

2019-10

Mathematical Model for the Transmission Dynamics of Bovine Tuberculosis in Human and Livestock with Control Strategies

Sabini, Theresia Shirima

International Journal of Advances in Scientific Research and Engineering (ijasre)

10.31695/IJASRE.2019.33535

Downloaded from Nelson Mandela-AIST's institutional repository

Mathematical Model for the Transmission Dynamics of Bovine Tuberculosis in Human and Livestock with Control Strategies

Theresia Shirima Sabini¹, Nkuba Nyerere², Jacob Ismail Irunde³ and Dmitry Kuznetsov⁴

^{1,4}School of Computational and Communication Science and Engineering, The Nelson Mandela African Institution of Science and Technology, P.O.Box 447, Arusha, Tanzania

³Department of Mathematics, Mkwawa University College of Education, P.O. Box 2513 Iringa, Tanzania

²Department of Mathematics, Informatics and Computational Sciences, Sokoine University of Agriculture, P.O.Box 3038 Morogoro, Tanzania

Abstract

A deterministic mathematical model for bovine tuberculosis (bovine TB) in humans and livestock is formulated and used to assess the effectiveness of dairy products inspection, human treatment and quarantine of infected livestock as the control of the disease transmission. The computed effective reproduction number shows that the disease can be cleared from the population if Re is less than unity and it persists if Re is greater than unity. It means that if $Re < 1$ the disease-free equilibrium is asymptotically stable which means the disease can be cleared from the population and endemic when $Re > 1$ which implies the disease persists in the population. Numerical analysis was carried out to investigate how the controls can help to minimize the spread of the disease.

Key Words: Bovine tuberculosis, Dairy products, Mathematical model, Livestock.

1. INTRODUCTION

Tuberculosis (TB) is a bacterial disease which caused by mycobacterium tuberculosis. The disease is listed among the diseases that kills large number of people around the world. Based on the global TB report of 2018, it is estimated that around 1.3 million people die annually due to TB of which 1.2-1.4 million people are HIV negative while 300,000 people are HIV positive [1]. Globally, it is estimated that about 10 million which range between 9.0-11.1 million people are exposed to TB where, 5.8 millions are men, 3.2 millions are women and 1.0 million are children [1]. Africa is leading by having large percentage of individuals who develops into active TB, followed by India, China, Indonesia, the Philippines, Pakistan, Nigeria and Bangladesh: (27%), (9%), (8%), (6%), (5%), (4%) and (3%) respectively [1]. Although TB seems to be very important health problem worldwide, extra-pulmonary TB is not given attention and this makes difficult to attain the goals of ending TB around 2020. This attracts many researchers to find out about TB especially the extra pulmonary TB especially bovine tuberculosis.

Bovine tuberculosis (bovine TB) is a bacterial disease which have great impacts in agricultural sector [2]. It is animal's disease which affects both wild and domesticated animals such as cattle, goats, sheep, pig and pet animals [3]. Bovine TB is transmitted through inhalation of aerosols, sharing of grazing areas, water sources, and several contacts between susceptible and infected animals [3]. Though bovine TB plays very important role in animal's health and human's health it has given very little attention [4]. The disease have great negative impacts on economy of many countries since agriculture sector play big role in the growth of economy [5], [6]. However, many people loses their job especially livestock keepers and those who works in beef sectors due to the outbreak of the disease [4], [5], [7]. Since bovine TB causes economic, social and health problems worldwide it attracts much attention for many scholars to control the transmission of the disease [8]–[10].

It is not easy to diagnose animals with Bovine TB at once because an animal can be in latent state for long time and sometimes it can be diagnosed in old age or during the stress [11]. Some associated symptoms of bovine TB includes reducing productivity, gradual loose of weight, coughing, weakness and some animal's lymph nodes enlarge which may end up bursting [5], [11], [12]. Bovine TB is endemic in many developing countries while in developed countries they have controlled the disease in large extent

[13]. In non-industrialized countries the disease seems to be endemic due to hygienic situation, economy and their living style [13].

Bovine TB is transmitted to human being through consumption of infected or contaminated dairy product, inhalation of aerosols and through scratches when someone contact infected dairy products [3]. However, it is difficult to control the spread of the disease especially in developing countries when the disease enters wild animals because many livestock keepers graze their animals in the bush where they interact with wild animals [14]. The fact that both domestic animals and wild animals share watering areas and grazing areas complicates the dynamics of the disease and consequently makes it difficult to control it in both animals and humans.

Various mathematical studies such as Liu et al., [9], Augusto et al., [5], Hassan et al., [15] have been conducted to understand the dynamics of bovine TB and suggested ways of controlling the disease. Many of these studies recommended test and slaughter to be the best way of controlling the transmission. However, test and slaughter is almost impossible in developing countries because it needs farmers willingness and government readiness for compensation of farmers. More importantly, the fact that the environment can host mycobacterial bovis is not given much attention in most of these studies. Furthermore, international organizations such as, World Health organization (WHO), World Organization for Animal Health (OIE), Food and Agriculture Organization (FAO) discussed how reduce the spread of bovine TB up to 2030 [16]. They suggested different approaches like “one health approach, together we can save lives and secure live hoods” as strategies to control the spread of bovine TB but the problem is still there. However, very few of them highlighted the environment as a hindering factor for the control of the disease. This study uses mathematical model to assess the impacts of environment on the transmission of bovine TB to livestock and humans.

