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Modelling and optimisation of water loss management strategies in a water distribution system: a case of Moshi Urban Water Supply and Sanitation Authority (MUWSA)

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MODELLING AND OPTIMISATION OF WATER LOSS MANAGEMENT STRATEGIES IN A WATER DISTRIBUTION SYSTEM: A CASE OF MOSHI URBAN WATER SUPPLY AND SANITATION AUTHORITY (MUWSA)

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A Dissertation Submitted in Partial Fulfilment of the Requirements for the Degree of Master's in Mathematical and Computer Sciences and Engineering of the Nelson Mandela African Institution of Science and Technology

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ABSTRACT

Water loss in water distribution systems (WDS) is a serious problem in developing countries. A lot of water is lost on its way from the sources before reaching the consumers due to leakage, illegal use, and theft of infrastructures among others. The effect of water loss in the WDS includes reduction of revenue, water shortage, disruption of water quality, and inflation of operation and maintenance cost of the water authorities. The control of water loss in the WDS is closely dependent on the commitment of the decision-makers, the strategies used and budget set for water loss management (WLM).

This study presents a combined model of Multi-Criteria Decision Making (MCDM) and Integer Linear Programming (ILP) methods which may help decision-makers to prioritise and select the best strategies for WLM. The MCDM family methods; the MAVT, SMARTER, SAW, and COPRAS were used to evaluate and prioritize the strategies, while ILP was used to select the best strategies. Additionally, the study compared the SAW and COPRAS methods in prioritising and selecting the strategies. The data used were collected at MUWSA.

The results show that the COPRAS and SAW methods rank the given alternatives differently while when integrated with the ILP technique, the formulated models select the same portfolios of alternatives. Thirteen alternatives which cost 97% of the total budgets set for WLM were selected. Furthermore, the ILP models showed robustness in selecting the portfolio of alternatives as they select the same alternatives despite the ranking of alternatives and change of weights of evaluation criteria.

Finally, the study proposed the decision model framework which can be used by decision-makers to evaluate and select the best strategies for WLM in WDS.

DECLARATION

I, **Maselle Joseph Kadenge** do hereby declare to the Senate of Nelson Mandela African Institution of Science and Technology that, this dissertation is my original work and that it has neither been submitted nor being concurrently submitted for degree award in any Institution.

___________________________ **27 March 2020**

Maselle Joseph Kadenge **Date**

Name and signature of the candidate

The above declaration is confirmed by

___________________________ **27 March 2020**

Prof. Verdiana Grace Masanja **Date**

Name and signature of the supervisor

___________________________ **27 March 2020**

Dr. Mashaka James Mkandawile **Date**

Name and signature of the supervisor

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CERTIFICATION

We, the undersigned, certify that we have read and hereby recommend for acceptance by the Nelson Mandela Institution of Science and Technology, a dissertation entitled, “Modelling and Optimisation of Water Loss Management Strategies in a Water Distribution System: A case of Moshi Urban Water Supply and Sanitation Authority”, submitted in Partial Fulfilment of the Requirement for the Degree of “Master’s in Mathematical and Computer Science and Engineering”.

______________________________	**27 March 2020**
Prof. Verdiana Grace Masanja	**Date**
Name and signature of the supervisor	

______________________________	**27 March 2020**
Dr. Mashaka James Mkandawile	**Date**
Name and signature of the supervisor	

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DEDICATION

I dedicate this work to my beloved wife Ms. Christina Kambwa, my sons Jesse, Jensen and Jovine, and my daughter Janise for their patience when I was out of home busy with studies.

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LIST OF ABBREVIATIONS AND SYMBOLS

AHP	Analytic Hierarch Process
ARAS	Additive Ratio Assessment
BIP	Binary Integer Programming
BOT	Bank of Tanzania
COPRAS	Complex Proportional Assessment
CRITIC	Criteria Importance Through Inter-Criteria Correlation
DAWASA	Dar es Salaam Water Supply and Sanitation Authority
DM	Decision-Maker
DO	Deterministic Optimisation
DP	Dynamic Programming
ELECTRE	Elimination Et Choix Traduisant la Realite or Elimination and Choice Translating Reality
FAVAD	Fixed and Variable Area Discharge
GDP	Gross Domestic Product
IP	Integer Programming
ILP	Integer Linear Programming
IWA	International Water Association
LP	Linear Programming
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MADM	Multi-Attribute Decision Making
MAU	Multi-Attribute Utility
MAUT	Multi-Attribute Utility Theory
MAVT	Multi-Attribute Value Theory

MCDM	Multi-Criteria Decision Making
MCDA	Multi-Criteria Decision Analysis
MDDM	Multi-Dimensions Decision Making
MILP	Mixed-integer Linear Programming
MINLP	Mixed-Integer Non-Linear programming
MODM	Multi-Objective decision making
MOWI	Ministry of Water and Irrigation
MUWSA	Moshi Urban Water Supply and Sanitation Authority
NBS	National Bureau of Statistics
NM-AIST	Nelson Mandela Institution of Science and Technology
NRW	Non-Revenue Water
NLP	Non-Linear programming
PROMETHEE	Preference Ranking Organisation Method for Enrichment Evaluation
PRV	Pressure Reducing Valves
QP	Quadratic Programming
RTO	Real-Time Optimisation
SAW	Simple Additive weighting
SMARTS	Simple Multi-Attribute Rating Technique Swings
SMARTER	Simple Multi-Attribute Rating Technique Extended to Ranking
SO	Stochastic Optimisation
SP	Strategic Planning
TBS	Tanzania Bureau of Standards
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

UAAL	Unavoidable Annual Apparent Losses
UARL	Unavoidable Annual Real Losses
UWSSA	Urban Water Supply and Sanitation Authorities
WDS	Water Distribution System
WLTF	Water Loss Task Force
WLM	Water Loss Management
WSA	Water Supply Authority
$\succ$	Succeed or preferable

CHAPTER ONE

INTRODUCTION

1.1 Background information

Water is very important for both animals and plants' survival and critical to the economic development of all countries in the world. In Tanzania, for instance, the economy is highly dependent on water resources. According to reports from (Bank of Tanzania, 2018; National Bureau of Statistics, 2018), about one-quarter of Tanzania's Gross Domestic Product (GDP) depends on agriculture, fishing, and livestock activities which are highly dependent on water resources. Developing countries worldwide are facing big challenges in water management due to the increasing demand for urban water supply caused by industrialisation, urbanization, improvement of living standards, and impact of global warming in fresh-water supply (Mahoo *et al.*, 2015; Makaya, 2016). Worldwide water demand is increasing while water resources are decreasing. In addition to the mentioned challenges, not all the produced water reaches to the consumers; some amount of water is lost on its way from the source before reaching the consumers. The loss of water in the distribution systems is due to leakage, illegal use, and theft of water infrastructure such as pipes, fittings, etc. In Tanzania, for instance, the average loss of the water in the distribution system is 32.3% (Ministry of Water and Irrigation, 2019). Water loss in the distribution network is a problem facing water authorities but in developing countries, the problem is worse due to poor infrastructure and poor sanitation. Water loss is the difference between the water produced and water billed or consumed:

$$\text{Water loss} = \text{water produced} - \text{water billed or consumed} \quad (1)$$

Water loss in the water distribution system (WDS) is grouped into two categories: Apparent (commercial) and real (physical) losses. Apparent losses include water theft (illegal connection and meter tampering), meter inaccuracies, and unbilled water for firefighting and public use such as water used in public toilets. Physical losses, on the other hand, include water leakage from pipes, fittings, and joints, and water leakage from tank overflow, tank floor, and walls (Selek *et al.*, 2018). Water leakage is the major contributing factor for water loss in WDS About 70% of the total loss of water in developing countries is caused by water leakage (Ndunguru & Hoko, 2016; Samir *et al.*, 2017). The leakage can be caused by the ageing of pipes, poor network design and construction, damage to exposed pipes, poorly sealed connections, and theft of pipes.

The total water loss represents the non-revenue water (NRW), which is the unbilled water or water which does not bring revenue to the water authorities (IWA). The NRW is the indicator of the efficient operation of WDS in which the higher NRW indicates the poor performance of the WDS. Water losses negatively affect the operation of water authorities as no authority can operate efficiently if it does not realise all its revenue. Moreover, water losses in the WDS have economic, environmental, public health, and social effects. Besides, water losses in the WDS reduce revenue, interrupt the quality of water, and inflate the operation and maintenance cost of the authority (Makaya, 2016; Selek *et al.*, 2018).

There are various water loss control strategies set by the International Water Association (IWA). The Water Loss Task Force (WLTF) of IWA has developed and promotes water management strategies that apply worldwide to reduce water loss in the WDS and increase the revenue generation of the water authorities (Charalambous *et al.*, 2014). The developed methodologies broadly focus on the accuracy of water meters, water stability, and the managing of apparent and physical losses.

To manage the physical losses, the WLTF emphasises to the use of four approaches: active leakage control, pressure management, speed, and quality repair, and infrastructure management. All these strategies should be well-adjusted to get the most economical leakage management as illustrated in Fig. 1.

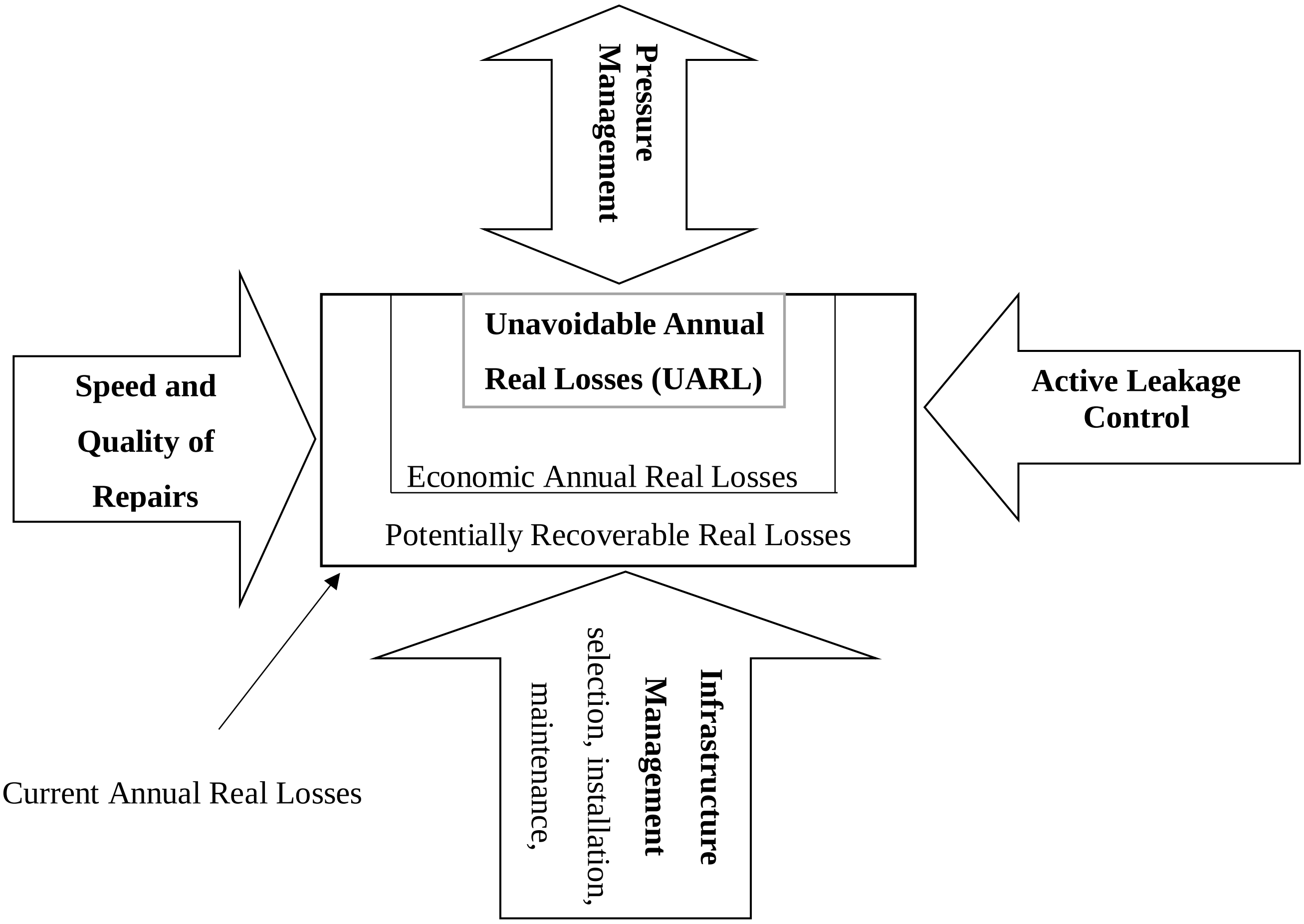

Figure 1: Four primary methods for real loss management (Selek *et al.*, 2018)

As with real losses, apparent losses control also uses four basic methodologies: Customer meter accuracy, meter reading errors, data handling, and billing errors, and illegal use as shown in Fig. 2.

