for a fixed flow speed, where the entire cost corresponds to the expenses of the demand flow variable. The method is built on the Granados method, which is an instinctive and practical gradient-based technique. To become familiar with the regular application, the idea of a similar velocity and flow volume is presented and used in a simple case study.

In Cassiolato *et al.* (2019), the author used disjunctive programming approach to find the minimum cost for distributing water where the hydraulic parameters are used to formulate constraints of the objective function. The method is suggested and the entire variables are optimized simultaneously, escaping the use of external software for pressure and velocity control. Two situations (case) studies were used to check the model applicability and coded in GAMS. The results presented only for the two overall optimum cases achieved, showing the possibility of solving the problem, individually from the external hydraulic simulator.

The optimization model based on Mansouri *et al*. (2015) was described as the Mixed Integer Linear Programming (MILP). Pressure and velocity constraints were used to determine optimal water distribution networks.

The Study of Samani and Zanganeh (2010) develops an LP model consisting of three objective function terms: (a) the cost of the pipes, which includes the cost of transport and installation (b) the cost of the reservoirs and (c) the cost of the pumps and the constraints used are speed constraints and pressure constraints to determine the minimum cost of WDNs.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Study Area

The model developed for this study was applied to the water distribution networks of Dodoma city. Dodoma is located at an altitude ranging from 1030 m to 1320 m above sea level and has the savannah type of climate. The average maximum and minimum temperatures for Dodoma are 31 ^{0}C and 18 ^{0}C respectively. The area of Dodoma city is 2769 square kilometers (276 910 Ha) and the city in the year 2010 was composed of 30 Wards and 119 villages. In the year 2018, the city comprises 41 Wards and 18 villages 170 mitaa and 89 hamlets. The present Dodoma City wards comprise urban, mixed and rural areas. According to Tanzania National Bureau of Statistics (NBS, 2013), Dodoma City has a population of 410 956 from 93 887 households. The classification of the distribution of population is as follows 57% in urban wards, 15% in mixed wards and 28% in rural wards.

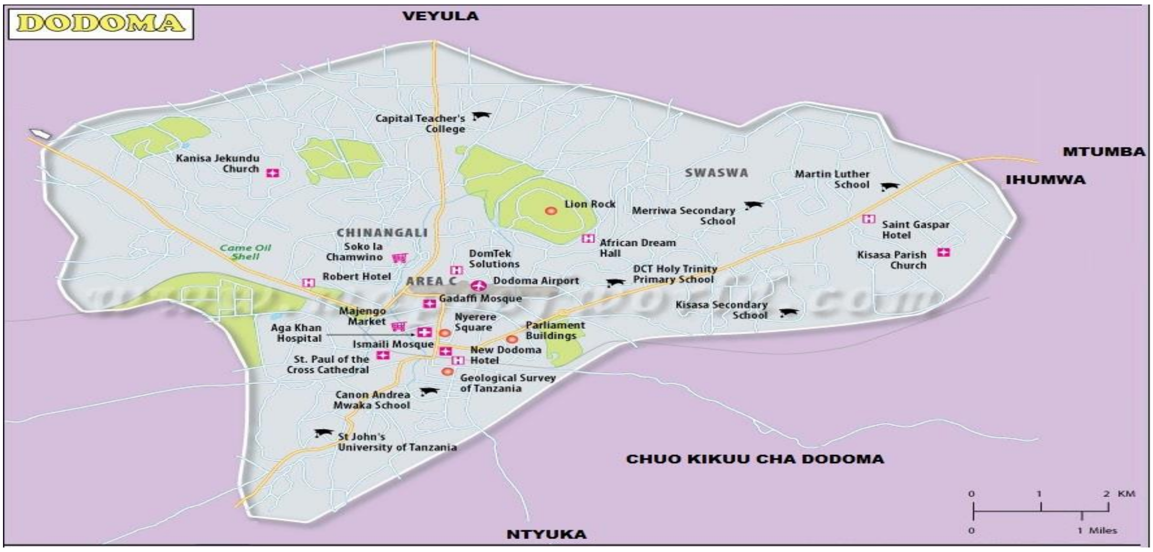

Figure 2: The DUWASA WDS coverage areas

3.1.1 Dodoma Water Supply System

The water supply for Dodoma city is currently under DUWASA. The water supply for Dodoma city currently depends on groundwater sources. The present main source of water for the city is the Makutopora Well field (about 25 km) from Dodoma town along Arusha road. The current water supply facilities for Dodoma City consist of Makutopora Well field with 25 drilled boreholes and Collector system, Pumping Stations facilities at Mzakwe, Transmission System including Booster Stations, Distribution Network including Booster Stations, Reticulation Systems and Storage Reservoirs.