1.1 Model Formulation

In this section, a mathematical model for the dynamics of bovine TB in human and animal populations is developed. Human population is divided into susceptible class S_h exposed class E_h and infected class I_h . Animals' population is divided into susceptible class S_a , exposed class E_a and infectious class I_a . Susceptible individuals S refers to the individuals who are likely to be infected with the disease. Exposed individuals or latent class E refers to the individuals who are exposed into the disease but they do not have any symptoms of the disease and they cannot spread the disease into others. Lastly, the infectious class these are individuals who are infected with the disease this is when they developed symptoms of the disease and at this stage an individual can spread the disease into others.

Susceptible class S_h increases due to birth and recovery at the rates Λ_h and Π_h respectively, and decreases when they acquire disease and become latent after come into contact with infectious humans and by consuming infectious dairy products from infectious animals at a rate X_h .

Exposed human class E_h increases when susceptible class acquire bovine TB and moves into the class at the rate X_h . However, they decreases by dying naturally at the rate μ_h and when their TB status progress to active stages at the rate γ_h by either endogenous reactivation or exogenous reinfection

Infectious class I_h increases when individual from exposed class progress into infectious class at a rate γ_h and decreases due to natural mortality rate μ_h and disease induced death rate of α_h .

Susceptible animals S_a increases through birth and recovery at a rate Λ_a and Π_a respectively. They acquire bovine tuberculosis latent infection following contacts with infectious humans and animals, and after consuming infectious dairy products during breastfeeding at a rate X_a .

Exposed animals E_a increase following latent infection of susceptible animals S_a at a rate X_a . However, they decreases due to natural death μ_a and by progressing into infectious class at a rate γ_a .

Infectious animals I_a increase at a rate γ_a and diminish following quarantine of infected animals τ_a , natural and disease mortality at the rates α_a ma respectively.

Dairy products are produced by infectious animals at a rate ρ where, susceptible human consumed the products at rate β_3 , susceptible livestock consume dairy products at rate β_6 through breastfeeding from infectious livestock and the remaining products leak at rate ω . However the rate of producing dairy products decreases following quarantine of infected animals and inspection of dairy products at the rates τ_a , and ϵ respectively.

1.2 Model Flow Diagram

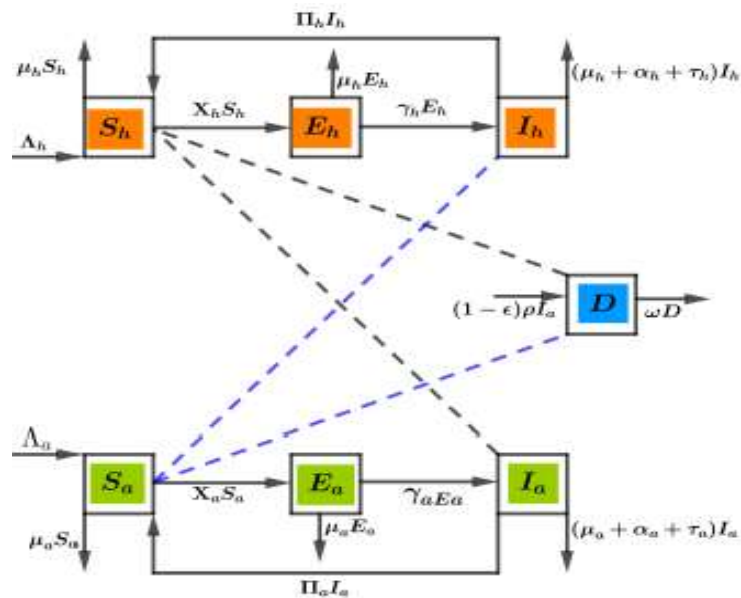


Figure 1: Model Flow Diagram

From the model flow diagram we have the following system of differential equations;

$$\frac{dS_h}{dt} = \Lambda_h + \Pi_h - \left(\frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\epsilon)\beta_3 D}{N_h} \right) S_h - \mu_h S_h \tag{1.1a}$$

$$\frac{dE_h}{dt} = \left(\frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\epsilon)\beta_3 D}{N_h} \right) S_h - (\gamma_h + \mu_h) E_h \tag{1.1b}$$

$$\frac{dI_h}{dt} = \gamma_h E_h - (\alpha_h + \mu_h + \tau_h) I_h \tag{1.1c}$$

$$\frac{dS_a}{dt} = \Lambda_a + \Pi_a - \left(\frac{(1-\tau_h)\beta_4 I_h + (1-\tau_a)\beta_5 I_a + (1-\epsilon)\beta_6 D}{N_a} \right) S_a - \mu_a S_a \tag{1.1d}$$

$$\frac{dE_a}{dt} = \left(\frac{(1-\tau_h)\beta_4 I_h + (1-\tau_a)\beta_5 I_a + (1-\epsilon)\beta_6 D}{N_a} \right) S_a - (\gamma_a + \mu_a) E_a \tag{1.1e}$$

$$\frac{dI_a}{dt} = \gamma_a E_a - (\alpha_a + \mu_a + \tau_a) I_a \tag{1.1f}$$

$$\frac{dD}{dt} = (1-\epsilon)\rho I_a - (\omega + \theta) D \tag{1.1g}$$

Subject to their initial conditions;

$$S_h(0) > 0, E_h(0) > 0, I_h(0) > 0, S_a(0) > 0, S_a(0) > 0, E_a(0) > 0, I_a(0) > 0 \text{ \& } D(0) > 0$$

Where $X_h = \frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\epsilon)\beta_3 D}{N_h}$, and $X_a = \frac{(1-\tau_h)\beta_4 I_h + (1-\tau_a)\beta_5 I_a + (1-\epsilon)\beta_6 D}{N_a}$ given τ_h is humans treatment rate, τ_a is quarantine of infected animals and ϵ is dairy products inspection rate.