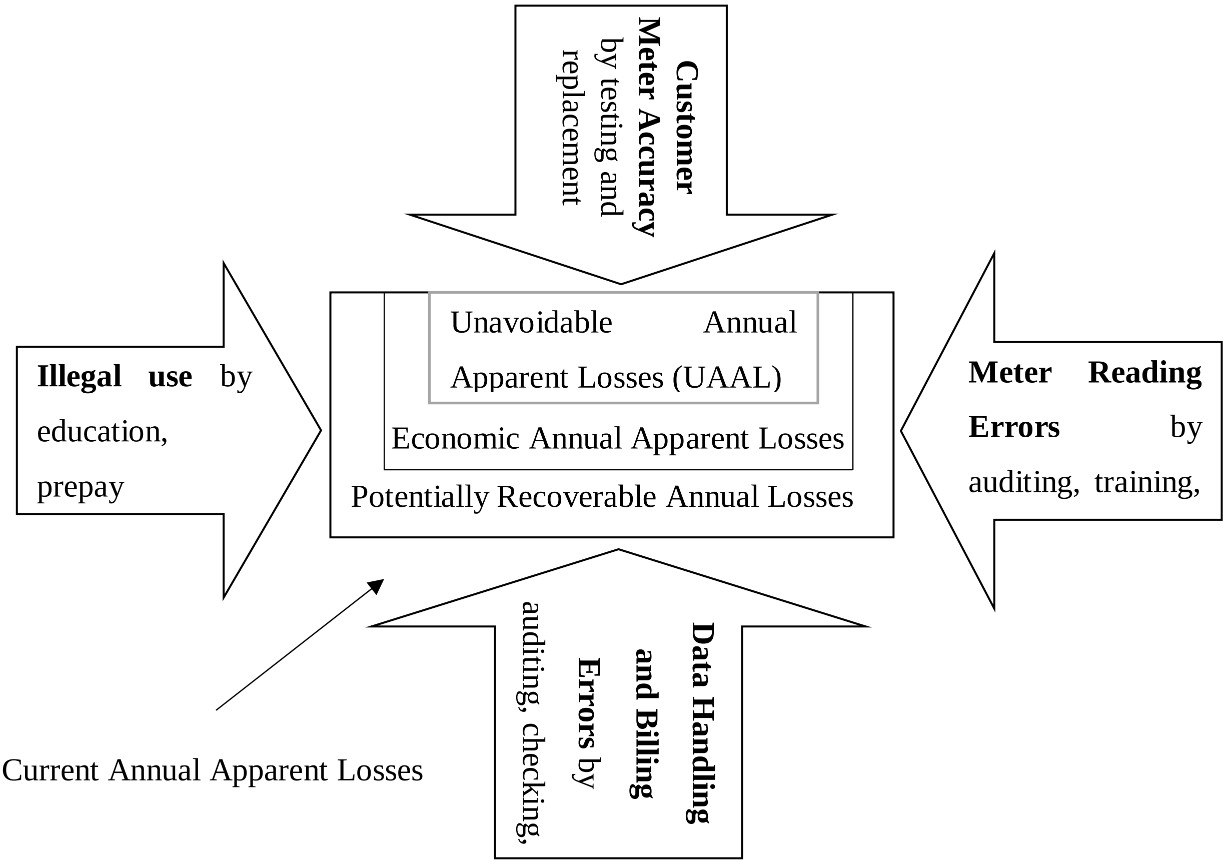

Figure 2: Four basic methods of managing apparent loss (Selek *et al.*, 2018)

1.2 Statement of the problem

A study done by Ndunguru and Hoko (2016) showed that water loss in the WDS worldwide ranges from 15% to 60% of the total water supplied. The situation is critical in third world countries where an amalgamation of ageing infrastructure, illegal connections, and theft of pipes worsen the problem. While the World Bank (WB) recommends that NRW should be less than 23% (Makaya, 2016), the Southern Africa countries have set a standard of 20% as optimal for a well-performing water authority (Ministry of Water, 2014). In Tanzania, the average of NRW for the Urban Water Supply and Sanitation Authorities (UWSSAs) was recorded to be 46% for the year 2013/14, 43.6% for the year 2014/15, 41.6% for the year 2015/16, 38.4% for the year 2016/17 and 32.3% for the year 2018/19 (Energy and Water Utility Regulatory Authority, 2017; Ministry of Water and Irrigation, 2019; Ministry of Water, 2014).

Despite the Ministry of Water and Irrigation (MoWI) setting a target for reducing NRW from 46% in 2013 to 25% in 2019, water loss in the WDS is still high. The MoWI in its 2019/20 budget report shows that Dar es Salaam Water Supply and Sewage Authority (DAWASA) had the highest per cent of 40% of NRW while other regional UWSSAs have an average of 32% of NRW and District Water Utilities have 25% of NRW. In particular, MUWSA has a loss of 22% of NRW (Research data, 2019) in which the authority wants to reduce to 15% of NRW in five years to come. This high loss of water can be caused by poor management of the present strategies and on how to prioritise them. Moreover, the control or reduction of water loss in the WDS is closely dependent on the commitment of the decision-makers and on the implementation of the present strategies and budget, they set for that purpose.

This research intends to prepare a decision model for water loss management (WLM) which will assist the decision-makers (DMs) to rank and select the best strategies present to control or reduce the water loss in the WDS.

1.3 Rationale of the study

Water loss in WDS is a serious challenge to many water supply authorities (WSA) worldwide. Studies show that about 15% to 60% of total water produced lost on its way from the sources before reaching the consumers. The losses are due to leakage, illegal use, and infrastructure theft among others. International organisations like IWA through WLTF, water practitioners, and researchers have developed control strategies and methodologies for the design and operations of the WDS to increase the revenue and reduce water loss and operation costs of the WSA. Despite there are recommended strategies and methodologies, water loss in the WDS is still a challenge to many WSA. This challenge raises a need for more researches on water loss in the WDS.

This study focuses on the control/reduction of water loss in the WDS by considering the managerial aspect using mathematical techniques. The study discusses the actions taken by the management of the WSA to control water loss and how they implement the strategies used in WLM. Furthermore, the study provides a mathematical model for evaluating and selecting the portfolio of the best strategies to be applied by the WSA for WLM. It recommends the use of the MCDM family methods; MAVT, SMARTER, SAW and COPRAS, and ILP, a numerical optimisation technique in evaluation and selection of strategies.

1.4 Objectives

1.4.1 General objective

To develop a decision model for WLM to assist DMs to prioritise identified strategies and select a portfolio of the best strategies required to reduce NRW in the urban WDS.

1.4.2 Specific objectives

(i) To investigate and evaluate the WLM strategies used at MUWSA by applying MCDM methods.

(ii) To develop and evaluate decision models for selecting strategies used in water loss management in the WDS by using MCDM and ILP methods for the collected data.

(iii) To compare two MCDM methods (COPRAS and SAW) in prioritising and selecting the strategies to assess their appropriateness and efficiency in water loss management.

(iv) To perform a sensitivity analysis to assess the robustness of the models.

1.5 Research questions

(i) What strategies are used at MUWSA for the management of water loss?

(ii) How can the combined model of MCDM methods and ILP be applicable for selecting the portfolio of strategies for water loss management?

(iii) What MCDM ranking methods between COPRAS and SAW is more appropriate and efficient in water loss management?

(iv) How stable is the model in selecting strategies for water loss management?

1.6 Significance of the study

Results obtained in this study could:

(i) Be of great help to DMs in planning and selecting the best strategies for management of water loss in the WDS in all UWSSAs in Tanzania.

(ii) Be used as an additional reference to researchers on the application of MCDM methods and numerical optimisation.

1.7 Delineation of the study

Data used in this study were collected from MUWSA, other UWSSAs were excluded in the sample due to limitations of time and financial constraints. It is believed that similar strategies for WLM can be applied even in other water authorities that operate under similar conditions.

CHAPTER TWO

LITERATURE REVIEW

2.1 Theoretical bases of the research

2.1.1 Multi-criteria decision-making methods

Multi-criteria decision-making (MCDM) is the study of methods and procedures by which multiple and contradicting criteria can be unified in the decision process. The main objective of MCDM methods is to provide a tool to DMs to assist them to advance in solving MCD problems, where several conflicting criteria are taken into account (Zardari *et al.*, 2015). Several names can be used to refer to MCDM: Multi-Criteria Decision Analysis (MCDA), Multi-Objective Decision Making (MODM), Multi-Attribute Decision Making (MADM), and Multi-Dimensions Decision Making (MDDM).

The MCDM weighting methods are classified into two main groups. The first group includes compensatory weighting methods such as Multi-Attribute Utility Theory (MAUT), Multi-Attribute Value Theory (MAVT), Analytic Hierarchy Process (AHP), Simple Multi-Attribute Rating Technique Swings (SMARTS), SMARTER, COPRAS, and SAW. Others are Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). These methods are used in Multi-Attribute Utility (MAU) methods (Zardari *et al.*, 2015).

The second category includes non-compensatory weighting methods like Elimination Et Choix Traduisant la Realite (French) or Elimination and Choice Translating Reality (English) (ELECTRE). Others are Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) and their families which are used in outranking methods (Fontana & Morais, 2016).

In a compensatory approach, the evaluation of strategies considers trade-offs between criteria while in non-compensatory methods, a loss of an alternative on a criterion cannot be compensated by other criteria.

Furthermore, Zardari *et al.* (2015) put various MCDM methods into three categories based on their similarities:

(i) Multi-attribute theory

This group of methods aggregates different ideas into a single function for optimisation. The methods use numbers to represent the preference of the considered action. The methods in this category are MAUT, MAVT, SMARTS, AHP, and TOPSIS.

(ii) Outranking methods

These methods are based on a pair-wise comparison of action. They develop a relationship which is called outranking relationship which represents DMs' preferences to solve their problems. The methods in this category are ELECTRE and PROMETHEE.

(iii) Elementary methods

These are the simplest MCDM methods that use simple preference models. They include weighted sum, weighted product, conjunctive, disjunctive, and linear assignment.

Weighting to the evaluation criteria in MCDM methods is vital as the final result of decision making mainly depends on the weights. The study by Zardari *et al.* (2015) classifies weighting methods into three groups.

(i) Subjective weighting methods

These determine the weights of the criteria basing on the choices of the DMs. The elicitation process is subjective methods are explained clearly and mostly used for MCDM in water resource management. Common subjective weighting methods are direct rating, ranking method, SMART/SMARTER, point allocation, swing method, ratio method, pair-wise comparison, SIMOS method, graphical weighting, and Delphi method.

(ii) Objective weighting methods

The criteria weights in these methods are obtained through mathematical approaches in which DMs have no part in the determination of the importance of criteria. Popular objective weighting methods include: mean weight, standard deviation, Statistical variance procedure, Criteria Importance Through Inter-criteria Correlation (CRITIC), entropy method, TOPSIS, least mean square, Multi-Objective Optimisation.