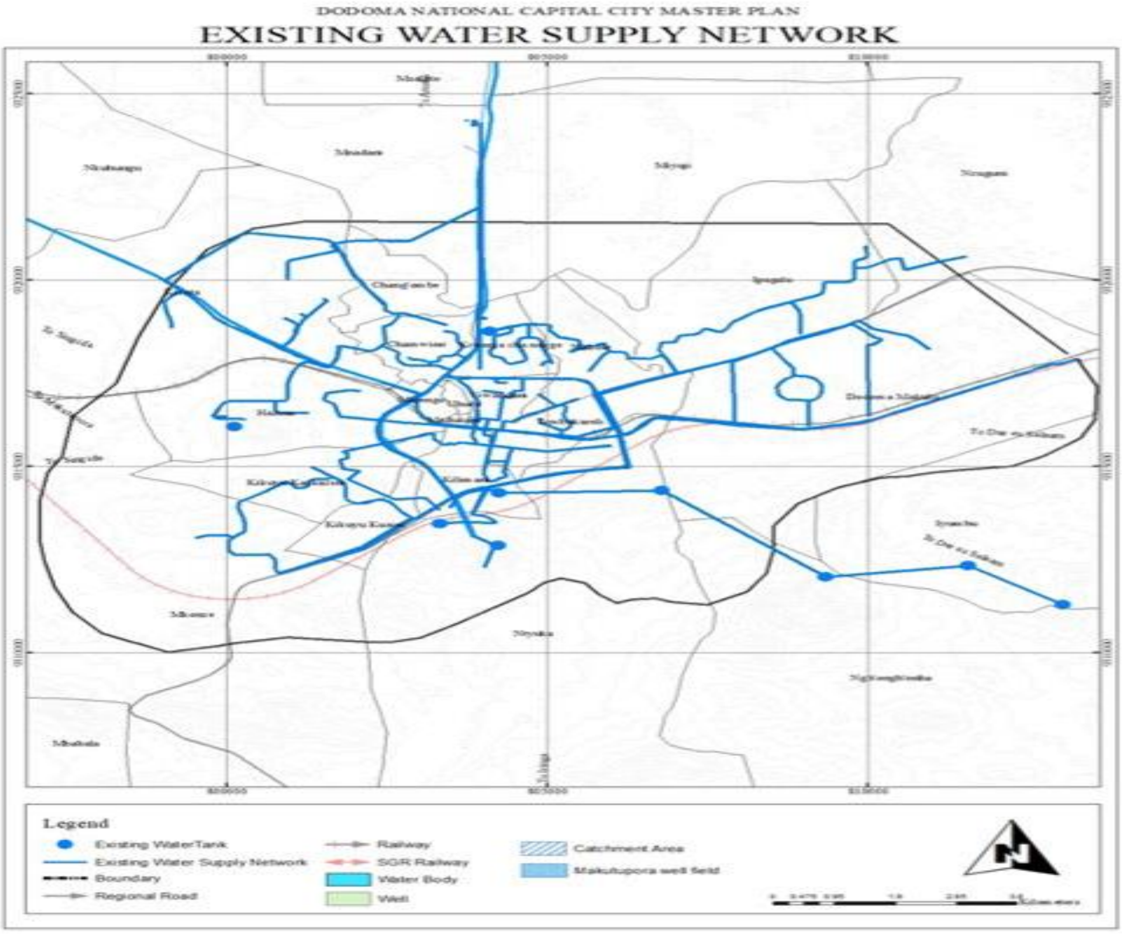

Figure 3: Existing Water Supply Networks in Dodoma city

3.2 Study Data Type

This study uses secondary hydraulic data and cost data collected through documentary reviews and water project reports i.e. upcoming, ongoing and completed projects. The data includes hydraulics parameters such as pressure, water velocity, length of pipes, reservoirs and pipe diameters as well as cost data such as pipe cost operation cost, maintenances cost, electricity cost, chemical cost, the quantity of electricity, the quantity of chemical used for treatment and the number of personnel.

3.3 Development of Mathematical Model for Optimizing Clean Water Distribution Networks

3.3.1 Model Assumptions

(i) The cost data are in Tanzania shillings.

(ii) Water distribution should be treated for domestic use and the cost treatment is included in the model.

(iii) Hazen – William Formula was used to calculate the pipe friction head loss.

3.3.2 Objective Function

The sum of all pipes diameters and their costs and the cost of treatment must be considered in the objective function. The objective function of the optimization model.

$$Min(C(d_n) + T(M_c, E_c, C_c, P_c)) \tag{3}$$

Where $C(d_n)$ denotes the cost of the pipes which consists of transportation and installation, d_n is the diameter and $T(M_c, E_c, C_c, P_c)$ the treatment costs, which include maintenance costs M_c, energy cost E_c, chemical cost C_c and personnel cost P_c. These cost functions are given by Equations (4) and (5).

The cost of the pipes is given as in Equation (2).

$$C(d_n) = \sum_{n=1}^{np} L_n CP_n\,(d_n) \tag{4}$$

The treatment cost is given as in Equation (5).

$$T(M_c, E_c, C_c, P_c) = \beta Y_1 + \alpha Y_2 + \delta Y_3 + \lambda Y_4 \tag{5}$$

From Equations (3) and (4) np represents a number of the pipe L_n represents the length of pipe number n and n represents pipe number. CP_n represent the cost of pipe n per unit length, where pipe n varies with diameter. β is the Maintenance cost coefficient, α is the energy cost coefficient, δ the chemical cost coefficient and λ is the personnel cost coefficient,

Y_1 is the number of maintenances in a month, Y_2 is the average quantity of energy, Y_3 is the average quantity of chemicals, and Y_4 is the number of personnel.

Now the objective function which is the total cost of distributing clean water is given as Equation (6).

$$Min(\sum_{n=1}^{np} L_n CP_n\,(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \lambda Y_4 \qquad (6)$$

Maintenance cost coefficient (β) is given as in Equation (7).

$$\beta = \frac{AMC}{NM} \qquad (7)$$

where AMC is the average maintenance Cost and NM Number of maintenances in a month.

The energy cost coefficient(α) is given as in Equation (8)

$$\alpha = \frac{AEC}{AQE} \qquad (8)$$

where AEC average energy cost and AQE average quantity of energy in kWh.

The chemical cost coefficient (δ) is given as in Equation (9).

$$\delta = \frac{ACT}{AQC} \qquad (9)$$

where ACT average chemical cost for treatment and AQC average quantity of chemicals.

The personnel cost coefficient (λ) is given as in Equation (10).

$$\lambda = \frac{APC}{NP} \qquad (10)$$

where APC personnel average cost and NP number of personnel.

Note that, to convert the objective function from non-linear to a linear problem we have to multiply the first term summations of Equation (6) by non-zero unity variables such as X_n

and when all commercially available pipes are added together, the following Equation (11) is obtained.

$$Min(\sum_{j=1}^{NPA}\sum_{n=1}^{np} L_n CP_n(d_n)X_n + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \lambda Y_4) \tag{11}$$

where NPA is the number of pipe sizes available on the marketplace, np is the number of pipe, Y_1 is the number of maintenances in a month, Y_2 is the average quantity of energy, Y_3 is the average quantity of chemicals and Y_4 is the number of personnel.

3.3.3 Constraints of the Model

The following constraints apply to the objective function:

(i) Pressure Constraint

Equation (12) gives the energy Equation between the reference node and any other node.

$$P_i = P_j + \Delta Z - HL_j \tag{12}$$

Where P_i represent pressure head at the node i, P_j represent reference node pressure, ΔZ is the height difference between the reference node and node i, HL_j is the head-losses from reference node and end at the node i.

Head-losses: refers to the frictional loss in the pipe expressed as a friction of the available head. In fluid flow, major head loss or friction loss is the loss of pressure or head in pipe flow due to the effect of the viscosity of the fluid near the surface of the pipe and is due friction in straight pipe. Minor head loss occurs due to components such as valves and bends of pipe. For this study Hazen – William formula was used to calculate head loss which is given by Equation (13).

$$HL_j = \frac{10.67 * L_n * Q_n^{1.85}}{C^{1.85} * d_n^{4.87}} \tag{13}$$

where Q_n discharge in a pipe (m^3/sec), d_n is pipe diameter, L_n is the length of pipe n and C is Hazen-Williams coefficient.

Equations (14) and (15) are the upper and lower limit of pressure.

$$P_i \geq Pr_{min} \tag{14}$$

$$P_i \leq Pr_{max} \tag{15}$$

Equation (12) is substituted into Equations (14) and (15), respectively, to give Equations (16) and (17).

$$P_j + \Delta Z - HL_j \geq Pr_{min} \tag{16}$$

$$P_j + \Delta Z - HL_j \leq Pr_{max} \tag{17}$$

The Equations (16) and (17) are multiplied by summation and non-zero unit variables in the head loss to make them linear constraints as in Equations (18) and (19).

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j X_n \geq Pr_{min} \tag{18}$$

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_J X_n \leq Pr_{max} \tag{19}$$

Therefore, Equations (18) and (19) are the model pressure constraints. Where NPR is the number of pipes connected to the reference node.