Table1: Model Variable descriptions

| Symbol | Descriptions |
|--------|--------------------------------------------|
| S_h | Number of susceptible human at time t. |
| E_h | Number of exposed human at time t. |
| I_h | Number of infected human beings at time t. |
| S_a | Number of susceptible animals at time t. |
| E_a | Number of Exposed animals at time t. |
| I_a | Number of infected animals at time t. |
| D | Dairy products at time t |

Table2: Parameters descriptions

| Parameter | Descriptions |
|--------------------------------|---------------------------------------------------------------------------------------|
| Λ_h | Human recruitment rate. |
| μ_h | Human natural death. |
| μ_a | Animals' natural death. |
| γ_h | Progression rate from E_h to I_h |
| γ_a | Progression rate from E_a to I_a |
| α_h | Humans mortality rate due to disease |
| α_a | Animals mortality rate due to disease |
| ρ | Rate of producing infectious dairy products |
| ε | Rate of inspecting dairy products |
| τ_h | Infected humans treatment rate |
| τ_a | Quarantine rate of Infected animals |
| Π_h | Human recovery rate |
| Π_a | Animals recovery rate |
| β_1, β_2 & β_3 | Humans infection rates from infected humans, animals and dairy products respectively |
| β_4, β_5 & β_6 | Animals infection rates from infected humans, animals and dairy products respectively |
| ω | Decay rate of unused dairy products |
| N_h | Human total populations |
| N_a | Animals total populations |

1.3 Analysis of the Model

1.4 Invariant Region

Invariant region shows the feasibility of the model solutions. To find the invariant region, we denote humans and livestock populations by N_h and N_a respectively.

Beginning with human population we have:

$$N_h = S_h + E_h + I_h,$$

$$\frac{dN_h}{dt} \leq \Lambda_h - \mu_h S_h \quad (1.2)$$

Solving inequality (1.2), we have

$$N_h \leq \frac{\Lambda_h}{\mu_h} + \left(N_h(0) - \frac{\Lambda_h}{\mu_h} \right) e^{-\mu_h t} \leq \frac{\Lambda_h}{\mu_h},$$

$$\lim_{t \rightarrow \infty} \left(N_h(0) - \frac{\Lambda_h}{\mu_h} \right) e^{-\mu_h t} \rightarrow 0,$$

Then,
$$0 \leq N_h \leq \frac{\Lambda_h}{\mu_h}. \quad (1.3)$$

The total animals population is given by;

$$N_a = S_a + E_a + I_a,$$

$$\frac{dN_a}{dt} \leq \Lambda_a - \mu_a S_a \quad (1.4)$$

Solving equation (1.4) we obtain,

$$N_a \leq \frac{\Lambda_a}{\mu_a} + \left(N_a(0) - \frac{\Lambda_a}{\mu_a} \right) e^{-\mu_a t} \leq \frac{\Lambda_a}{\mu_a},$$

$$\lim_{t \rightarrow \infty} \left(N_a(0) - \frac{\Lambda_a}{\mu_a} \right) e^{-\mu_a t} \rightarrow 0,$$

Then,
$$0 \leq N_a \leq \frac{\Lambda_a}{\mu_a}. \quad (1.5)$$

For dairy products we have;

$$\frac{dD}{dt} \leq (1 - \varepsilon)\rho I_a - (\omega + \theta)D,$$

Since $I_a \leq \frac{\Lambda_a}{\mu_a}$

Then, $\frac{dD}{dt} \leq (1 - \varepsilon)\frac{\Lambda_a}{\mu_a}\rho - (\omega + \theta)D,$ (1.6)

From (1.6) we have

$$D(0) \leq \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho + \left(D(0) - \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho\right) e^{-(\omega+\theta)t},$$

$$\lim_{t \rightarrow \infty} D(t) \leq \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho e^{-(\omega+\theta)t} \rightarrow \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho,$$
 (1.7)

Therefore the model (1.1) is positive invariant in the region;

$$Z = \left\{ (S_h, E_h, I_h, S_a, E_a, I_a, D) \in R_+^7 : 0 \leq N_h \leq \frac{\Lambda_h}{\mu_h}; 0 \leq N_a \leq \frac{\Lambda_a}{\mu_a}; 0 \leq D \leq \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho \right\}$$
 (1.8)

1.5 Positivity of solution

Theorem 1: Let the initial values for the state variables for the model (1.1) be $S_h(0) \geq 0, E_h(0) \geq 0, I_h(0) \geq 0, S_a(0) \geq 0, E_a(0) \geq 0, I_a(0) \geq 0$ & $D(0) \geq 0$ then the solutions of the model (1.1) are positive $\forall t \geq 0$.