(iii) Combination or optimal weighting methods

These are hybrid methods that comprise multiplication and additive synthesis. The weights are obtained from both subjective and objective facts on criteria weights. The methods in this category are swing weights, ranking, rating, pair-wise comparison, trade-off analysis, and qualitative translation.

2.1.2 Optimisation approaches

Mathematical programming is the selection of the best component with regards to some criteria from the set of available alternatives. Optimisation arises in several disciplines such as computer science, engineering, operational research, and economics. Optimisation problems comprise maximization or minimization of the actual function by analytically choosing input values from within the acceptable set and computing the value of the function. Moreover, optimisation takes account of finding the best solution to the objective function by the given inputs or restrictions (Onwubolu & Babu, 2013).

Optimisation approaches are categorized as deterministic, non-gradient, and real-time optimisation as explained below (Balekelayi & Tesfamariam, 2017; Kumar, 2014; Savić & Mala-Jetmarova, 2018).

(i) Deterministic (Numerical) optimisation

These are classical methods used to find the optimal solution or unrestricted maximum or minimum of continuous and differentiable functions. The methods are analytical which makes use of differential calculus in locating the optimal solution. The methods have limited scope in practical applications as some of them involve objective functions which are not continuous and/or differentiable. Numerical optimisation has the following sub-fields:

Convex optimisation*:* this deals with the problem of convex (minimization) or concave (maximization) objective function over convex sets of constraints. Convex optimisation comprises linear programming, conic programming, and geometric programming.

- **Linear programming (LP)**: is the branch of mathematical programming that comprises the linear objective function *(f)* and the set *(A)* of linear equalities and/or inequalities (constraints) used to find the optimum allocation of limited resources among competing activities. The constraints could reflect financial, technological, marketing, organizational, or any other deliberations;

- **Non-linear programming (NLP)**: is the branch of mathematical optimisation in which the objective function or the constraints or both contain non-linear parts. In water management, NLP is used in the design of water networks;

Integer programming (IP)*:* is the mathematical programming in which some or all variables are limited to take on integer values. IP are grouped as;

- **Pure integer linear programming (ILP)**: is the programming which occurs when all the variables problem are limited to take on integer values;
- **Binary integer programming (BIP)**: is the programming that occurs when all the variables of the problem are limited to take on binary values (0 or 1). The BIP is the special case of a pure ILP problem;
- **Mixed-integer linear programming (MILP)**: is the programming that occurs when only some of the variables are restricted to take on integer values and some are allowed to take only real values;
- **Mixed-integer non-linear programming problem (MINLP)**: is the programming which occurs when the objective function and /or constraints are non-linear functions;

Quadratic programming (QP): is the mathematical programming used to solve the particular form of mathematical optimisation problem specifically, a quadratic optimization problem, that is, the problem of optimising (minimizing or maximizing) a quadratic function of several variables subject to linear constraints on these variables. Quadratic programming is a special form of non-linear programming in which the objective function has quadratic terms and constraints are stated with linear equalities and/ or inequalities.

Dynamic programming (DP): is the optimisation approach that is based on dividing the problem into smaller sub-problems.

Stochastic programming (SP): is the mathematical programming in which some of the constraints depend on random variables. It gives a framework for modelling optimisation problems that involve uncertainty.

Deterministic optimisation (DO): is the optimisation in which the formulated problems include the known parameters and are real-life problems that contain some unknown parameters.

(ii) Non-gradient optimisation

These are optimisation techniques that do not need gradient information to converge to the solutions, but they only use function evaluations of the objective functions to converge to the solutions (Hare *et al.*, 2013; Kumar, 2014). Non-gradient methods are heuristic or meta-heuristic. The heuristic and Meta-heuristic methods are used in computer science, artificial intelligence, and mathematical optimisation. The heuristic technique is used to find an estimated solution when the classical methods fail to find any exact solution. Meta-heuristic is higher-level techniques of heuristic used to find, generate, or select a heuristic that can provide an appropriately good solution to an optimisation problem, particularly with inadequate or deficient information or restricted computation capacity (Balekelayi & Tesfamariam, 2017).

Non-gradient optimisation techniques are categorized as:

Evolution algorithm: Includes genetic algorithm and evolutionary techniques.

Physical algorithm*:* Includes harmony search, simulated annealing, Tabu search, and ray optimisation.

Swarm algorithm*:* Includes ant-colony optimisation, particle swarm optimisation, artificial bee colony, and shuffled frog- leaping.

Direct search methods: Includes directional direct search, simplicial direct search, simplex gradient methods, and trust-region methods.

(iii) Real-time optimisation (RTO)

Real-time optimisation is the process that intends to optimise the process performance in real-time systems. It is the combination of deterministic and meta-heuristic algorithms coupled to accelerate decision-making.

Most of the optimisation methods are designed principally to be applied in decision-making.

2.2 Empirical analysis of the past studies

2.2.1 Review of broad studies in water loss in WDS

Studies on water loss problems in the WDS are of interest worldwide as the problem affects water authorities and consumers all over the world. Many studies in the aspect of technical and managerial have been done to reduce water loss, design, and operation of the WDS, and improving services. Most of the studies carried in this area are from the points of view of science, engineering, and mathematics. The focus of these studies has mostly been on water loss/leakage control or reduction, network design, and operation of WDS.

(i) Water leakage

In water leakage researchers focus on the causes of leaks (e.g. pressure) and control measures. Some optimisation techniques such as dynamic, linear, non-linear, and mixed-integer non-linear programming are used in leakage optimisation of the WDS. Studies on leakage in WDS have attracted both practitioners and researchers, for instance:

Gupta *et al.* (2017) studied on optimisation methods used in leakage management in the urban WDS. They used a pressure management approach to optimise the water level in a storage tank and optimising the pressure reducing valve (PRV) in the system. Furthermore, a Multi-Objective genetic algorithm (NSGA-II) was applied to find out the optimised operational control setting of PRV for leakage minimization. Results showed that the algorithm reduced leakage for 26.51% in any-town WDS and 20.81% in a modified benchmark WDS.

Samir *et al.* (2017) studied on pressure management to minimize leakage in WDS. He modelled the leakage as a function of pressure and length of pipe, calibrated leakage coefficient, used fixed PRVs to develop pressure fluctuation and waterCAD program to simulate water flow. The result shows that the approach reduces leakage by 37%.

Deyi *et al.* (2014) studied leakage control by the use of the pressure management technique by the use of FAVAD (Fixed and Variable Area Discharge) equation and leakage number. The findings show that the FAVAD equation and leakage number give results on the behaviour of the leaking system with pressure in real networks than the conventional (orifice) equation.
Covelli *et al.* (2016) studied on how to get the optimum position and location of PRVs in the WDS for Leakage minimization. The study used a genetic algorithm coupled with modelling

of leakage at joints for simulation of hydraulic. The methodology showed success in positioning and sizing the PRVs within the WDS and in reducing joints' leakage when compared with the sophisticated hydraulic models.

Other studies on leakage management are from Roshani and Filion (2014). The study discusses the combination of pressure control and pipe rehabilitation by using a multi-objective optimisation method as a better way of reducing water leakage and reduction of operational costs. The results show that the approach reduces the leakage rate by 80% and operational cost by 53%.

In general, pressure management and pipes rehabilitation are the major techniques recommended by the researchers and practitioners for reduction or control of water leakage in the WDS.

(ii) Design and operation of WDS

In the design and operation of WDS researchers focused on the proper design and operation of the WDS to reduce costs. This includes optimisation of network design, optimisation of operational cost of the pumping system, and real-time control. The following are some of the studies done in this area:

The study by Bagloee *et al.* (2018) studied the reduction of electricity usage of water pumps. He used a hybrid method of regression models with optimisation techniques. The problem regression models were solved using MATLAB codes and machine learning, GAMS and CPLEX solvers were used to solve the optimisation model part of the problem, while Ms-Excel and MS-Access, respectively, were used as a user interface and as a database. The approach showed promising results in the minimization of power usage.

The study by Balekelayi and Tesfamariam (2017) reviewed and compared the optimisation techniques used in the design and operations of WDS. In the review, they compared three main optimisation techniques namely deterministic, non-gradient, and real-time optimisation techniques. The review concluded that the non-gradient approaches give better results compared to deterministic techniques. Furthermore, real-time optimisation which combines the deterministic and the non-gradient techniques gives a fair result in a reasonable time of computation.

The study by Świtnicka *et al.* (2017) deliberated on optimising the WDS by using Bentley WaterGEMS software for simulation of the hydraulic flow and generic algorithm a meta-heuristic method for optimisation the water flow velocity, pressure head and pump energy consumption. The results show that the software and optimisation method are the useful and common tool which can be applied to enhance the decision-making process in maintenance planning of the WDS.

The study by Bohórquez *et al.* (2015) discussed on optimisation of the pumping system to reduce the operation costs of the WDS. The study used a genetic algorithm method for optimising the energy consumption of the pump and EPANET software program for the simulation of hydraulic flow. The study reveals that the high operation costs of the WDS are due to the energy consumption of the pump and leakage of the system. To minimize the energy usage of the pump, the study recommends the optimisation process which reduces excessive pressure in the system implying the reduction of water lost due to leakage.

The study by Abdul Gaffoor (2017) discussed on optimisation of water supply and WDS and real-time control. In his research, he developed an intelligence platform called Real-time Dynamically Dimensioned Schedulers (RT-DDS) which was used to control and optimise WDS operations. By using this methodology, results show that the energy-saving was up to 25% per day which leads to a cost-saving of over 2.3 million US dollars over ten years. Furthermore, a stable flow of water in the system was experienced.

The study by Savić and Mala-Jetmarova (2018) discussed the history of optimisation in WDS Analysis. The study explains the number of published studies covering the study of WDS optimisation, outlines the problems, applications, and optimisation methods. The major WDS optimisation problems discussed in the study are WDS design or component sizing (includes pipe sizing, tanks, pumps and valves, and existing system design optimisation) and WDS operations which covers pump operation (planning and real-time operation), water quality management and valve control. Furthermore, the study discusses the main methods used in the optimisation of WDS as deterministic which comprises LP, DP, and NLP; and meta-heuristic which includes genetic algorithms, particle swarm optimisation, simulated annealing, evolutionary algorithm, etc. Besides, the study discusses other WDS applications that require the different formulation of the optimisation problem as; WDS model calibration, system partitioning, reliability, robustness, and resilience of WDS.

The study by Porse *et al.* (2017) analysed and optimised the system of water supplies in Los Angeles. He developed an optimisation model to evaluate the degree to which the municipal of Los Angeles can minimize the dependence of imported water. The results showed that the model increases the storm-water capture by 300% and emphasise the reuse of water of the present facilities, and reduce the imported water by 30%.

In general, studies in this area are based on reducing the operational cost and energy consumption of the pumping system in the WDS.

2.2.2 Review of related studies in water loss in WDS by using MCDM methods

Studies in decision-making are guided by MCDM methods. These are tools developed in the arena of decision concepts to solve problems in operational research. The methods form a limited number of decision alternatives in which DMs evaluate and rank the alternatives based on the weights of the limited set of evaluation criteria (Mutikanga *et al.*, 2011). Many researchers have done researches in MCDM methods and their application in various fields, particularly in water loss and resource management, and planning. Some of the studies are discussed hereunder;

Studies by Banihabib *et al.* (2017), Cambrainha and Fontana (2018), and Yilmaz and Harmancioglu (2010) discussed the applications of MCDM methods in water loss and resource management: The study by Cambrainha and Fontana (2018) formulated a model based on a problem structuring method (PSM), PROMETHEE and ILP methods to assess strategies used to balance water supply – demand in WDS. Results show that the model was robust for decision making. Furthermore, the study summarised studies conducted on the application of MCDM methods in water management and planning. The study by Yilmaz and Harmancioglu (2010) used SAW, CP (compromise programming) and TOPSIS methods to formulate a model for water resource management. Results show that the best alternative does not depend on the MCDM method used but it depends on the weights of the evaluation criteria and data used in the analysis. The study by Banihabib *et al.* (2017) compares the compensatory and non-compensatory MCDM methods for water resources strategic management. The SAW and AHP represented compensatory methods and ELECTRE III represented non-compensatory methods. Findings show that the ELECTRE III technique has less sensitivity than SAW and AHP methods when changes in weights occur. Furthermore, the ranking found by the ELECTRE III method is more authentic for decision-making.