(ii) Velocity Constraint

The Flow velocity constraint is given as in Equation (20).

$$V_{min} \leq V_n \leq V_{max} \tag{20}$$

$V_{\min}$ is the minimum allowable flow speed in the pipe, $V_{\max}$ is the maximum allowable flow speed in the pipe and V_n is the pipe flow speed which is given by Equation (21).

$$V_n = \frac{4Q_n}{\pi d_n^2} \tag{21}$$

Substituting Equation (21) in Equation (20) and multiplying by summation and non-zero unit variable results in Equation (22) which is a velocity model constraint.

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_n \leq V_{max} \tag{22}$$

(iii) Maintenance Constraint

The products of maintenance coefficient cost and the number of maintenance in a month are greater or equal to the average maintenance cost and it is given by Equation (23).

$$\beta Y_1 \geq M \tag{23}$$

where M is the average maintenance cost.

(iv) Energy Constraint

The product of energy coefficient cost and the average quantity of electricity used is greater or equal to the average cost of electricity and it is given by Equation (24).

$$\alpha Y_2 \geq E \tag{24}$$

where E is the average cost of electricity.

(v) Chemical Constraint

The product of chemical coefficient cost and the average quantity of chemical is greater or equal to the average cost in chemical and it is given by Equation (25).

$$\delta Y_3 \geq Z \tag{25}$$

where Z is the average cost in chemical.

(vi) Personnel Constraint

The product of personnel coefficient cost and the number of personnel is greater or equal to average personnel cost and it is given by Equation (26).

$$\lambda Y_4 \geq P \tag{26}$$

where P is the average personnel cost.

3.4 Developed Optimization Model

The optimization model developed in this study:

Objective function

$$Minimize(\sum_{j=1}^{NPA}\sum_{n=1}^{np} L_n CP_n(d_n)X_n + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \lambda Y_4 \quad (27)$$

Subject to the constraints

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j\, X_n \geq Pr_{min} \quad (28)$$

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_J\, X_n \leq Pr_{max} \quad (29)$$

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_n \leq V_{max} \quad (30)$$

$$\beta Y_1 \geq M \quad (31)$$

$$\alpha Y_2 \geq E \quad (32)$$

$$\delta Y_3 \geq Z \quad (33)$$

$$\mu Y_4 \geq P \quad (34)$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \quad (35)$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0 \quad (36)$$

3.5 Computations and Model Data

This section contains tables of model data and the calculated model variables from the data. The model developed in this study for analyzing the clean water distribution cost has been applied to the water treatment data for DUWASA (these are secondary data) and hydraulic data (raw data) collected from ongoing water projects in Dodoma city.

Table 1: Details of the Pipe Laid, Ihumwa, Dodoma

Junction	Pipe size	Length(m)	Discharge(l/s)	Head-loss (m)	Velocity(m/s)
Tank- J1	300	279.8	48.9	0.37	0.69
J1-J2	300	643.9	48.9	0.85	0.69
J1-J11	250	450.5	48.9	1.44	0.99
J2-J17-J30	200	774.7	48.9	6.95	1.5
J10-J12-J16	160	768.1	48.9	21.66	2.42
J11-J10-J9	160	224.2	48.9	6.32	2.42
J9-J15-J8	160	265.3	48.9	7.48	2.42
J8-J7-J6	160	355	48.9	10.01	2.42
J6-J2	160	277.8	48.9	7.84	2.42
J2-J3	160	271.8	48.9	7.67	2.42
J19-J20	160	259	48.9	7.3	2.42
J19-J30	160	293.4	48.9	8.28	2.42
J21-J22-J20	160	634.1	48.9	17.88	2.42
J16-J14-J5	75	1353.6	48.9	1533.33	11
J5-J4	75	332.7	48.9	376.88	11
J4-J3	75	335.6	48.9	380.16	11
J5-J6	75	240.1	48.9	271.87	11
J9-J13	75	414.6	48.9	469.65	11
J27-J28	75	639.9	48.9	724.86	11
J21-J23	75	333.1	48.9	377.33	11
J23-J24	75	175.7	48.9	199.03	11
J24-J25-J26	75	345.7	48.9	391.7	11
J19-J25	75	341.4	48.9	386.73	11
J26-J27	75	411.4	49.9	466.02	11
J28-J29	75	367.4	48.9	416.18	11
Total	**Length**	**10788.8**			

Table 1 shows the details of pipes laid at Ihumwa water supply network in Dodoma region. Discharges head-lose and velocities of the pipes from Table 1 are calculated using computer water software.

Table 2: Average Discharge and Head-loss (HL) for the Pipe Used

Diameter (mm)	Pressure (N/m^2)	Discharge (Q)(l/s)	Headloss (HL)(m)	Elevation (m)
300	12	49	3.22	0
250	12	48.9	1.44	0
200	10	47.4	6.95	0
160	10	48.9	10.5	0
75	10	48.9	499.4	0

Table 2 shows the averages of pipe discharges, head-loss; elevation of pipes calculated from Table 1.

Table 3: Cost of Pipe

Diameter (mm)	Length (m)	Cost per unit length (TZS)	Total Cost (TZS)
300	924	78260	72312240
250	451	52440	23650440
200	751	44230	33349420
160	3283	41190	13945310
75	5380	12680	67089880
Total length	**10788**	**Total Cost**	**334347290**

Table 3 shows the cost of pipe used in the network.

Table 4: Cost of Energy, Chemicals Cost, Maintenances/Operation and Personnel Cost

Month	Energy cost (TZS)	Treatment cost (TZS)	Maintenance cost (TZS)	Personnel cost (TZS)
January	1437812.58	550154.3	142815.66	2739500
February	1159542.47	482400.45	138948.17	3353560
March	1529840	482400.45	162890.08	2758905
April	1340012.36	592573	162861.16	2258900
May	1487451.4	770491.33	164424.94	3590680
June	1342699.26	770491.33	164795.29	930800
July	1726131.58	779536.98	167046.38	3442000
August	1649303.26	736698.3	157902.79	2852770
September	1561136.88	933843.9	170776.23	5234190
October	1706424.45	836497.95	171417.747	4538500
November	1324500	1670468.16	922190.85	172909.84
December	1664566.2	887812.35	169732.44	3426700
Total	**16765388.6**	**8745091.19**	**1946520.72**	**33024305**
Average	**1397115.717**	**728757.6**	**162210**	**2752025.417**
Total	**Average**	**5040108.317**		

Table 4 shows the cost of energy, chemicals, maintenance and personnel cost together with their total and averages which are used to calculate cost coefficients for energy, chemicals, maintenance and personnel cost.

Maintenance coefficient cost (β) is given as Equation (37).

$$\beta = \frac{AMC}{NM} = \frac{16210.06}{6} = 27035 \tag{37}$$

where AMC is the average maintenance cost and NM is the number of maintenance in a month whereby maintenance is done after five days which makes a total of 6 maintenance in a month.