Proof: Let's consider the equations (1.1a) of the model system (1.1) which is

$$\frac{dS_h}{dt} = \Lambda_h + \Pi_h - \left(\frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\varepsilon)\beta_3 D}{N_h}\right) S_h - \mu_h S_h,$$

$$\frac{dS_h}{dt} \geq - \left(\frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\varepsilon)\beta_3 D}{N_h} + \mu_h\right) S_h,$$
 (1.9)

Solving differential equation (1.9) and apply initial conditions we get;

$$S_h(t) \geq S_h(0) e^{\int_0^t - \left(\frac{(1-\tau_h)\beta_1 I_h(s) + (1-\tau_a)\beta_2 I_a(s) + (1-\varepsilon)\beta_3 D(s)}{N_h(s)} + \mu_h\right) ds},$$
 (1.10)

Then, $S_h(t) \geq 0, \forall t \geq 0.$

By following the same procedure to the remaining equations of the model system (1.1) we obtain the following;

$$E_h(t) \geq E_h(0) e^{-(\gamma_h + \mu_h)t}$$
 (1.11)

$$I_h(t) \geq I_h(0) e^{-(\alpha_h + \mu_h + \tau_h)t}$$
 (1.12)

$$S_a(t) \geq S_a(0) e^{\int_0^t - \left(\frac{(1-\tau_h)\beta_4 I_h(s) + (1-\tau_a)\beta_5 I_a(s) + (1-\varepsilon)\beta_6 D(s)}{N_a(s)} + \mu_a\right) ds}.$$
 (1.13)

$$D(t) \geq D(0) e^{-(\omega + \theta)t}$$
 (1.14)

Therefore solutions of the model satisfies the conditions hence a model is positive and bounded since;

$$S_h(0) \geq 0, E_h(0) \geq 0, I_h(0) \geq 0, S_a(0) \geq 0, E_a(0) \geq 0, I_a(0) \geq 0 \text{ & } D(0) \geq 0, \forall t \geq 0$$

1.6 Disease free equilibrium

The disease free equilibrium point is the state when there is no disease in the population. When there is no bovine TB in human and animal populations, the disease free equilibrium is given by:

$$DF^0 = (S_h, E_h, I_h, S_a, E_a, I_a, D) = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_a}{\mu_a}, 0, 0, 0\right)$$
 (1.15)

1.7 Endemic Equilibrium of Model With Control

Stability of an endemic equilibrium of model with control is analyzed using force of infection due to complexity of the model equations. By equating the right side of the model system (1.1) equals to zero and solve simultaneously we obtain;

$$S_h^* = \frac{\Lambda_h(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(X_h^* + \mu_h) - \Pi_h X_h^* \gamma_h}, \tag{1.16}$$

$$E_h^* = \frac{(\alpha_h + \mu_h + \tau_h)\Lambda_h X_h^*}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(X_h^* + \mu_h) - \Pi_h X_h^* \gamma_h}, \tag{1.17}$$

$$I_h^* = \frac{\Lambda_h \gamma_h X_h^*}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(X_h^* + \mu_h) - \Pi_h X_h^* \gamma_h}, \tag{1.18}$$

$$S_a^* = \frac{\Lambda_a(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a}, \tag{1.19}$$

$$E_a^* = \frac{(\alpha_a + \mu_a + \tau_a)\Lambda_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a}, \tag{1.20}$$

$$I_a^* = \frac{\Lambda_a \gamma_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a} \text{ and} \tag{1.21}$$

$$D^* = \frac{(1-\varepsilon)\gamma_a \Lambda_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - (\omega + \theta)\Pi_a X_a^* \gamma_a} \tag{1.22}$$

1.8 Effective Reproduction Number R_e

To determine effective reproduction number when control parameters are administered, we use next generation approach by [14]. The control strategies are effective when the effective reproduction number $R_e < 1$ and they are ineffective if reproduction number $R_e > 1$. If infectious and transfer terms are denoted by H_i and P_i respectively, then the effective reproduction number R_e is given as the maximum eigenvalue. That is;

$$R_e = \rho(HP)^{-1} \tag{1.23}$$

Where, $H = \frac{\partial H_i}{\partial X_j}(DF^0)$ and $P = \frac{\partial P_i}{\partial X_j}(DF^0)$

From the model system (1.1) we therefore computed the effective reproduction number R_e and obtain the following;

$$R_e = \frac{1}{2} \left(\frac{(1-\tau_a)(\omega + \theta)\beta_5 \gamma_a + (1-\varepsilon)^2 \beta_6 \rho \gamma_a}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(\omega + \theta)} + \frac{(1-\tau_h)\beta_6 \gamma_h}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)} + \sqrt{\left(\frac{(1-\tau_a)(\omega + \theta)\beta_5 \gamma_a + (1-\varepsilon)^2 \beta_6 \rho \gamma_a}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(\omega + \theta)} - \frac{(1-\tau_h)\beta_6 \gamma_h}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)} \right)^2 + 4rt} \right) \tag{1.24}$$

Where; $rt = \frac{(1-\tau_a)(1-\tau_h)(\omega + \theta)\beta_2 \beta_4 \gamma_h \gamma_a + (1-\tau_h)(1-\varepsilon)^2 \rho \beta_3 \beta_4 \gamma_h \gamma_a}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(\omega + \theta)}$

The effective reproduction number decreases as we increases human’s treatment rate τ_h , animals quarantine rates τ_a and dairy products inspection ε . The transmission of the disease can be minimized if these control strategies are used effectively and this is when effective reproduction number R_e become less than a unit.