In this study, the discussion is made on the water loss control or reduction in the aspect of management with a focus on WLM in WDS. Water loss management is the management of the quantity of water which is the difference between supply and consumption. Water loss management is vital in deciding matters to meet the strategic objectives as outlined in the company Strategic Planning (SP). Strategic planning is a process of setting long-term goals for the future of the company which is centred on predictions, analysis of strategies, and the key decisions. It is the best tool for sustainable water resource management. Most of the problems facing the water authorities in the third world countries are caused by a lack of strategic planning.

2.3 Finding from the literature review

From the reviewed literature, it can be summarised that many studies in MCDM methods and their applications, particularly in water resource management and planning have done to help the decision-making of the organisations.

It is noted that the reviewed studies used AHP, PROMETHEE, ELECTRE, TOPSIS, and SAW, MCDM methods to assess and prioritise the present strategies of the companies. Furthermore, it has noted that few studies have included the optimisation techniques to optimise the alternatives/strategies used in decision-making.

The interest of this study is to use a new ranking method, COPRAS, to evaluate and prioritise the strategies used in WLM to check its appropriateness with other ranking methods such as SAW. Moreover, the study went further on selecting the portfolio of the best alternatives using the combined model of MCDM family methods (MAVT, SMARTER, COPRA, and SAW) and the ILP method.

Finally, this study has compared two integrated decision models to evaluate their appropriateness and efficiency in decision-making and solving issues in water management and planning. The compared models are formulated from two ranking methods COPRAS and SAW. The COPRAS and SAW methods use SMARTER to assign weights to evaluation criteria (Velasquez & Hester, 2013). These are the methods of Multi-Attribute Utility Theory (MAUT) (Volvačiovas *et al.*, 2013).

The MAUT and MAVT are two related methods in compensatory weighting methods, which permit compensation of poor performance of the criteria to the good performance of other criteria. The methods use additive weighted value function which integrates multiple criteria into a single measure of the overall value of each alternative. All methods use SMARTS or SMARTER to assign weights to the evaluation criteria (Velasquez & Hester, 2013). The COPRAS method uses both benefit (maximization) and cost (minimization) criteria values for evaluation of MCDM alternatives while the SAW method requires conversion of cost criteria to benefit criteria in the evaluation of alternatives (Stanujkić *et al.*, 2013).

2.4 Workflow for model formulation

To answer the questions stated in section 1.5, this study has developed a model framework to evaluate, prioritise, and select the best strategies used for WLM in the WDS using MCDM methods and numerical optimisation techniques. The workflow for the model formulation was adopted from the decision-making process that was developed by Yoe (2002), Zardari *et al.* (2015) and Fontana and Morais (2016)

The workflow is divided into two main phases as illustrated in Fig. 3. Phase one: MCDM family methods approach and Phase Two: ILP, Numerical Optimisation family methods approach.

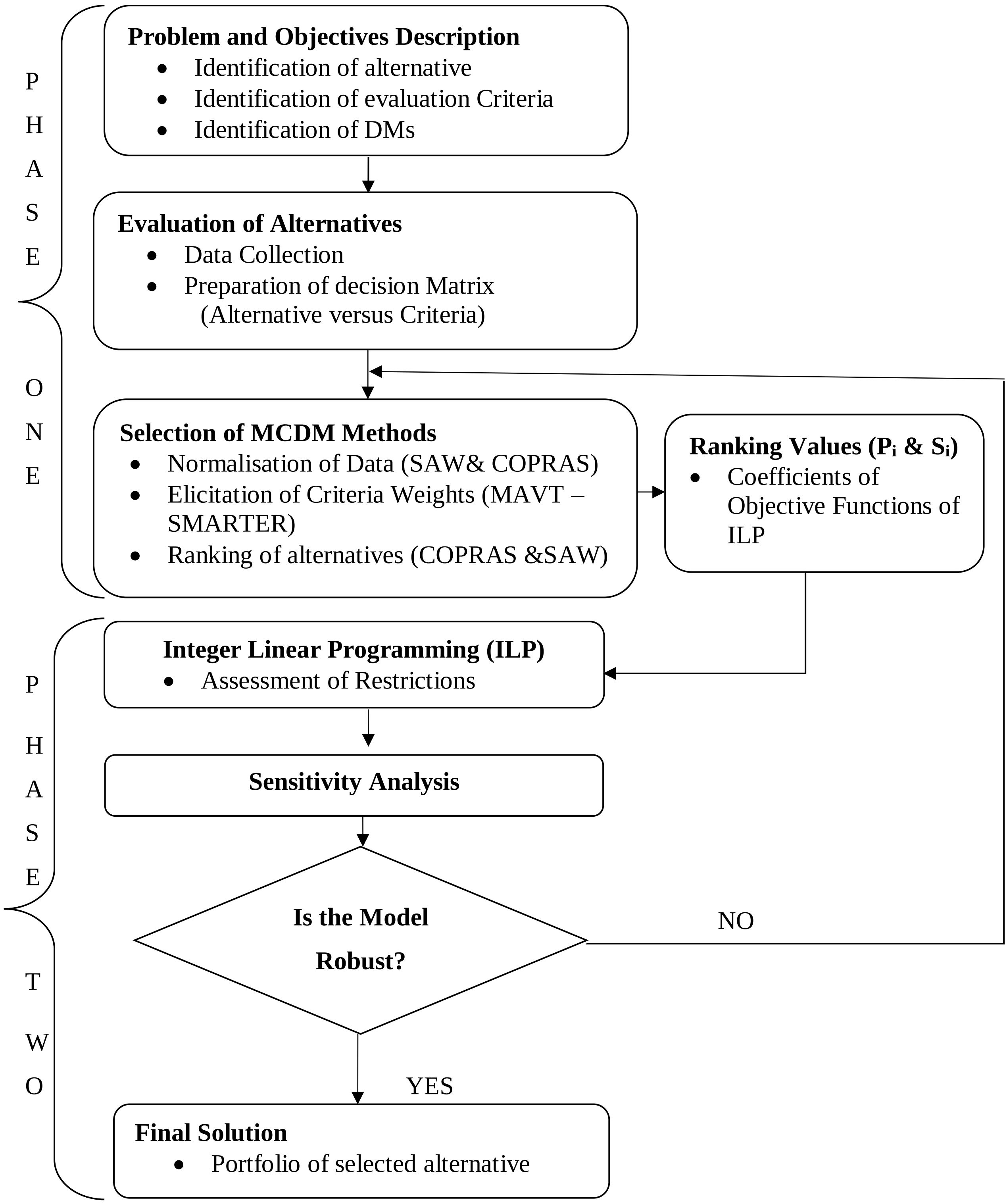

Figure 3: The workflow for the model formulation (Fontana & Morais, 2016; Zardari *et al.*, 2015)

CHAPTER THREE

MATERIALS AND METHODS

3.1 Case study

The model developed in this study was applied to MUWSA which is located in Moshi municipality located at the foot of Mt. Kilimanjaro in the northern part of Tanzania. The water distribution system of MUWSA covers 590.3 km and as of 30 April 2019 it was serving 36,625 customers. The MUWSA WDS is divided into 12 zones and has 19 sources in which 12 are springs and 7 are boreholes. The authority experiences the loss of water of about 2.6 million m^3 per year in the WDS which is equal to 22% of the total water produced per year (Research data, 2019). The Authority aims to reduce this loss to 15% in five years to come.

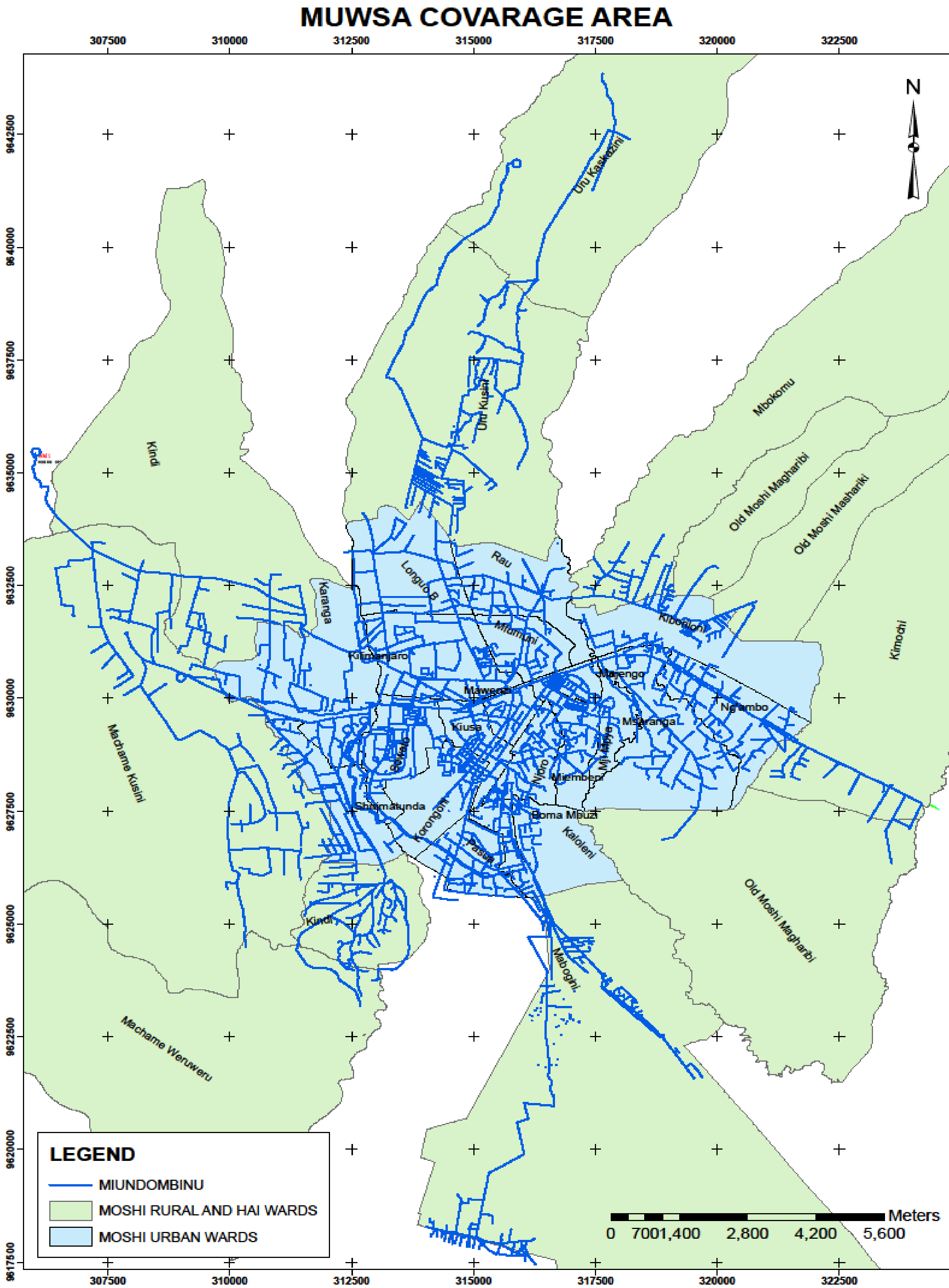

Figure 4: The MUWSA WDS coverage areas (Research data, 2019)

3.2 Data collection and tools used

Primary data were collected from knowledgeable and experienced DM by using questionnaires and face-to-face interviews, while secondary data were collected through documentary reviews and content analyses from reports, brochures, flyers, and posters. The collected data were; the strategies/ alternatives used by the company in WLM, evaluation criteria employed, budgets set for implementation, and the quantity of water produced and lost, water sources, number of customers connected, the coverage area of the system, and company five years strategic plan.