Table 5: Quantities of Chemicals and Electricity

Month	Chemicals (kg)	Electricity (KWh)
January	676 390	4179.68
February	598 725	3370.76
March	721 710	4447. 21
April	595 075	3895.38
May	776 845	4323.99
June	678 925	3903.20
July	780 265	5017.82
August	731 695	4794.49
September	725 530	4538.19
October	647 515	4960.54
November	690 345	4856.01
December	669 420	4838.86
Total	**8 292 440**	**53 126.04**
Average	**691 036.67**	**4427.17**

Table 5 shows the quantity of chemicals utilized for water treatment and the quantity of electricity utilized annually. The Average Quantity of energy (AQE) is in kWh and the chemicals are in Kg. The Average Quantity of energy and the chemicals are used to calculate the energy coefficient and chemical coefficient.

The energy cost coefficient(α) is calculated as in Equation (38)

$$\alpha = \frac{AEC}{AQE} = \frac{397115.717}{4427.17} = 315.58 \tag{38}$$

Where AEC is the average energy cost and AQE average quantity of energy.

The chemical cost coefficient (δ) is calculated as in Equation (39).

$$\delta = \frac{ACT}{AQC} = \frac{728757.6}{691695.8} = 1.05 \tag{39}$$

Where ACT is the average chemical cost for treatment and AQC is the average quantity of chemicals.

The personnel cost coefficient (λ) is calculated as in Equation (40).

$$\lambda = \frac{APC}{NP} = \frac{2752025.417}{80} = 34400.3 \tag{40}$$

Where APC is the average personnel cost and NP is the number of personnel.

Table 6: Calculated Cost Coefficients

Maintenance cost coefficient (β)	27 035
Energy cost coefficient (α)	315.58
Chemical cost coefficient (δ)	1.05
Personnel cost coefficient (μ)	34 400.3

Table 6 Shows the Calculated water treatment cost coefficient.

3.6 Resulting Optimization Model

The resulting optimization model is given from Equations 41 to 51 as shown below:

Objective function

Minimize

$$72312240X_1 + 23650440X_2 + 33349420X_3 + 13945310X_4 + 67089880X_5 + 27035Y_1 + 315.58Y_2 + 1.05Y_3 + 34400.3Y_4 \tag{41}$$

Subject to the constraints

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \leq 700 \tag{42}$$

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \geq 650 \tag{43}$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \geq 0.5 \tag{44}$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \leq 2 \tag{45}$$

$$27035Y_1 \geq 16210.06 \tag{46}$$

$$315.58Y_2 \geq 1397115.717 \tag{47}$$

$$1.05Y_3 \geq 728757.6 \tag{48}$$

$$34400.3Y_4 \geq 2752025.417 \tag{49}$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \tag{50}$$

$$X_1, X_2, X_3, X_4, X_5, \geq 0 \tag{51}$$

The optimization model that resulted produced an optimization problem that was solved using LINGO (linear, interactive, discrete optimizer) software. The model shows that the unknowns for pipes and water treatment give optimal cost for clean water distribution networks. The unknown parameters for pipes give the optimal solution under controlled minimum and maximum pressure and velocity of water in the pipes.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Costs Comparison

From Table 3 shows the original total cost of pipes which is TZS 334 347 290 and Table 4 shows the original cost of water treatment which is TZS 5 040 108.317. After solved the resulted optimization model by Lingo the cost of the pipes is reduced from TZS 334 347 290 to TZS 322 664 634.4 which is equal to 3.5% of the total pipes cost. This cost reduction is for values of $X_1=0, X_2=0$, $X_3=7.260143$, $X_4=0$ and $X_5=1.200525$ since there are five type pipes in the network i.e. 300 mm, 250 mm, 200 mm, 160 mm and 70 mm. The percentage of the cost of pipes reduced after solve the resulted optimization model is given as

$$\text{PPRC} = \frac{\text{PRC}}{\text{TPC}} \times 100\% \tag{52}$$

where PPRC represent percentage pipe reduced cost, *PRC* pipe reduced cost, *TPC* total pipe cost. Therefore:

$$\text{PPRC}=\frac{11\ 682\ 655.6}{334\ 347\ 290} \times 100\%=3.5\% \tag{53}$$

While the cost of treatment is reduced from TZS 5 040 108.317 to TZS 4 904 932.224 which is equivalent to 2.9% of the total treatment cost. This cost reduction is for values of $Y_1=0.5995957 \approx 1, Y_2=4427.136$, $Y_3=694054.9$ and $Y_4=80.00004 \approx 80$.

The percentage of the cost of water treatment reduced is given as in Equation (54).

$$\text{PWTRC}=\frac{\text{WTRC}}{\text{TTWC}} \times 100\% \tag{54}$$

where PWTRC represent percentage water treatment reduced cost, WTRC, represent water treatment reduced cost and TTWC represent the total cost of water treatment. Therefore:

$$\text{PWTRC}=\frac{135\ 176.09}{5\ 040\ 108.317} \times 100\%=2.7\% \tag{55}$$

The total cost of distributing clean water is reduced from TZS 339 387 428.7 to TZS 327 558 700 (Objective value) which is equivalent to 3.48% of the total cost of distributing clean water in the given network which is the general percent of the reduced cost of distributing clean water.

The general percentage of reducing the total cost of distributing clean water is given as in equation (56):

$$GPRC=\frac{TRC}{TC}\times 100\% \tag{56}$$

where GPRC represent the general percentage of reducing the total cost, TRC represent the total reduced cost and TC represent the total original cost of distributing clean water. Therefore:

$$GPRC=\frac{11\ 817\ 831.7}{339\ 387\ 398.317}\times 100\%=3.48\% \tag{57}$$

Table 7: Comparison of the Original and l Optimal costs of the Decision Variables of the Model

	Original Cost	Optimal Cost	Difference
Pipe Cost	334 347 290	322 664 634.4	11 682 655.6
Treatment Cost	5 040 108.317	4 904 932.224	135 176.1
Total Cost	**339 387 398.317**	**327 569 566.6**	**11 817 831.7**

4.2 Maximum and Minimum Pressure and Velocity for Optimal Cost

The maximum and minimum pressures and velocity is obtained by testing different intermediate values in pressure and velocity constraint that gives the minimum cost for distributing clean water. According to the model, the minimum and maximum pressures that yield the best solution are 650 N/m^2 and 700 N/m^2, respectively, while the minimum and maximum velocity are 0.5 m/s and 2 m/s. The maximum and minimum velocity and pressure are shown in Table 8.

Table 8: Maximum and Minimum Velocity (m/s) and Pressure (N/m2) for Optimal Cost

	Velocity (m/s)	Pressure (N/m^2)
Maximum	2	700
Minimum	0.5	650

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this study, the optimization model was used to minimize the cost of the clean water distribution network. The developed optimization model is characterized by non-linearity in the first term and linear in the second term. The non-zero unit variable is multiplied in the first term and its associated constraints so that to make the model linear which can be solved as a linear programming problem to find the optimal cost of distributing clean water.