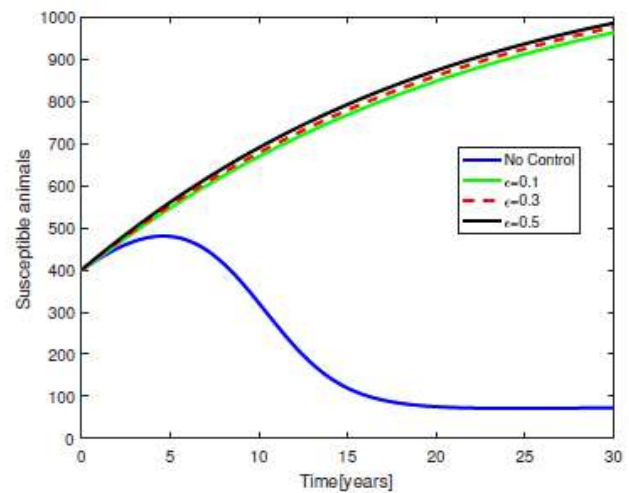
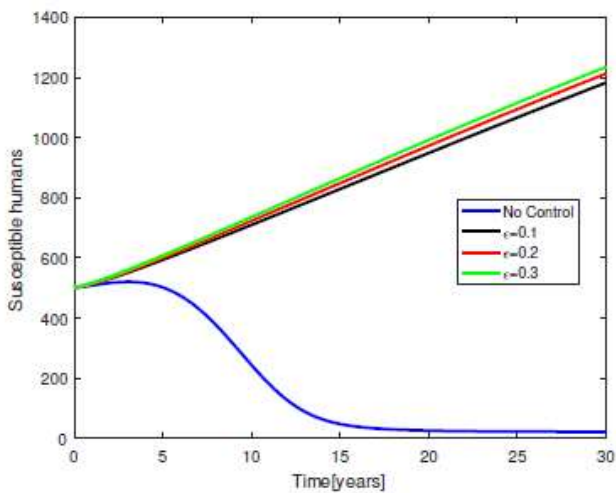
2. Numerical Simulation

We performed a numerical simulation to discuss how bovine TB can be eliminated from the population using different strategies such as dairy products inspections, treatment of infected humans and quarantine of infected animals. In numerical simulation we used some parameters from the related studies and some are estimated. Some of the parameters are estimated because the disease is neglected and there is no surveillance data despite the negative impacts it brings to the health of people as well as to the economy.

Table 3: Summarized the parameter values

| Parameter | Descriptions | Values/ y^{-1} | Source |
|---------------|---------------------------------------------|------------------|-----------|
| γ_a | Progression rate from E_a to I_a | 0.18 | [11] |
| μ_a | Animals’ natural death. | 0.05 | Estimated |
| γ_h | Progression rate from E_h to I_h | 0.18 | Estimated |
| α_h | Humans mortality rate due to disease | 0.139 | [6] |
| α_a | Animals mortality rate due to disease | 0.12 | Estimated |
| μ_h | Human natural death. | 0.01 | [6] |
| ρ | Rate of producing infectious dairy products | 0.69 | Estimated |
| ε | Rate of inspecting dairy products | 0.5 | Estimated |
| τ_h | Infected humans treatment rate | 0.58 | [6] |

| | | | |
|-----------|------------------------------------------------------|-------|-----------|
| τ_a | Quarantine rate of Infected animals | 0.65 | Estimated |
| Π_h | Human recovery rate | 0.01 | [6] |
| Π_a | Animals recovery rate | 0.013 | [11] |
| β_1 | Humans infection rates from infected humans | 0.35 | Estimated |
| β_4 | Animals infection rates from infected humans | 0.25 | Estimated |
| ω | Decay rate of unused dairy products | 0.4 | Estimated |
| β_2 | Humans infection rates from infected animals | 0.55 | Estimated |
| β_5 | Animals infection rates from infected animals | 0.6 | Estimated |
| β_3 | Humans infection rates from infected dairy products | 0.999 | Estimated |
| β_6 | Animals infection rates from infected dairy products | 0.34 | Estimated |

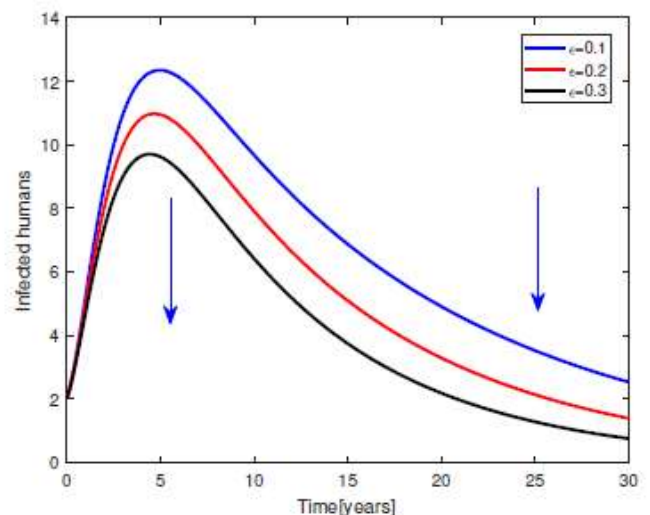
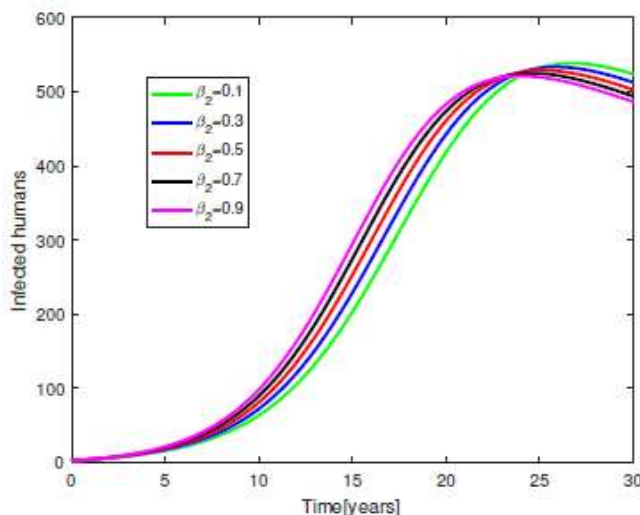