This study has designed the questionnaire and interview guides to assess and measure the strategies used by the authority in WLM and to elicit information that will provide answers to the research questions. The study has used the Likert scale of five levels to evaluate the decision-making strategies as it can be referred from Appendix 5. The study included open-ended questions in the questionnaire to give the respondents more freedom to present their views.

3.3 Data analysis

Data have been analysed through various steps of the integrated model of MCDM family methods and the ILP technique: (a) The MAVT method was used to aggregate the performance of strategies through all the criteria to obtain a cumulative evaluation value; (b) The SMARTER, the compensatory techniques were used to calculate and allocate the weights to the evaluation criteria according to the DM's ranking; (c) The COPRAS and SAW methods were used to rank the strategies; (d) The ILP technique was used to formulate the decision model for the selection of the best strategies for WLM in a WDS; (e) The LINDO 6.1 software package was used to solve the ILP equations, and (f) The Python software was used to draw the bar charts (Histograms) for comparison of the ranking between COPRAS and SAW methods.

3.4 Model formulation

The formulation of the models followed the framework shown in Fig. 3.

3.4.1 Model assumptions

(i) The model considers only one decision-maker

(ii) The model does not account for revenue constraints

3.4.2 Model formulation phase I - MCDM methods

(i) Description of problem and objectives

The first step in decision making is to define explicitly the problem and objectives of what is needed in the study. In doing so, the researcher identified the root causes of the problem and understood the context of the company under study. The alternatives applied in WLM and their evaluation criteria were identified. Expert DMs were engaged in the evaluation of alternatives.

Thus, based on MUWSA's information, the problem of the study was on how the company manages the strategies used in controlling or reducing water loss in the WDS. The objectives were to identify, prioritise, and select the portfolio of the best alternative used at MUWSA for WLM. The goals of the study were derived from the quality policy statement of the company which states that: "*MUWSA aims to achieve the provision of adequate, sustainable and competitive clean and safe water supply and sanitation service which consistently satisfies the needs and expectations of its customers"*. The derived goals were to: maximize the revenue, minimize the operational cost, maximize water-saving, maximize water supply reliability, and maximize water quality.

Identification of alternatives

The surveyed alternatives for WLM in this study were:

- **Alternative (A_1): Education** to the community on the effective usage of water to facilitate saving of water at home and outside, and to inspire people to report the visible leakages and faults to the water authority to speed up the repairs process and avoid unnecessary water loss. This comprises of four options used for education: A_{11} – Advertising campaigns, A_{12} – Educational campaigns in schools, A_{13} - Ward meetings with the society, and A_{14} – Meeting with local leaders.
- **Alternative (A_2): Illegal use control**. A measure intended to control losses that are caused by illegal use of water from end-users (illegal connection, a setback of the meter and damage or theft of the infrastructure). This has one option: A_{21} - Illegal use control.
- **Alternative (A_3): Network zoning**. This has one option: A_{31} **-** Network zoning and establishment of District Metering Areas (DMA).

- **Alternative (A_4): Indicators to measure/quantify** the losses to give essential information for planning the actions needed to control losses. This has one option: A_{41} – 24 hours Zone Measuring.
- **Alternative (A_5): Strategies used to control inaccuracy meter**. This has two options: A_{51} - Calibration of the meter, and A_{52} - Replacement of the defect meters.
- **Alternative (A_6): Detection of apparent/physical losses.** This has three options: A_{61} – Visual inspection of the WDS, A_{62} – Comparison between the bulk water meter and customer water meter readings, and A_{63} – Report from the community on the detected leak through a toll-free telephone.
- **Alternative (A_7): Pipes replacement**. This has one option: A_{71} – Replacement of deteriorated pipes with new pipes.
- **Alternative (A_8): Quality pipes.** This has one option: A_{81} – Installation of quality pipes.
- **Alternative (A_9): Repairs.** This has one option: A_{91} - Timely repair of pipe leaks (active leakage control).
- **Alternative (A_{10}): Pressure management.** This has one option: A_{101} – Pressure management.

Evaluation criteria

The identified evaluation criteria for this study were:

- **Criteria (C_1): Income generation**. The capacity of an alternative to improve income. The highest score value, the best of the alternative is.
- **Criteria (C_2): Investment cost**. The cost required to execute the alternative. The lowest score value the best of the alternative is.
- **Criteria (C_3): Operation &Maintenance cost**. The cost related to the implementation of the alternative. The lower the score value the most preferable the alternative is.
- **Criteria (C_4): Saving of Water**. The capacity of an alternative to reduce water losses. The highest score value, the best of the alternative is.
- **Criteria (C_5): Quality of Water**. The ability of an alternative for retaining water quality. The highest score value, the best of the alternative is.
- **Criteria (C_6): Water supply reliability**. The capacity of an alternative to reduce flow disruptions. The least number of disruptions (burst, leaks, and illegal uses), the best of the alternative is.

- **Criteria (C_7): Efficiency**. The capacity of the alternative to minimize water losses. The highest score value, the best of the alternative is.

Identification of the DM

This study considered only one DM who is knowledgeable about both technical and managerial issues. The DM filled a questionnaire, evaluated the Alternatives against the Criteria, and responded to the interview questions.

(ii) Evaluation of alternatives

Data collection

The DM filled the questionnaire to assess the Alternatives versus the Criteria. The score of each criterion over the alternatives was given according to the Likert scale as 5 – the highest; 4 – the higher; 3 – fair; 2 – the lower; and 1 – the lowest. Table 1 indicates the scores of the alternatives given by the DM against each criterion.

Decision matrix

Table 1: Score evaluation matrix: Criteria versus Alternative

Alternative	**Criteria**						
	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	$\mathbf{c_4}$	$\mathbf{c_5}$	$\mathbf{c_6}$	$\mathbf{c_7}$
A_{11}	4	3	2	4	3	3	4
A_{12}	3	1	1	3	3	3	3
A_{13}	5	2	2	5	3	3	4
A_{14}	4	2	2	4	3	3	4
A_{21}	4	3	2	4	2	3	3
A_{31}	4	3	2	4	2	2	3
A_{41}	4	2	1	4	2	3	3
A_{51}	5	3	2	5	1	2	4
A_{52}	5	4	2	5	1	2	4
A_{61}	4	2	1	4	3	3	4
A_{62}	4	1	1	4	3	3	4
A_{63}	4	2	1	4	3	3	4
A_{71}	5	4	2	5	4	4	4
A_{81}	4	4	2	4	4	4	4
A_{91}	4	3	3	4	4	4	4
A_{101}	4	3	1	4	3	3	3
Benefit/Cost	benefit	Cost	Cost	benefit	benefit	benefit	benefit

Source**:** Research data (2019)

(iii) Selection of MCDM methods

This study used compensatory weighting methods MAVT, SMARTER, SAW, and COPRAS. The MAVT method was used to aggregate the performance of alternatives through all the criteria to obtain a cumulative evaluation value, the SMARTER was used to calculate and allocate the weights to the evaluation criteria according to the DM’s ranking, while the COPRAS and SAW methods were used to rank the alternatives.

Normalisation of data

The normalisation process is done to convert the data to the commensurable unit. Data normalisation is an important part of the decision-making process because it converts the input data into numerical of the same unit and comparable data, which allow MCDM methods to rate and rank the strategies (Mukhametzyanov & Pamucar, 2018; Vafaei *et al.*, 2016). The value of normalised data ranges from 0 to 1, where 0 is the least alternative value and 1 the highest alternative value in every criterion if its objective is to do maximization and/or minimization (Fontana & Morais, 2016). The study done by Stanujkić *et al.* (2013) explains normalisation methods for COPRAS and SAW methods as follows:

- **The COPRAS method**

The normalisation of data for the COPRAS method is done without transforming the cost to benefit type criteria by using the linear transformation – sum method.

$$a_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{2}$$

Where $0 \leq a_{ij} \leq 1, x_{ij}$ *is the score of i -th alternative for j -th criterion before normalisation, and* a_{ij} *is the normalised value.*

- **The SAW method**

SAW method uses four normalisation techniques which are: Linear scale transformation - max method, linear scale transformation – sum method, linear scale transformation – MaxMin method, and vector normalisation.

Linear scale transformation – sum method

$$a_{+ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad \text{Benefit criteria} \tag{3}$$

$$a_{-ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \quad \text{Cost criteria} \tag{4}$$

Linear scale transformation - max method

$$a_{+ij} = \frac{x_{ij}}{max\ x_j} \qquad \text{Benefit criteria} \qquad (5)$$

$$a_{-ij} = \frac{min\ x_j}{x_{ij}} \qquad \text{Cost criteria} \qquad (6)$$

Linear scale transformation – MaxMin method

$$a_{+ij} = \frac{x_{ij} - min\ x_j}{max\ x_j - min\ x_j} \qquad \text{Benefit criteria} \qquad (7)$$

$$a_{-ij} = \frac{max\ x_j - x_{ij}}{max\ x_j - min\ x_j} \qquad \text{Cost criteria} \qquad (8)$$

Vector normalization

$$a_{+ij} = \frac{x_{ij}}{\sqrt{\left(\sum_{i=1}^{m} x_{ij}^2\right)}} \qquad \text{Benefit criteria} \qquad (9)$$

$$a_{-ij} = 1 - \frac{x_{ij}}{\sqrt{\left(\sum_{i=1}^{m} x_{ij}^2\right)}} \qquad \text{Cost criteria} \qquad (10)$$

Where $0 \leq a_{+ij}, a_{-ij} \leq 1$, x_{ij} *is the score of i-th alternative for j-th criterion before normalisation, and* a_{+ij} , a_{-ij} *are the normalised values for benefit and cost criteria.*

This study carried out the normalisation for the SAW method by using the linear scale transformation - sum method (equations 3 and 4) which separates the benefit (maximum) and the cost (minimum) criteria.

That means the method transforms the cost (minimum) criteria to the benefit (maximum) criteria.

Table 2 and Table 3 show the normalised matrices for COPRAS and SAW methods respectively, which have been carried out in this study.