The model representation of the clean water distribution system was solved using Lingo by testing different intermediate values of pressure and velocity to find out the maximum and minimum pressures and velocities that give the minimum cost of distributing clean water in a given network. The maximum and minimum pressure that gives an optimal cost for distributing clean water are 700 N/m^2 and 650 N/m^2, respectively, while the maximum and minimum velocity are 2 m/s and 0.5 m/s, respectively. Finally, the output of the resulted optimization model shows that the cost of distributing clean water was reduced by 3.48% as the obtained objective value cost compared with the original value cost of distributing clean water.

5.2 Recommendations

This study focused on the water treatment cost and hydraulic design optimization of the clean water distribution system, but additional studies in other areas of the water distribution system should be conducted. Pump/valve location and reservoir optimization are two of these areas that should be researched further to further cost optimization in clean water distribution systems. Due to the non-linearity of head loss and the discrete nature of pipe sizes, there are several complexities present in the optimizations of a clean water distribution system. However, due to the massive costs of the system, the optimal cost design of a clean water distribution system has always been a focus of research.

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APPENDICES

Appendix 1: LINDO Software codes for a resulted model

```
Minimize
72312240X1+23650440X2+33349420X3+137945310X4+67089880X5+27035Y1+315.58Y2+1.
05Y3+34400.3Y4

Subject to
3.22X1+1.44X2+6.95X3+10.95X4+499.4X5<=700
3.22X1+1.44X2+6.95X3+10.95X4+499.4X5>=650
0.69X1+0.993X2-1.56X3+2.44X4+11.1X5>=0.5
0.69X1+0.993X2-1.56X3+2.44X4+11.1X5<=2
X1+X2+X3+X4+X5>=0
27035Y1>=16210.06
315.58Y2>=1397115.717
1.05Y3>=728757.6
34400.3Y4>=2752025.417
Y1>=0
Y2>=0
Y3>=0
Y4>=0
X1>=0
X2>=0
X3>=0
X4>=0
X5>=0
```

Appendix 2: LINDO Software Output of the resulted model

```
Global optimal solution found.
  Objective value:                              0.3275587E+09
  Infeasibilities:                               0.000000
  Total solver iterations:                              2
  Elapsed runtime seconds:                           0.49

  Model Class:                                         LP

  Total variables:                    9
  Nonlinear variables:                0
  Integer variables:                  0

  Total constraints:                 19
  Nonlinear constraints:              0

  Total nonzero:                    47
  Nonlinear nonzero:                 0
```

Variable	Value	Reduced Cost
X1	0.000000	0.8357238E+08
X2	0.000000	0.4162659E+08
X3	7.260143	0.7335734E-08
X4	0.000000	0.1780059E+09
X5	1.200525	0.000000
Y1	0.5995953	0.000000
Y2	4427.136	0.000000
Y3	694054.9	0.000000
Y4	80.00004	0.000000

Row	Slack or Surplus	Dual Price
1	0.3275587E+09	-1.000000
2	50.00000	0.000000
3	0.000000	-554582.8
4	1.500000	0.000000
5	0.000000	0.1890710E+08
6	8.460668	0.000000
7	0.000000	-1.000000
8	0.000000	-1.000000
9	0.000000	-1.000000
10	0.000000	-1.000000
11	0.5995953	0.000000
12	4427.136	0.000000
13	694054.9	0.000000
14	80.00004	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	7.260143	0.000000
18	0.000000	0.000000
19	1.200525	0.000000

RESEARCH OUTPUTS

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Modeling and Optimization of Clean Water Distribution Networks

***Lucas Edward*[1*], *Mashaka James Mkandawile*[2] and *Verdiana Grace Masanja*[1]**

[1]Department of Applied Mathematics and Computational Science
Nelson Mandela African Institution of Science and Technology
P.O. Box 447, Arusha, Tanzania.
[2]College of Natural and Applied Sciences, University of Dar-es-salaam
P.O. Box 35065, Dar-es-salaam, Tanzania.
[*]Corresponding author. Email: edwardl@nm-aist.ac.tz

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Abstract. In this study, an optimal clean water distribution system cost model has been developed to find the minimum cost to distribute clean water. The model was then tested with real data collected from Ihumwa water distribution network of Dodoma city and other treatment cost data from literature to test the workability of the model. Hydraulic parameters such as head losses of the pipes, flow velocity and pipe pressure are calculated using water flow software. The resulted model was solved using LINGO software and the optimal cost of clean water distribution system was found by testing the different maximum and minimum velocity and pressure that give an optimal cost.

Keywords: Modeling, optimization model, clean water distribution

AMS Mathematics Subject Classification (2010): 46N10

1. Introduction

The first and most important public service that people demand is a consistent supply of clean and safe water. A network of pipes, tanks, pumps, valves and other hydraulic elements consists of the water distribution system. The goal is to supply quality water under specific pressure conditions and a range of specifications to customers.

The UN Water Development Report of 2018 shows that many people will be affected by drinking water shortages by 2050. This is due to increased demand for water, reduced water resources and increasing water pollution driven by spectacular population and economic increase [1].

It is complex to manage and allocate water from the multi-reservoir systems and thus requires dynamic modeling systems to obtain optimal performance [2].

The water distribution network consists of pumps, pipes, valves and node sets of a reservoir and pipe connections. A set of stationary points, some of which are nonlinear, determines the flow pressure in the network. The experimentally determined relationship between the pressure and the flow rate is associated with nonlinear conditions (i.e. the discrete pressure from one point of the pipe to the other is a flow rate/time nonlinear function).

Based on [3], the problem of classical pipe network analysis is based on finding a set of flows and pressures in the water distribution pipe network when inputs and withdrawals are known. New water systems (NWAs) are difficult to manage due to increased urbanization, changing consumer needs, old infrastructure, operating costs and lack of water resources.

The study of [4] contends that in a wide range of industrial processes and urban centers, WDNs are present. WDNs are formed by reservoirs, pipes, nodes, loops and pumps and their design can be formulated as an optimization problem. The primary goal is to minimize the cost of distributing water, which depends on pipe diameters and flow directions, in the given network.

In the study of [5] used an integrated model of Multi-Criteria Decision Making (MCDM) and Integer Linear Programming (ILP) to optimize of water loss management strategies.

The study of [6], shows a design method for a fixed flow speed, where the entire cost corresponds to that expenses of the demand flow variable. The method is built on the Granados method, which is an instinctive and practical gradient based technique. To familiarize it to regular demand, the idea of similar flow speed and volume is presented and used in a simple case study.

Effective decision-making when it comes to water and wastewater services requires a comprehensive approach that ensures the best return at an acceptable level of risk, taking into account the costs of constructing, operating, maintaining and disposing of capital assets over their lifetime [7].