(a) The effects of dairy products inspection on susceptible humans

(b) The effects of dairy products inspection on susceptible animals

Figure 2: The impacts of control parameters on susceptible humans and animals

Figure 2 shows the impacts of the inspection of dairy products on the transmission of bovine TB in humans and animals. The results shows how inspection of dairy products can helps to reduce the spread of the disease to both humans and animals since susceptible classes are increasing. The blue line graph indicates susceptible humans and animals before inspection of dairy products.

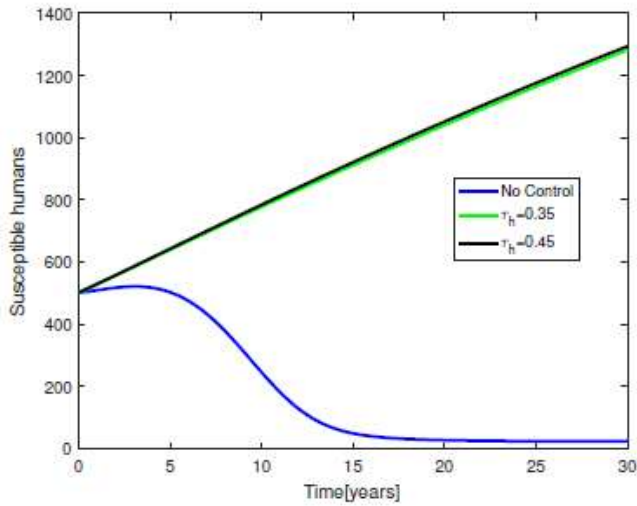


(a) Infected humans before controls

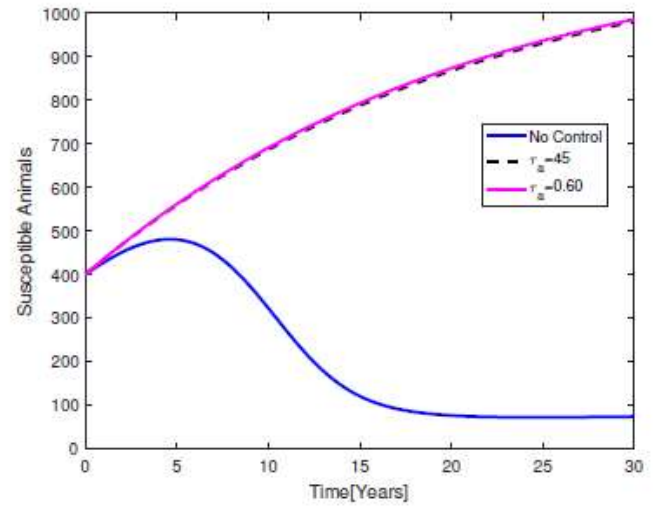
(b) Infected humans after dairy products inspection

Figure 3: Infected humans before and after controls

Infected humans class increases before any controls strategies as shown in Figure 3(a). However the class of infected humans decreases after inspection of dairy products as shown in Figure 3(b). Infected individual class decreases the more we inspect dairy products as we can see from 10% to 30%.



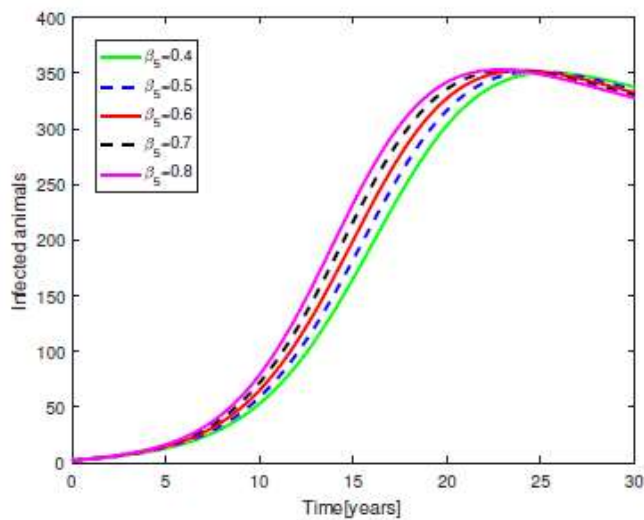
(a) Effect of human treatment on susceptible humans



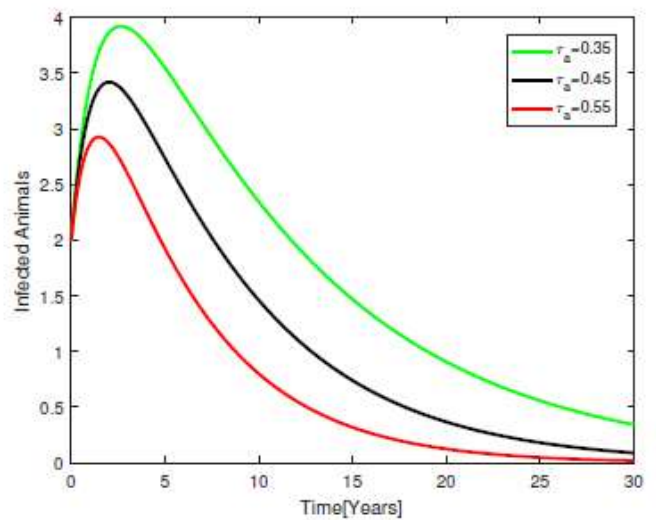
(b) Effect of animals quarantine on susceptible animals

Figure 4: The effects of quarantine infected animals rate into susceptible humans and animals

Figure 4 shows that if infected humans and animals treated once they diagnosed having bovine TB the transmission of the disease into humans and animals decrease hence susceptible individual classes increasing.