Table 2: Normalised value matrix: Criteria versus Alternative – COPRAS method

Alternative	**Criteria**						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
A_{11}	0.059 70	0.071 43	0.074 07	0.059 70	0.068 18	0.062 50	0.067 80
A_{12}	0.044 78	0.023 81	0.037 04	0.044 78	0.068 18	0.062 50	0.050 85
A_{13}	0.074 63	0.047 62	0.074 07	0.074 63	0.068 18	0.062 50	0.067 80
A_{14}	0.059 70	0.047 62	0.074 07	0.059 70	0.068 18	0.062 50	0.067 80
A_{21}	0.059 70	0.071 43	0.074 07	0.059 70	0.045 45	0.062 50	0.050 85
A_{31}	0.059 70	0.071 43	0.074 07	0.059 70	0.045 45	0.041 67	0.050 85
A_{41}	0.059 70	0.047 62	0.037 04	0.059 70	0.045 45	0.062 50	0.050 85
A_{51}	0.074 63	0.071 43	0.074 07	0.074 63	0.022 73	0.041 67	0.067 80
A_{52}	0.074 63	0.095 24	0.074 07	0.074 63	0.022 73	0.041 67	0.067 80
A_{61}	0.059 70	0.047 62	0.037 04	0.059 70	0.068 18	0.062 50	0.067 80
A_{62}	0.059 70	0.023 81	0.037 04	0.059 70	0.068 18	0.062 50	0.067 80
A_{63}	0.059 70	0.047 62	0.037 04	0.059 70	0.06818	0.062 50	0.067 80
A_{71}	0.074 63	0.095 24	0.074 07	0.074 63	0.090 91	0.083 33	0.067 80
A_{81}	0.059 70	0.095 24	0.074 07	0.059 70	0.090 91	0.083 33	0.067 80
A_{91}	0.059 70	0.071 43	0.111 11	0.059 70	0.090 91	0.083 33	0.067 80
A_{101}	0.059 70	0.071 43	0.037 04	0.059 70	0.068 18	0.062 50	0.050 85
Sum	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00
Benefit/Cost	benefit	cost	Cost	benefit	Benefit	benefit	benefit

Table 3: Normalised value matrix: Criteria versus Alternative – SAW method

Alternative	Criteria						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
A_{11}	0.059 70	0.045 98	0.046 15	0.059 70	0.068 18	0.062 50	0.067 80
A_{12}	0.044 78	0.137 93	0.092 31	0.044 78	0.068 18	0.062 50	0.050 85
A_{13}	0.074 63	0.068 97	0.046 15	0.074 63	0.068 18	0.062 50	0.067 80
A_{14}	0.059 70	0.068 97	0.046 15	0.059 70	0.068 18	0.062 50	0.067 80
A_{21}	0.059 70	0.045 98	0.046 15	0.059 70	0.045 45	0.062 50	0.050 85
A_{31}	0.059 70	0.045 98	0.046 15	0.059 70	0.045 45	0.041 67	0.050 85
A_{41}	0.059 70	0.068 97	0.092 31	0.059 70	0.045 45	0.062 50	0.050 85
A_{51}	0.074 63	0.045 98	0.046 15	0.074 63	0.022 73	0.041 67	0.067 80
A_{52}	0.074 63	0.034 48	0.046 15	0.074 63	0.022 73	0.041 67	0.067 80
A_{61}	0.059 70	0.068 97	0.092 31	0.059 70	0.068 18	0.062 50	0.067 80
A_{62}	0.059 70	0.137 93	0.092 31	0.059 70	0.068 18	0.062 50	0.067 80
A_{63}	0.059 70	0.068 97	0.092 31	0.059 70	0.068 18	0.062 50	0.067 80
A_{71}	0.074 63	0.034 48	0.046 15	0.074 63	0.090 91	0.083 33	0.067 80
A_{81}	0.059 70	0.034 48	0.046 15	0.059 70	0.090 91	0.083 33	0.067 80
A_{91}	0.059 70	0.045 98	0.030 77	0.059 70	0.090 91	0.083 33	0.067 80
A_{101}	0.059 70	0.045 98	0.092 31	0.059 70	0.068 18	0.062 50	0.050 85
Sum	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00
Benefit/cost	Benefit	Cost	Cost	Benefit	Benefit	Benefit	Benefit

Elicitation of weights

The SMARTER method was chosen in the elicitation of weights to the evaluation criteria because it uses a swing procedure to attain a constant scale, also uses linear function values in evaluation. Furthermore, the SMARTER method is more precise in generating weights to the evaluation of criteria than the weights of evaluation criteria given by the DMs.

Studies by Sureeyatanapas (2016), Barfod and Leleur (2014), and Roszkowska (2013) identified various methods in which SMARTER uses in generating the weights to the evaluation criteria. The common methods are rank-sum (RS), rank inverse or reciprocal (RR), rank exponent (RE), rank order centroid (ROC), an equal weight (EW).

- **In the ROC weight method**, the weights are given by:

$$W_i(ROC) = \frac{1}{i}\sum_{j=1}^{n}\frac{1}{j} \quad (11)$$

$$i = 1, 2, 3\ldots n$$

*Where **n** represents the number of evaluation criteria; **i** represent the rank*

- **In the RS method**, the weights are the individual ranks normalised by dividing by the sum of ranks. The RS weights are given by:

$$W_i(RS) = \frac{2(n+1-i)}{n(n+1)} \quad (12)$$

$$i = 1, 2, 3, \ldots., n$$

- **In the RR method**, the weights are the inverse of the rank which is normalised by dividing each term by the sum of the inverses. The RR weights are given by:

$$W_i(RR) = \frac{\frac{1}{i}}{\sum_{j=1}^{n}\frac{1}{j}} \quad (13)$$

Where i is the rank of i-th criterion, i = 1, 2,… n.

- **In the EW method**, the weights are given in a uniform distribution of n criteria. The EW is given by:

$$W_i(EW) = \frac{1}{i} \quad (14)$$

Where i the rank, i= 1, 2, 3,…, n.

- **The RE weight method** is a generalisation of the rank sum method. The RE weights are given by:

$$W_i(RE) = \frac{(2(n+1-i))^p}{n(n+1)^p} \quad (15)$$

Where i is the rank of the i-th criterion, p - parameter describing the weights, i= 1,.,n, and n is the number of evaluation criteria.

The parameter *p* may be estimated by the decision-maker using the weight of the most important criterion or through interactive scrolling. *p = 0* results to equal weight method, and *p = 1*, results to rank sum method. Generally, as *p* increases, the distribution of weights becomes steeper.

The criteria are assigned weights with vector $W_j = [w_1, w_2, \ldots\ldots., w_n]$, in which $w_1 \succ w_2 \succ w_3 \succ w_4 \succ \ldots\ldots.. \succ w_n$, which satisfies,

$$\sum_{j=1}^{n} W_j = 1 \tag{16}$$

This study chose the ROC method to estimate the weights of the evaluation criteria because the weights given by this method represents the centre of mass of all weights of the rank order of the criteria. Besides, the method has much less error for ranked criteria and has a clear statistic basis (Roszkowska, 2013; Zardari *et al.*, 2015).

Thus, the weights generated by the SMARTER – ROC weights method for the seven criteria are; $w_1 = 0.3704$, $w_2 = 0.2276$, $w_3 = 0.1561$, $w_4 = 0.1085$, $w_5 = 0.0728$, $w_7 = 0.0204$. The DM ranked the evaluation criteria as $c_1 \succ c_4 \succ c_7 \succ c_3 \succ c_2 \succ c_6 \succ c_5$. This means that; $c_1 \rightarrow w_1, c_4 \rightarrow w_2, c_7 \rightarrow w_3, c_3 \rightarrow w_4, c_2 \rightarrow w_5, c_6 \rightarrow w_6, c_5 \rightarrow w_7$.

The weighted normalised values for the alternative *($V_{ij}(a), V_{+ij}(a)$* and *$V_{-ij}(a)$)* over each criterion was calculated by:

$$V_{ij}(a) = W_j a_{ij} \tag{17}$$

$$V_{+ij}(a) = W_{+j} a_{+ij} \tag{18}$$

$$V_{-ij}(a) = W_{-j} a_{-ij} \tag{19}$$

i = 1, 2,…,n, and j = 1, 2, 3, …., m

Where w_j represents the weight of j criteria, $a_{ij}$and b_{ij} represent the normalised value of alternative i concerning criteria j, *$V_{ij}(a)$, $V_{+ij}(a)$ and $V_{ij}(a)$ represents the weighted normalised values of the alternatives for the COPRAS method, benefit, and cost criteria for the SAW method respectively.*

The weighted normalised matrices for the COPRAS and SAW methods were obtained as given in Table 4 and Table 5 respectively.

Table 4: Weighted normalised matrix: Criteria versus Alternative – COPRAS method

	Criteria						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Alternative	**0.3704**	**0.0728**	**0.1085**	**0.2276**	**0.0204**	**0.0442**	**0.1561**
A_{11}	0.022 11	0.005 20	0.008 04	0.013 59	0.001 39	0.002 76	0.010 58
A_{12}	0.016 59	0.001 73	0.004 02	0.010 19	0.001 39	0.002 76	0.007 94
A_{13}	0.027 64	0.003 47	0.008 04	0.016 99	0.001 39	0.002 76	0.010 58
A_{14}	0.022 11	0.003 47	0.008 04	0.013 59	0.001 39	0.002 76	0.010 58
A_{21}	0.022 11	0.005 20	0.008 04	0.013 59	0.000 93	0.002 76	0.007 94
A_{31}	0.022 11	0.005 20	0.008 04	0.013 59	0.000 93	0.001 84	0.007 94
A_{41}	0.022 11	0.003 47	0.004 02	0.013 59	0.000 93	0.002 76	0.007 94
A_{51}	0.027 64	0.005 20	0.008 04	0.016 99	0.000 46	0.001 84	0.010 58
A_{52}	0.027 64	0.006 93	0.008 04	0.016 99	0.000 46	0.001 84	0.010 58
A_{61}	0.022 11	0.003 47	0.004 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{62}	0.022 11	0.001 73	0.004 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{63}	0.022 11	0.003 47	0.004 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{71}	0.027 64	0.006 93	0.008 04	0.016 99	0.001 85	0.003 68	0.010 58
A_{81}	0.022 11	0.006 93	0.008 04	0.013 59	0.001 85	0.003 68	0.010 58
A_{91}	0.022 11	0.005 20	0.012 06	0.013 59	0.001 85	0.003 68	0.010 58
A_{101}	0.022 11	0.005 20	0.004 02	0.013 59	0.001 39	0.002 76	0.007 94
Benefit/Cost	benefit	cost	Cost	Benefit	Benefit	benefit	Benefit

Table 5: Weighted normalised matrix: Criteria versus Alternative – SAW method

	Criteria						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Alternative	**0.3704**	**0.0728**	**0.1085**	**0.2276**	**0.0204**	**0.0442**	**0.1561**
A_{11}	0.022 11	0.003 35	0.005 01	0.013 59	0.001 39	0.002 76	0.010 58
A_{12}	0.016 59	0.010 04	0.010 02	0.010 19	0.001 39	0.002 76	0.007 94
A_{13}	0.027 64	0.005 02	0.005 01	0.016 99	0.001 39	0.002 76	0.010 58
A_{14}	0.022 11	0.005 02	0.005 01	0.013 59	0.001 39	0.002 76	0.010 58
A_{21}	0.022 11	0.003 35	0.005 01	0.013 59	0.000 93	0.002 76	0.007 94
A_{31}	0.022 11	0.003 35	0.005 01	0.013 59	0.000 93	0.001 84	0.007 94
A_{41}	0.022 11	0.005 02	0.010 02	0.013 59	0.000 93	0.002 76	0.007 94
A_{51}	0.027 64	0.003 35	0.005 01	0.016 99	0.000 46	0.001 84	0.010 58
A_{52}	0.027 64	0.002 51	0.005 01	0.016 99	0.000 46	0.001 84	0.010 58
A_{61}	0.022 11	0.005 02	0.010 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{62}	0.022 11	0.010 04	0.010 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{63}	0.022 11	0.005 02	0.010 02	0.013 59	0.001 39	0.002 76	0.010 58
A_{71}	0.027 64	0.002 51	0.005 01	0.016 99	0.001 85	0.003 68	0.010 58
A_{81}	0.022 11	0.002 51	0.005 01	0.013 59	0.001 85	0.003 68	0.010 58
A_{91}	0.022 11	0.003 35	0.003 34	0.013 59	0.001 85	0.003 68	0.010 58
A_{101}	0.022 11	0.003 35	0.010 02	0.013 59	0.001 39	0.002 76	0.007 94
Benefit/cost	Benefit	cost	cost	Benefit	Benefit	benefit	benefit

Note: The columns C_1, C_4, C_5, C_6, and C_7 on both Tables 2 and 3, and Tables 4 and 5 have the same data because the score values have normalised with the same formulas as given in equations (2) and (3).

Ranking of alternatives

This study performed the ranking of alternatives through COPRAS and SAW methods. These methods are among the MAUT methods used for ranking the alternatives as explained by Volvačiovas *et al.* (2013). The COPRAS method uses both the maximizing (benefits) and minimizing (costs) criteria values in evaluation separately, while SAW method uses only benefit criteria after converting cost to benefit criteria (Mondal *et al.*, 2017; Podvezko, 2011; Stanujkić *et al.*, 2013; Velasquez & Hester, 2013; Volvačiovas *et al.*, 2013).