In the study of [8] the optimization model known as deterministic mathematical programming proposed to determine the minimum cost of looped WDNs. The model optimization taking into account pipe lengths and a discrete set of commercially available diameters and the constraints is mass balances in nodes, energy balances in loops and hydraulic equations. The discrete optimization problem is reformulated by generalized disjunctive programming to a non-linear integer-blended programming problem (MINLP). The problem is solved by General Algebraic Modeling System (GAMS) environment.

Moreover, to exercise the modeling and optimization of water distribution one needs to have the best model that will optimize the cost of clean water distribution networks in the selected area or data of a given area. In many literatures, the available models for optimization of water distribution network have been used to optimize the cost based on hydraulic parameters, example in the study of [9] which took place in the southern area of Italy (Crotone), it uses nonlinear optimization model to optimize drinking water distribution systems in relation to the effects of climate change. PSO method was used in the study of optimization of tree Pipe networks layout and size, by [10]. [11] Used mixed integer Linear Programming (MLP) to optimize the allocation of water and the location of one more reservoirs.

Although many studies have been conducted on various cost optimizations for water distribution networks, some of these studies do not consider the cost of water treatment parameters and hydraulic parameters in the same model while formulating water cost optimization models.

The aim of this study is to formulate an optimization model to optimize the cost of clean water distribution network using [12] model for the hydraulic parameter used, i.e. pressure, velocity and flow rate and model by [13] for the case of water treatment parameters. With a slight modification of the parameters for these two models, a new model has been developed in this study to optimize the cost of a clean water distribution system.

2. Optimization model

The LP model is based on the papers of [12] and [13]. The objective function to be minimized is the price of clean water distribution networks cost, composed of pipes diameters cost and the associated water treatment parameter. The constraints are pressures and velocity limits, maintenance cost, energy cost, chemical for water treatment cost and personnel cost.

2.1. Definition of the model parameters and variables

Table 1 defines parameters and variables of the model:

Table 1: Description model parameters and variables

Symbols	Definition
L_n	The length of pipe n
CP_n	The cost of unit length of pipe n
$C(d_n)$	Represents the cost of the pipes
d_n	Diameter of pipe n
M_c	Maintenance costs
E_c	Energy cost
C_c	Chemical cost
P_C	Personnel cost
β	Maintenance cost coefficient
α	Energy cost coefficient
δ	Chemical cost coefficient
μ	Personnel cost coefficient
Y1	Number of Maintenances
Y2	Average quantity of energy

Y3	Average quantity of chemicals
Y4	Number of personnel.
P_i	Represent pressure head at node i
P_j	Represent reference node pressure
Prmax	Maximum pressure
Prmin	Minimum pressure
V_{min}	Minimum velocity
V_{max}	Maximum velocity
Q_n	flow discharge
APC	Average person cost
ACT	Average chemical cost
AQC	Average quantity of chemicals
AQE	Average Quantity of energy in KWh

2.2. Objective function.

The sum of all tube diameters and their costs and the cost of treatment must be considered in the objective function.

$$Min(C(dn) + T(m_c + E_c + C_c + P_c)) \tag{1}$$

where $C(d_n)$ represents the cost of the pipes which includes transportation and installation cost and $T(m_c, E_c, C_c, P_c)$ is the treatment cost which include maintenance cost, energy cost, chemical cost and personnel cost.

The cost of the pipes is given as in equation (2)

$$C(d_n) = \sum_{n=1}^{NP} L_n CP_n\,(d_n) \tag{2}$$

The treatment cost is given as in equation (3)

$$T(m_c + E_c + C_c + P_c) = \beta Y_1 + \alpha Y_2 + \sigma Y_3 + \mu Y_4 \tag{3}$$

Now the objective function which is the total cost of distributing clean water is given as equation (4)

$$Min(\sum_{n=1}^{NP} L_n CP_n\,(d_n) + \ \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4 \tag{4}$$

Maintenance cost coefficient (β) is given as in equation (5)

$$\beta = \frac{AMC}{NM} \tag{5}$$

Energy cost coefficient(α) is given as in equation (6)

$$\alpha = \frac{AEC}{AOF} \tag{6}$$

Chemical cost coefficient (δ) is given as in equation (7)

$$\delta = \frac{ACT}{AQC} \tag{7}$$

Personnel cost coefficient (μ) is given as in equation (8)

$$\mu = \frac{APC}{NP} \tag{8}$$

The first term of the objective function has the non-linearity property therefore is multiplied by the summations and non-zero unit variables such as X_N. The addition of all commercially available pipes gives the general objective function as in equation (9).

$$Min(\sum_{j=1}^{NPA}\sum_{n=1}^{NP} L_n CP_n(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4) \tag{9}$$

where NPA is the number of tube sizes available on the marketplace.

2.3. Constraints of the objective function

The following constraints apply to the objective function:

2.3.1. Pressure constraint

The pressure constraint for this study is upper and lower pressure that gives the optimal cost for clean water distribution which is given by equation (10) and equation (11).

$$P_i \geq Pr_{min} \tag{10}$$

$$P_i \leq Pr_{max} \tag{11}$$

where P_i is the pressure head at node i, which is given by equation (12)

$$P_i = P_j + \Delta Z - HL_j \tag{12}$$

where HL_j are head-losses from reference node and end at node i, which are calculated using Hazen-Williams formula for this study and they are given by equation (13) below.

$$HL_j = \frac{10.67 * L_n * Q_n^{1.85}}{C^{1.85} * d_n^{4.87}} \tag{13}$$

Equation (12) substituted into equation (10) and (11), respectively, to give equations (14) and (15).

$$P_j + \Delta Z - HL_j \geq Pr_{min} \tag{14}$$

$$P_j + \Delta Z - HL_j \leq Pr_{max} \tag{15}$$

The equations (14) and (15) are multiplied by summation and non-zero unit variable in the head loss to make them linear constraints as in equation (16) and (17)

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j X_{NJ} \geq Pr_{min} \tag{16}$$

$$P_j = \Delta Z - \sum_{j=1}^{NPR} HL_J X_{NJ} \leq Pr_{max} \tag{17}$$

Therefore, equation (16) and (17) are the model pressure constraints. Where NPR is the number of pipes connected to the reference node.

2.3.2. Velocity constraint

The Flow velocity constraint is given as in equation (18).

$$V_{min} \leq V_n \leq V_{max} \tag{18}$$

$V_{\min}$ is the minimum allowable flow speed in the pipe, $V_{\max}$ is the maximum allowable flow speed in the pipe and V_n is the pipe flow speed which is given by equation (19).

$$V_n = \frac{4Q_n}{\pi d_n^2} \tag{19}$$

Substituting equation (19) in equation (18) and multiplying by summation and non-zero unit variable results in equation (20) which is a velocity model constraint.