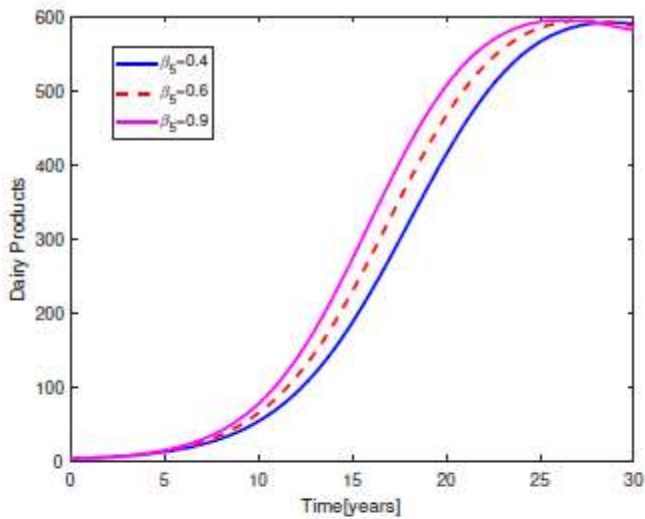
(a) Infected animals before animals quarantine



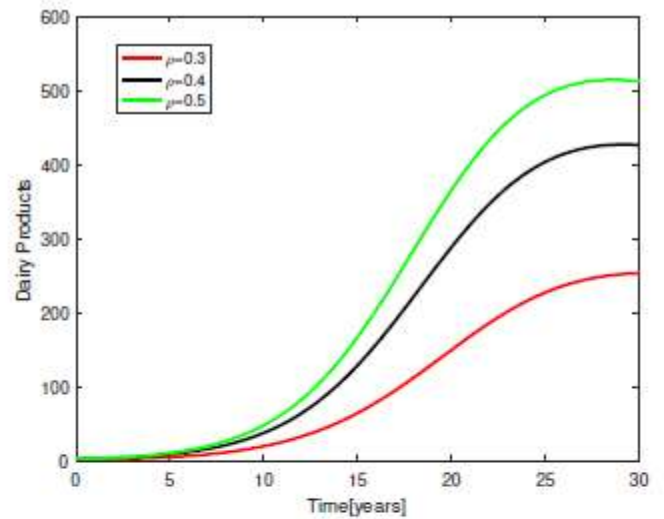
(b) Infected animals after animals quarantine

Figure 5: Infected animals before and after quarantine of infected animals

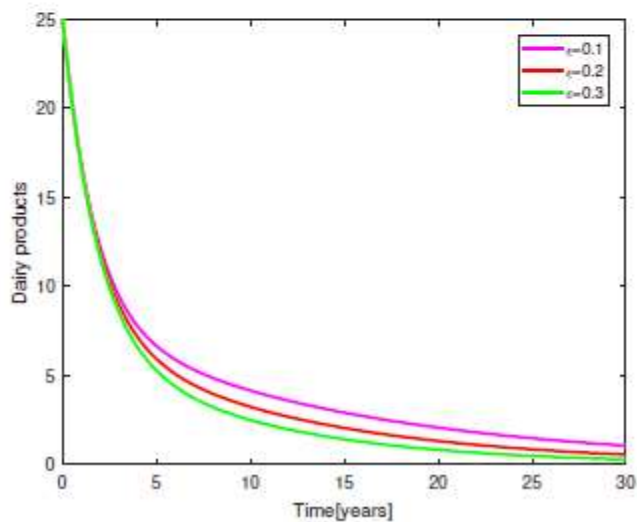
Before quarantine of infected individuals animal class were increasing as shown in Figure 5(a). After quarantine of infected individuals the infected individual class started decreasing as shown in Figure 5(b). This shows that the quarantine of infected individuals helps to reduce the spread of the disease.



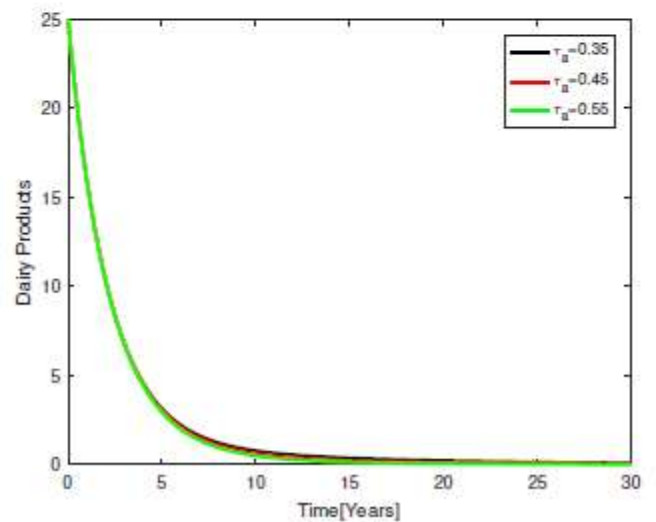
(a) Production of dairy products before controls



(b) Production of dairy products before controls



(c) Production of dairy products after controls



(d) Production of dairy products after controls

Figure 6: The impacts of control parameters on the production of dairy products

Figure 6(a) and (b) shows how production of infected dairy products was increasing before control, while Figure 6(b) and (c) shows that, after quarantine of infected animals and inspection of dairy products the production decreases. This helps to minimize the rate of consuming infected dairy products hence the spread of the disease also decreases.