- **Ranking of Alternatives using the COPRAS method**

The ranking process passes through the following steps:

Firstly, the sum of weighted normalised values for both the maximization (Benefit) and minimization (cost) criteria were found. The performance of each alternative through all criteria is the weighted normalised sum of each criterion which is given by the MAVT value function as shown in equation 20 and equation 21.

$$S_{+i} = \sum_{j=1}^{n} W_{+j}\, a_{+ij} = \sum_{j=1}^{n} V_{+ij}\,(a) \qquad (20)$$

$$S_{-i} = \sum_{j=1}^{n} W_{-j}\, a_{-ij} = \sum_{j=1}^{n} V_{-ij}\,(a) \qquad (21)$$

$$i = 1, 2, 3, \ldots, n$$

Where S_{+i} *and* S_{-i} *are sums of maximizing (benefit) and minimizing (cost) weighted normalized criteria respectively*.

The highest value of S_{+i} represents the best alternative, and the least value of S_{-i} represents the best alternative. The S_{+i} and S_{-i} values explain the degree of importance achieved by each alternative. The significance of the alternative based on the positive alternatives S_{+i} and negative alternatives S_{-i} features are established as given in Table 6.

Table 6: Sums of the weighted normalised values – COPRAS method

Alternative	S_{+i}	Value	S_{-i}	Value
A_{11}	S_{+11}	0.050 44	S_{-11}	0.013 24
A_{12}	S_{+12}	0.038 87	S_{-12}	0.005 75
A_{13}	S_{+13}	0.059 36	S_{-13}	0.011 50
A_{14}	S_{+14}	0.050 44	S_{-14}	0.011 50
A_{21}	S_{+21}	0.047 33	S_{-21}	0.013 24
A_{31}	S_{+31}	0.046 41	S_{-31}	0.013 24
A_{41}	S_{+41}	0.047 33	S_{-41}	0.007 49
A_{51}	S_{+51}	0.057 52	S_{-51}	0.013 24
A_{52}	S_{+52}	0.057 52	S_{-52}	0.014 97
A_{61}	S_{+61}	0.050 44	S_{-61}	0.007 49
A_{62}	S_{+62}	0.050 44	S_{-62}	0.005 75
A_{63}	S_{+63}	0.050 44	S_{-63}	0.007 49
A_{71}	S_{+71}	0.060 75	S_{-71}	0.014 97
A_{81}	S_{+81}	0.051 82	S_{-81}	0.014 97
A_{91}	S_{+91}	0.051 82	S_{-91}	0.017 26
A_{101}	S_{+101}	0.047 79	S_{-101}	0.09 22

Secondly, the study determined the relative significances or priorities ***(P_i)*** of the alternative by using the relationship given by equation 22 below.

$$P_i = S_{+i} + \frac{S_{-min} \sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{S_{-min}}{S_{-i}}} \tag{22}$$

Where S_{-min} is the minimum value of S_{-i}, m is the number of alternatives.

The equation can be simplified to

$$P_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{1}{S_{-i}}} \tag{23}$$

The greatest value of P_i, represents the most prioritised alternative. The relative significance value of an alternative indicates the degree of fulfilment achieved by that alternative.

The alternative with the highest priority value (P_{max}) is the best choice among the evaluated alternatives. For example, *i = 1*

$$P_1 = S_{+1} + \frac{\sum_{i=1}^{16} S_{-i}}{S_{-1} \sum_{i=1}^{16} \frac{1}{S_{-i}}}$$

$$= 0.050\ 438 + \frac{0.181\ 3}{0.013\ 237(159\ 1.370\ 762\ 6)}$$

$$P_1 = 0.059\ 045$$

Note: $P_1 = P_{11}$, $S_{+1} = S_{+11}$, $S_{-1} = S_{-11}$

The relative significance of other alternatives was computed as shown in Table 7.

Thirdly, the quantitative Utilities (U_i) which are determined by comparison with the priorities of all alternatives with the one with maximum relative significance were computed by equation 24.

$$U_i = \left(\frac{P_i}{P_{max}}\right) x\ 100\ \% \qquad (24)$$

The U_i values of the alternatives range from 0 % to 100 %. Table 7 shows the U_i values that were computed in this study.

Table 7: Priorities (P_i), quantity utility (U_i), and ranking of alternative – COPRAS method

Rank	**Alt**	**P_i**	**U_i (%)**	**Rank**	**Alt**	**P_i**	**U_i (%)**
1.	A_{71}	0.073 31	100.000 00	9.	A_{41}	0.062 15	84.772 14
2.	A_{62}	0.070 37	95.994 98	10.	A_{91}	0.061 04	83.266 49
3.	A_{63}	0.070 13	95.658 05	11.	A_{14}	0.060 88	83.050 96
4.	A_{13}	0.069 88	95.319 74	12.	A_{11}	0.059 05	80.543 73
5.	A_{51}	0.066 52	90.745 89	13.	A_{12}	0.058 14	79.313 31
6.	A_{52}	0.065 13	88.845 69	14.	A_{101}	0.057 01	77.769 14
7.	A_{61}	0.064 34	87.772 14	15.	A_{21}	0.056 34	76.851 09
8.	A_{81}	0.063 69	86.882 74	16.	A_{31}	0.055 13	75.196 43

Basing on the results given in Table 7 above, the ranking of the alternatives for WLM is given as $A_{71} \succ A_{62} \succ A_{63} \succ A_{13} \succ A_{51} \succ A_{52} \succ A_{61} \succ A_{81} \succ A_{41} \succ A_{91} \succ A_{14} \succ A_{11} \succ A_{12} \succ A_{101} \succ A_{21} \succ A_{31}$ with A_{71} indicates the best alternative with 100% utility degree and A_{31} indicates the worst alternative with 75.196 43% utility degree.

- **Ranking of alternatives by using the SAW Method**

The sum S_i of the weighted values of all criteria were computed using equation 25.

$$S_i = \sum_{j=1}^{n}\left(W_{+j}a_{+ij} + W_{-j}a_{-ij}\right) = \sum_{j=1}^{n}\left(V_{+j}(a) + V_{-j}(a)\right) \quad (25)$$

$$i = 1, 2, 3, \ldots, n$$

*Where w_{+j} and w_{-j} represent the weight of **j** criteria for benefit and cost respectively, a_{ij} and a_{ij} are the normalised values of alternatives i for benefit and cost criteria j respectively and a is the alternative*

The highest value of S_i represents the best ranked or most prioritised alternative. Table 8 shows the computed values of S_i and ranking of alternatives.

Table 8: Value of S_i and ranking of Alternatives – SAW method

Rank	Alternative	Value (S_i)	Rank	Alternative	Value (S_i)
1.	A_{62}	0.070 49	9.	A_{101}	0.061 15
2.	A_{13}	0.069 39	10.	A_{14}	0.060 47
3.	A_{71}	0.068 27	11.	A_{81}	0.059 34
4.	A_{51}	0.065 87	12.	A_{12}	0.058 92
5.	A_{61}	0.065 47	13.	A_{11}	0.058 79
6.	A_{63}	0.065 47	14.	A_{91}	0.058 51
7.	A_{52}	0.065 03	15.	A_{21}	0.055 68
8.	A_{41}	0.062 36	16.	A_{31}	0.054 76

Basing on the results given in Table 8 above, the ranking of alternatives for WLM is given as $A_{62} \succ A_{13} \succ A_{71} \succ A_{51} \succ A_{61}$, $A_{63} \succ A_{52} \succ A_{41} \succ A_{101} \succ A_{14} \succ A_{81} \succ A_{12} \succ A_{11} \succ A_{91} \succ A_{21} \succ A_{31}$ with A_{62} indicates the best alternative and A_{31} indicate the worst alternative.

3.4.2 Model formulation phase II – Numerical optimisation method

(i) Integer linear programming (ILP)

This study used the ILP to assess the operational restrictions of the company and to select the portfolio of alternatives. A special form of ILP, the Binary Integer Programming (BIP) was used. In BIP, all decision variables are limited to deal with binary values (either 0 or 1). The BIP gives the model more meaning in decision making. Numbers 0 and 1 in this programming represent the selection choice of alternatives instead of their arbitrary values.

The formulation of ILP models for COPRAS and SAW methods are the same. The constraints of the two models are the same, the difference is on the objective functions in which they differ on their coefficients.

The study developed the ILP models as follows:

General mathematical model formulation

- **Objective function**

The objective function intends to maximize the relative preference weights or sum of weighted normalised values of alternatives for WLM which help to select the portfolio of alternatives that minimize the operation cost while maximizing revenue, water-saving, supply reliability, and water quality. The values P_i and S_i are considered as the coefficients of the objective function for the MAVT – SMARTER – COPRAS and MAVT – SMARTER - SAW techniques respectively.

The objective functions were expressed using equation 26 and equation 27.

$$Max\ Z = \sum_{i=1}^{m} P_i A_i \quad (26)$$

$$Max\ Z = \sum_{i=1}^{m} S_i A_i \quad (27)$$

Where: i = index. i= 1, 2, 3… m

A_i *= alternative for index i*

P_i *= preference weight of alternative i – COPRAS method*

S_i *= Sum of the weighted normalised value of alternative i – SAW method*

- **Constraints**

The constraints of this objective function are:

Implementation cost constraints: These represent the cost required to implement each alternative and the limit of resources available. This group includes the constraints given in equation 28.

$$\sum_{i=1}^{m} b_i A_i \leq c_i \tag{28}$$

Where: b_i = *amount of resources (funds) used to implement alternative i.*

c_i = *cost limit for alternative i.*

Conflicting constraint**:** These represent the multiple-choice alternatives found in one category of alternatives. The constraint ensures that at least one sub- alternative can be adopted in WLM, as expressed in equation 29.

$$\sum_{i=1}^{m} A_i \geq 1 \tag{29}$$

The optimal number of alternatives in a portfolio constraint: This represents the maximum number of alternatives to be selected to form a portfolio of alternatives for WLM, as expressed in equation 30.

$$\sum_{i=1}^{m} A_i \leq 16 \tag{30}$$

Binary (decision variables) constraint: This constraint restricts all decision variables to take on binary value (either 0 or 1), where 1 means that the alternative is part (selected) of the solution and 0 means the alternative is not part of the solution, as expressed in equation 31.

$$A_i = (0 \; or \; 1) \tag{31}$$

Specific mathematical model formulation

The models' equations 26 to 31 consider sixteen alternatives and seven evaluation criteria. For each criterion, the COPRAS and SAW methods were used to rank the alternatives. This study has formulated the specific mathematical ILP models, equations 32 and 33 are the objective functions subject to constraints equations 34 to 39.