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_{NJ} \leq V_{max} \tag{20}$$

2.3.3. Maintenance constraint

The products of maintenance coefficient cost and the number of maintenance in a month is greater or equal to the average maintenance cost and it is given by equation (21).

$$\beta Y_1 \geq M \tag{21}$$

2.3.4. Energy constraint

The product of energy coefficient cost and the average quantity of electricity used is greater or equal to the average cost of electricity and it is given by equation (22).

$$\alpha Y_2 \geq E \tag{22}$$

2.3.5. Chemical constraint

The product of chemical coefficient cost and the average quantity of chemical is greater or equal to the average cost in chemical and it is given by equation (23).

$$\delta Y_3 \geq Z \tag{23}$$

2.3.6. Personnel constraint

The product of personnel coefficient cost and the number of personnel is greater or equal to average personnel cost and it is given by equation (24).

$$\mu Y_4 \geq P \tag{24}$$

2.4. Developed optimization model

The optimization model developed in this study

Objective function

$$Minimize(\sum_{j=1}^{NPA}\sum_{n=1}^{NP} L_n CP_n(d_n) + \beta Y_1 + \alpha Y_2 + \delta Y_3 + \mu Y_4 \tag{25}$$

Subject to the constraints

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j\, X_{Nj} \geq Pr_{min} \tag{26}$$

$$P_j + \Delta Z - \sum_{j=1}^{NPR} HL_j\, X_{Nj} \leq Pr_{max} \tag{27}$$

$$V_{min} \leq \sum_{j=1}^{NPA} \frac{4Q_n}{\pi d_n^2} X_{Nj} \leq V_{max} \tag{28}$$

$$\beta Y_1 \geq M \tag{29}$$

$$\alpha Y_2 \geq E \tag{30}$$

$$\delta Y_3 \geq Z \tag{31}$$

$$\mu Y_4 \geq P \tag{32}$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \tag{33}$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0 \tag{34}$$

3. Model application

The model developed in this study for analyzing the clean water distribution cost has been applied to the water treatment data for DUWASA (these are secondary data) and hydraulic data (raw data) collected from ongoing water projects in Dodoma city.

Table 2: Details of the pipe laid, IHUMWA, DODOMA

Junction	pipe size	Length(m)	Discharge(l/s)	Head-loss (m)	Velocity(m/s)
Tank- J1	300	279.8	48.9	0.37	0.69
J1-J2	300	643.9	48.9	0.85	0.69
J1-J11	250	450.5	48.9	1.44	0.99
J2-J17-J30	200	774.7	48.9	6.95	1.5
J10-J12-J16	160	768.1	48.9	21.66	2.42
J11-J10-J9	160	224.2	48.9	6.32	2.42
J9-J15-J8	160	265.3	48.9	7.48	2.42
J8-J7-J6	160	355	48.9	10.01	2.42
J6-J2	160	277.8	48.9	7.84	2.42
J2-J3	160	271.8	48.9	7.67	2.42
J19-J20	160	259	48.9	7.3	2.42
J19-J30	160	293.4	48.9	8.28	2.42
J21-J22-J20	160	634.1	48.9	17.88	2.42
J16-J14-J5	75	1353.6	48.9	1533.33	11
J5-J4	75	332.7	48.9	376.88	11
J4-J3	75	335.6	48.9	380.16	11
J5-J6	75	240.1	48.9	271.87	11
J9-J13	75	414.6	48.9	469.65	11
J27-J28	75	639.9	48.9	724.86	11
J21-J23	75	333.1	48.9	377.33	11
J23-J24	75	175.7	48.9	199.03	11
J24-J25-J26	75	345.7	48.9	391.7	11
J19-J25	75	341.4	48.9	386.73	11
J26-J27	75	411.4	49.9	466.02	11
J28-J29	75	367.4	48.9	416.18	11
TOTAL	LENGTH	10788.8			

Table 2. Shows the details of pipes laid at Ihumwa water supply network in Dodoma region. Discharges head-lose and velocity of the pipes from table 2 is calculated using computer water software.

Table 3: Average Discharge and Head-loss (HL) for the pipe used

Diameter(mm)	Pressure(N/m^2)	Discharge(Q)(l/s)	Headloss(HL)(m)	Elevation(m)
300	12	49	3.22	0
250	12	48.9	1.44	0
200	10	47.4	6.95	0
160	10	48.9	10.5	0
75	10	48.9	499.4	0

Table 3. Shows the averages of pipe discharges, head-loss; elevation of pipes calculated from table 2.

Table 4: Cost of pipe used

Diameter(mm)	length(m)	Cost per unit length(TZS)	Total Cost(TZS)
300	924	78260	72312240
250	451	52440	23650440
200	751	44230	33349420
160	3283	41190	13945310
75	5380	12680	67089880
Total length	10788	Total Cost	334347290

Table 4. Shows the cost of pipe used in the network.

Table 5: Quantities of chemicals and electricity

MONTH	CHEMICALS (Kg)	ELECTRICITY (KWh)
January	676390	4179.68
February	598725	3370.76
March	721710	4447. 21
April	595075	3895.38
May	776845	4323.99
June	678925	3903.20
July	780265	5017.82
August	731695	4794.49
September	725530	4538.19
October	647515	4960.54
November	690345	4856.01
December	669420	4838.86
Total	8292440	53126.04
Average	691036.67	4427.17

Table 5. Shows the quantity of chemicals utilized for water treatment and the quantity of electricity utilized annually. The Average Quantity of energy (AQE) is in kWh.

Table 6: Cost of energy, treatment and maintenances/ operation, personnel cost

Month	Energy cost(TZS)	Treatment cost(TZS)	Maintenance cost(TZS)	Personnel cost(TZS)
January	1437812.58	550154.3	142815.66	2739500
February	1159542.47	482400.45	138948.17	3353560
March	1529840	482400.45	162890.08	2758905
April	1340012.36	592573	162861.16	2258900
May	1487451.4	770491.33	164424.94	3590680
June	1342699.26	770491.33	164795.29	930800

July	1726131.58	779536.98	167046.38	3442000
August	1649303.26	736698.3	157902.79	2852770
September	1561136.88	933843.9	170776.23	5234190
October	1706424.45	836497.95	171417.747	4538500
November	1324500	1670468.16	922190.85	172909.84
December	1664566.2	887812.35	169732.44	3426700
Total	16765388.6	8745091.19	1946520.72	33024305
Average	1397115.717	728757.6	162210	2752025.417

Table 6. Shows the cost of energy, chemicals, maintenance and personnel cost together with their total and averages which are used to calculate cost coefficients for energy, chemicals, maintenance and personnel cost.

Table 7: Calculated cost coefficients

Maintenance cost coefficient (β)	27035
Energy cost coefficient (α)	315.58
Chemical cost coefficient (δ)	1.05
Personnel cost coefficient (μ)	34400.3

Table 7. Shows the Calculated water treatment cost coefficients.

3.1. Resulting model

Objective function.