3. CONCLUSION AND RECOMMENDATION

3.1 Conclusion

A deterministic model of bovine TB for human and livestock is formulated to assess the impacts of control strategies on the spread of the disease. Control strategies that have been discussed includes human treatment, quarantine of infected animals as well as dairy products inspection. The results shows that, the treatment of infected humans and quarantine of infected animals helps at large to decreases the spread of the disease. Also the inspection of dairy products helps to reduce the production of infected dairy products. If these way are followed and implemented effectively we are not going to slaughter infected livestock anymore.

3.2 Recommendations

This study recommends that, public health workers to conduct more research in order to have surveillance data since Tanzania is mentioned among the countries which do have enough information. Veterinary they have to inspect dairy products so that individuals to avoid the consumption of contaminated meat or milk, also screen of livestock several time to make sure that livestock and human are safe and free from the disease infection. Policy makers they have to form a policy that can be helpful for the country in terms of controlling disease transmission. Also further studies should be done to assess the transmission dynamics

between wild animals, domestic animals and humans and determine ways of eradicating the disease. Also cost effective analysis should be carried out to determine the ways which have low cost and be affordable for all nations regardless the economy of the country.

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my supervisors (Dr. Jacob Ismail Irunde, Prof. Dmitry Kuznetsov and Mr. Nkuba Nyerere) for their supervision as well as Dr. Shirima who advised me and supported me during the study. I give a special thanks to African Development Bank (AfDB) for funding the study because without their support I could not reach this far.

REFERENCES

1. W. H. Organization and others, *Global tuberculosis report 2018*. World Health Organization, 2018.
2. A. Moustakas and M. R. Evans, "A big-data spatial , temporal and network analysis of bovine tuberculosis between wildlife (badgers) and cattle," 2016.
3. M. J. Thoen, C. O., Steele, J. H., and Gilsdorf, "Mycobacterium Bovis Infection in Animals and Humans." 2008.
4. C. O. Thoen, B. Kaplan, T. C. Thoen, M. J. Gilsdorf, and J. A. Shere, "Special Article Zoonotic Tuberculosis . A Comprehensive One Health Approach Pathogenesis," pp. 159–165, 2016.
5. F. B. Agosto, S. Lenhart, A. B. Gumel, and A. Odoi, "Mathematical analysis of a model for the transmission dynamics of bovine tuberculosis," *Math. Methods Appl. Sci.*, vol. 34, no. 15, pp. 1873–1887, 2011.
6. G. Ghielmetti *et al.*, "Epidemiological tracing of bovine tuberculosis in Switzerland , multilocus variable number of tandem repeat analysis of Mycobacterium bovis and Mycobacterium caprae," pp. 1–15, 2017.
7. H. C. J. Godfray *et al.*, "A restatement of the natural science evidence base relevant to the control of bovine tuberculosis in Great Britain," *Proc. R. Soc. B Biol. Sci.*, vol. 280, no. 1768, pp. 1–18, 2013.
8. A. M. Kilale, "Mycobacteria in northern Tanzania: Exposure and risk of disease among agropastoralists and programmatic challenges in investigation of re-treatment cases," p. 122, 2016.
9. S. Liu, A. Li, X. Feng, X. Zhang, and K. Wang, "A Dynamic Model of Human and Livestock Tuberculosis Spread and Control in Urumqi, Xinjiang, China," *Comput. Math. Methods Med.*, vol. 2016, 2016.
10. I. Van Rhijn, J. Godfruid, A. Michel, and V. Rutten, "Bovine tuberculosis as a model for human tuberculosis: advantages over small animal models," *Microbes Infect.*, vol. 10, no. 7, pp. 711–715, 2008.
11. D. F. Ramos, P. E. A. Silva, and O. A. Dellagostin, "Diagnosis of bovine tuberculosis: review of main techniques," *Braz. J. Biol.*, vol. 75, no. 4, pp. 830–837, 2015.
12. A. S. Hassan, S. M. Garba, A. B. Gumel, and J.-S. Lubuma, "Dynamics of Mycobacterium and bovine tuberculosis in a human-buffalo population," *Comput. Math. Methods Med.*, vol. 2014, 2014.
13. A. Ssematimba, J. Jores, and J. C. Mariner, "Mathematical modelling of the transmission dynamics of contagious bovine pleuropneumonia reveals minimal target profiles for improved vaccines and diagnostic assays," *PLoS One*, vol. 10, no. 2, pp. 1–15, 2015.
14. B. Z. Katala *et al.*, "Bovine tuberculosis at the human-livestock-wildlife interface: Is it a public health problem in Tanzania? A review," *Onderstepoort J Vet Res*, vol. 79, no. 2, pp. 1–8, 2012.
15. A. S. Hassan, S. M. Garba, A. B. Gumel, and J. M. Lubuma, "Dynamics of Mycobacterium and bovine tuberculosis in a Human-Bufferalo Population," vol. 2014, 2014.
16. W. H. Organization and others, "Global tuberculosis report 2017: World Health Organization; 2017," *Back to cited text*, no. 1, 2016.