Minimize

$$72312240X_1 + 23650440X_2 + 33349420X_3 + 13945310X_4 + 67089880X_5 + 27035Y_1 + 315.58Y_2 + 1.05Y_3 + 34400.3Y_4 \tag{35}$$

Subject to the constraints

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \leq 750 \tag{36}$$

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \geq 690 \tag{37}$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \geq 0.5 \tag{38}$$

$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \leq 3 \tag{39}$$

$$27035Y_1 \geq 16210.06 \tag{40}$$

$$315.58Y_2 \geq 1397115.717 \tag{41}$$

$$1.05Y_3 \geq 728757.6 \tag{42}$$

$$34400.3Y_4 \geq 2752025.417 \tag{43}$$

$$Y_1, Y_2, Y_3, Y_4 \geq 0 \tag{44}$$

$$X_1, X_2, X_3, X_4, X_5, \geq 0 \tag{45}$$

4. Discussion

The resulting optimisation model produced an optimisation problem which was solved with the help of LINGO (linear, interactive, discrete optimizer) software. The model shows that the unknowns for pipes and water treatment give optimal cost for clean water distribution networks. The unknown for pipes give the optimal solution under controlled minimum and maximum Pressure and velocity of water in the pipes.

4.1. Costs comparison

From table 4.1 the cost of the pipes is reduced from TZS 334,347,290 to TZS 322,664,634.4 which equals 1.5% of the total pipes cost, while the cost of treatment is reduced from TZS 5,040,138.734 to TZS 4,894,084.659 which equals 2.9% of the total treatment cost.

The total cost of distributing clean water is reduced from TZS 339,387,428.7 to TZS 327,558,700 which equals 4.4% of the total cost of distributing clean water in the given network.

Table 8: Comparison of the optimal and original costs of the decision variables of the model

	OPTIMAL COST	ORIGINAL COST	DIFFERENCE
PIPE COST	322664634.4	334347290	11682655.6
TREATMENT COST	4894084.659	5040138.734	146054.075
TOTAL COST	327558700	339387428.7	11828709.68

4.2. Maximum and Minimum pressure and velocity for optimal cost.

From the model the minimum and maximum pressure that gives the optimal solution are $690N/m^2$ and $750N/m^2$, respectively, while the minimum and maximum velocities are 0.5m/s and 3m/s, respectively, as shown in Table 9.

Table 9: Maximum and Minimum velocity pressure for optimal cost.

	velocity(m/s)	pressure(N/m^2)
Maximum	3	750
Minimum	0.5	690

5. Conclusion

The model developed in this study was used to optimize the cost of the clean water distribution network. Hydraulic data from the Dodoma region under DUWASA (Dodoma Urban water supply, Sanitation Authority) and treatment cost data from other literature are used to test the capabilities of the developed model.

The developed optimization model is characterized by non-linearity in the first term and it is linear in the second term. The non-zero unit variable is multiplied in the first term and its associated constrains in order to make the model linear which can be solved as a linear programming problem to find the optimal cost of distributing clean water.

The model representation of the delivery system for clean water was solved using LINGO software by testing different maximum and minimum pressure and velocities that gives minimum cost of distributing clean water in a given system. The maximum and minimum pressure that gives an optimal cost for distributing clean water are 700N/m^2 and 650N/m^2, respectively, while the maximum and minimum velocity are 3m/s and 0.5m/s, respectively.

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Poster Presentation

Modeling and Optimization of Clean Water Distribution Networks

Lucas Edward. Master's in Mathematical and Computer Sciences and Engineering . Supervisors: Prof. *Verdiana G. Masanja*; & *Dr. Mashaka J. Mkandawile*

Background

- A network of pipes, reservoirs, pumps, valves and other hydraulic components make up the water distribution system. The goal is to provide quality water to consumers under specific pressure levels and a variety of requirements.
- Increased population, the development of new technologies, increased user awareness and the environmental sustainability needs of clean water distribution must be increasingly effective. These goals encourage the scientific community to propose innovative approaches to addressing new challenges through the use of quantitative predictive tools (Boano *et al*., 2015).

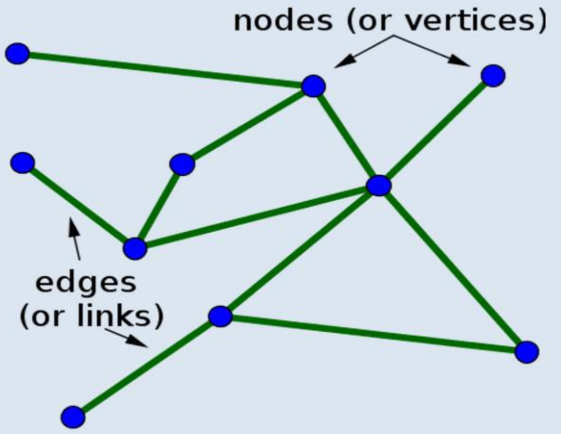

Figure 1: Simple network

Objectives

- **General Objective:** The general objective of the research is to formulate a linear programming model that proposes the distribution of clean water at a minimum cost in a given network.
- **Specific Objectives**
  - To develop a mathematical model that minimizes the cost of clean water distribution networks.
  - To analyze the appropriate pressure of the pipe, the flow velocity in the network will optimize the cost of the distribution of clean water.
  - Find the minimum cost for the distribution of clean water.

Materials and Methods

Objective function

Minimize

$$72312240X_1 + 23650440X_2 + 33349420X_3 + 13945310X_4 + 67089880X_5 + 27035Y_1 + 315.58Y_2 + 1.05Y_3 + 34400.3Y_4 \quad (1)$$

Subject to the constraints

$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \leq 700 \quad (2)$$
$$3.22X_1 + 1.44X_2 + 6.95X_3 + 10.95X_4 + 499.4X_5 \geq 650 \quad (3)$$
$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \geq 0.5 \quad (4)$$
$$0.69X_1 + 0.993X_2 - 1.56X_3 + 2.44X_4 + 11.1X_5 \leq 2 \quad (5)$$
$$27035Y_1 \geq 16210.06 \quad (6)$$
$$315.58Y_2 \geq 1397115.717 \quad (7)$$
$$1.05Y_3 \geq 728757.6 \quad (8)$$
$$34400.3Y_4 \geq 2752025.417 \quad (9)$$
$$Y_1, Y_2, Y_3, Y_4 \geq 0 \quad (10)$$
$$X_1, X_2, X_3, X_4, X_5, \geq 0 \quad (11)$$

Results

	ORIGINAL COST	OPTIMAL COST	DIFFERENCE
PIPE COST	334,347,290	322,664,634.4	11,682,655.6
TREATMENT COST	5,040,138.734	4,894,084.659	146,054.075
TOTAL COST	339,387,428.7	327,558,700	11,828,709.68

	velocity(m/s)	pressure(N/m^2)
Maximum	2	700
Minimum	0.5	650

Conclusion

The output of the resulted optimization model shows that the cost of distributing clean water was reduced by 3.48% as the obtained objective value cost compared with the original value cost of distributing clean water.