- **The COPRAS method**

Objective function

Maximize $Z = 0.059\,05A_{11} + 0.058\,14A_{12} + 0.069\,88A_{13} + 0.060\,88A_{14} + 0.056\,34A_{21} + 0.055\,13A_{31} + 0.062\,15A_{41} + 0.066\,52A_{51} + 0.065\,13A_{52} + 0.064\,34A_{61} + 0.070\,37A_{62} + 0.070\,13A_{63} + 0.073\,31A_{71} + 0.063\,69A_{81} + 0.061\,04A_{91} + 0.057\,01A_{101}$ (32)

- **The SAW method**

Objective function

Maximize $Z = 0.058\,79A_{11} + 0.058\,92A_{12} + 0.069\,39A_{13} + 0.060\,47A_{14} + 0.055\,68A_{21} + 0.054\,76A_{31} + 0.062\,36A_{41} + 0.065\,87A_{51} + 0.065\,03A_{52} + 0.065\,47A_{61} + 0.070\,49A_{62} + 0.065\,47A_{63} + 0.068\,27A_{71} + 0.059\,34A_{81} + 0.058\,51A_{91} + 0.061\,15A_{101}$ (33)

Subject to constraints

$14.58A_{11} + 14.58A_{12} + 14.58A_{13} + 14.58A_{14} + 31.59A_{21} + 29.16A_{31} + 2.43A_{41} + 75.33A_{51} + 75.33A_{52} + 4.86A_{61} + 4.86A_{62} + 4.86A_{63} + 29.16A_{71} + 51.03A_{91} + 4.86A_{101} \leq 243$ (34)

$A_{11} + A_{12} + A_{13} + A_{14} \geq 1$ (35)

$A_{51} + A_{52} \geq 1$ (36)

$A_{61} + A_{62} + A_{63} \geq 1$ (37)

$A_{11} + A_{12} + A_{13} + A_{14} + A_{21} + A_{31} + A_{41} + A_{51} + A_{52} + A_{61} + A_{62} + A_{63} + A_{71} + A_{81} + A_{91} + A_{101} \leq 16$ (38)

$A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{31}, A_{41}, A_{51}, A_{52}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, A_{101}$ =*[0 or 1]* (39)

Where: Equations 32 and 33 are the objective functions for COPRAS and SAW methods in which the coefficients are the relative preferences *(P_i)* and the sum of weighted normalised values *(S_i)* respectively. Equation 34 is the budget restriction constraint for the implementation of preventive actions which is in 1 000 000/= Tanzanian Shillings (TZS). Equations 35 to 37 represents the multiple-choice alternatives found in one category of alternative. The constraints ensure that at least one alternative can be adopted in WLM. Equation 38 is the constraint that represents the optimal number of alternatives to be selected in a portfolio, while equation 39 represents the binary decision variables (A_i) constraints with the values taking on integers 0 or 1, where 1 means that A_i is the selected alternative and 0 means otherwise.

(ii) Sensitivity analysis

This is a fundamental concept for the successive use and execution of quantitative decision models. Sensitivity analysis intends to assess the stability (robustness) of the optimum solution by changing some parameters (Goodridge, 2016). For that, the ranking of the criteria was changed after doing mathematical calculations which led to the change of weights of the criteria using mathematical formulae as explained by Triantaphyllou (2000).

Sensitivity analysis can be done by checking the following:

How critical each criterion is: This is done on the weights of the criteria to determine the smallest change in the existing weights of the criteria that can modify the current position of the strategies. It examines the effect of the alternatives in the weights of the decision criteria.

How critical are the various functional measures of the alternatives in the ranking of the alternatives*:* This examines the performance of alternatives by using a single evaluation criterion at a time on finding the final ranking of the alternatives.

This research considered the change of weights of the criteria to analyse the sensitivity of the model.

Two important definitions were considered when evaluating the sensitivity of the model.

- **The criticality degree of the criterion C_k, (D'_k):** This is the smallest percentage amount that causes the current value of the weight (w_k) to change, and results in the change of the existing ranking of strategies. The D'_k is calculated by equation 40.

$$D'_k = min\,|\delta'_{k,i,j}\,|, \forall\, n \geq k \geq 1 \;and\; 1 \leq \; i \; < \; j \; \leq \; m \qquad (40)$$

- **The sensitivity coefficient of criterion C_k, (sens(C_k))*:*** This is the inverse of criticality degree. The $sens\ (C_k)$ is calculated by equation 41.

$$sens\ (C_k) = \frac{1}{D'_k}, for\ any\ n \geq k \geq 1 \qquad (41)$$

The most important decision criterion is the one with the highest sensitivity coefficient. The minimum percentage change (min $\delta'_{k,i,j}$) in criteria weights are given by the formulae (42) to (45):

$$min\ \delta'_{k,i,j} = \frac{min\delta_{k,i,j}}{w_k}\ x\ 100, \quad for\ 1 \leq i < j \leq m\ and\ 1 \leq k \leq n \tag{42}$$

$min\delta_{k,i,j}$ represents the minimum absolute change in criteria weights w_k, and w_k is the current weight of criterion C_k

and

$$\delta_{k,i,j} < \left(\frac{P_j - P_i}{a_{j,k} - a_{i,k}}\right) \qquad if\ a_{j,k} > a_{i,k} \quad or \tag{43}$$

$$\delta_{k,i,j} > \left(\frac{P_j - P_i}{a_{j,k} - a_{i,k}}\right) \qquad if\ a_{j,k} < a_{i,k}$$

For $\delta_{k,i,j}$ to be achievable the condition in equation 44 must be met.

$$\left(\frac{P_j - P_i}{a_{j,k} - a_{i,k}}\right) \leq w_k \tag{44}$$

Where P_i, P_j, and a_{ik}, a_{jk} are relative preferences and normalised values of alternatives A_i and A_j respectively on weight w_k, and $\delta_{k,i,j}$ is the change in criteria weights.

Therefore, the percentage change in criteria weights ($\delta'_{k,i,j}$) is given as:

$$\delta'_{k,i,j} < \left(\frac{P_j - P_i}{a_{j,k} - a_{i,k}}\right) x\ \frac{100}{w_k}\ , if\ a_{j,k} > a_{i,k} \qquad or \tag{45}$$

$$\delta'_{k,i,j} > \left(\frac{P_j - P_i}{a_{j,k} - a_{i,k}}\right) x\ \frac{100}{w_k}\ , if\ a_{j,k} < a_{i,k}$$

Furthermore, this study calculated the new weights using equation 46.

$$w_k^* = w_k - min\delta_{k,i,j} \tag{46}$$

$$Where\ 0 \leq w_k^*,\ 0 \leq w_k - min\delta_{k,i,j},\ min\ \delta_{k,i,j} \leq w_k$$

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Investigation and evaluation of WLM strategies used at MUWSA

The study investigated and evaluated sixteen alternatives employed at MUWSA to control or reduce water losses in the WDS by using seven evaluation criteria.

To examine the strategies, the questionnaire was administered to the MUWSA DM and an interview was conducted. The DM identified 16 alternatives that they use in WLM in their WDS as explained in 3.4.2 (i).

The alternatives were evaluated by seven decision criteria as explained in 3.4.2 (i). The results obtained are shown in Table 7 and Table 8 for the COPRAS and SAW methods, respectively.

4.2 Development and analysis of the decision models for selecting the strategies

The models formulated in this study are given by equations 32 to 39 and have been solved using LINDO version 6.1 software whose codes are given in Appendices 1 and 3, the computations yielded the following results:

4.2.1 The ILP solutions

(i) The COPRAS method

OBJECTIVE FUNCTION VALUE 0.836 515

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
A11	1	-0.059 045	A52	0	-0.065 131
A12	1	-0.058 143	A61	1	-0.064 344
A13	1	-0.069 877	A62	1	-0.070 372
A14	1	-0.060 883	A63	1	-0.070 125
A21	0	-0.056 338	A71	1	-0.073 308
A31	0	-0.055 125	A81	1	-0.063 692
A41	1	-0.062 150	A91	1	-0.061 041
A51	1	-0.066 524	A101	1	-0.057 011

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	7.289 999	0.000 000

(ii) The SAW method

OBJECTIVE FUNCTION VALUE: 0.824 56

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
A11	1	-0.058 790	A52	0	-0.065 030
A12	1	-0.058 920	A61	1	-0.065 470
A13	1	-0.069 390	A62	1	-0.070 490
A14	1	-0.060 470	A63	1	-0.065 470
A21	0	-0.055 680	A71	1	-0.068 270
A31	0	-0.054 760	A81	1	-0.059 340
A41	1	-0.062 360	A91	1	-0.058 570
A51	1	-0.065 870	A101	1	-0.061 150

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	7.289 999	0.000 000

In both models, the results establish that; variables A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101} give values 1, while variables A_{21}, A_{31}, and A_{51} give 0 values. This means variables with values 1 are alternatives selected to form a portfolio of the best alternatives and those with 0 values are eliminated. The eliminated alternatives are of less importance and their roles can be performed by the remaining alternatives.

Furthermore, the selected alternatives represent a total cost of TZS 235.71 million which is 97% of the total cost budgeted by the Authority. This means the Authority will save TZS 7.29 million if the alternatives of this portfolio are implemented. The maximum sum of values of the selected alternatives are Z = 0.836 515 and Z = 0.824 56, respectively for COPRAS and SAW methods.

4.3 Comparison between COPRAS and SAW methods in prioritising and selecting the strategies

When considering Table 7, the values of U_i show no alternatives which are below 50%, and when comparing the values of S_i of alternatives versus the value of the most preferable alternative in Table 8 it is observed that there are no alternatives which are below 50%. This implies that all alternatives are important in WLM.

Table 7 shows the rank of alternatives as $A_{71} \succ A_{62} \succ A_{63} \succ A_{13} \succ A_{51} \succ A_{52} \succ A_{61} \succ A_{81} \succ A_{41} \succ A_{91} \succ A_{14} \succ A_{11} \succ A_{12} \succ A_{101} \succ A_{21} \succ A_{31}$ with A_{71} (replacement of deteriorated pipes) indicating the best alternative with 100% utility degree and A_{31} (network zoning and DMA) as the least important alternative with 75.196 43% utility degree. Table 8 ranks $A_{62} \succ A_{13} \succ A_{71} \succ A_{51} \succ A_{61}$, $A_{63} \succ A_{52} \succ A_{41} \succ A_{101} \succ A_{14} \succ A_{81} \succ A_{12} \succ A_{11} \succ A_{91} \succ A_{21} \succ A_{31}$ with A_{62} (comparison between the readings of bulk meter and customers' meter on detection of physical or apparent water losses) indicating the best alternative and A_{31} (network zoning) indicating the least important alternative in WLM.

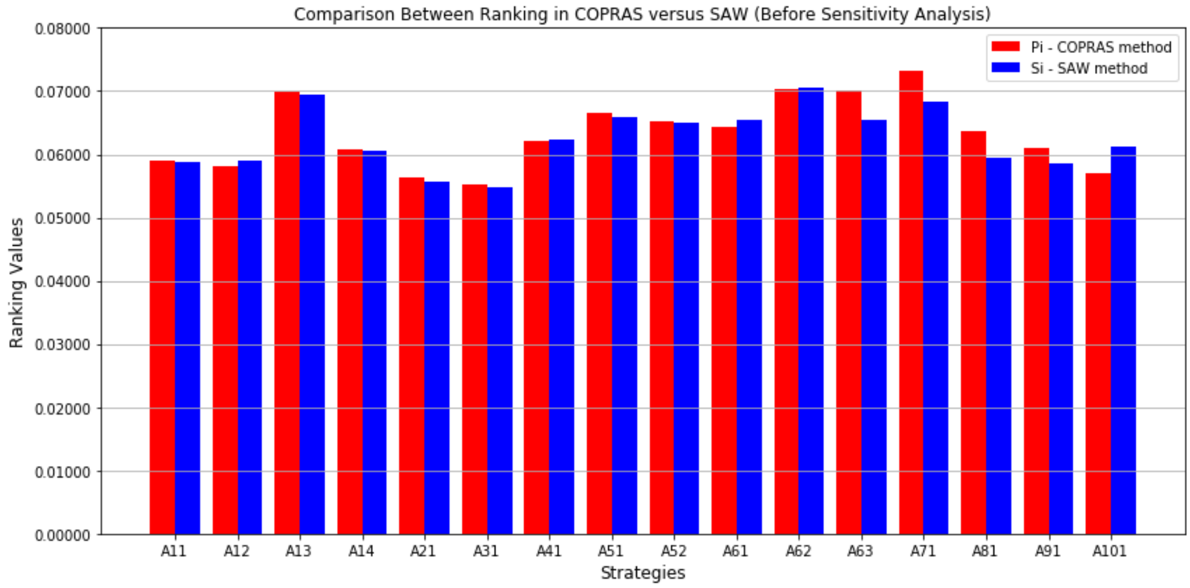

Figure 5: Ranking comparison between COPRAS and SAW methods (Before sensitivity analysis)

Moreover, in both ranking of alternatives, it is established that illegal use (A_{21}) and pressure management (A_{101}) are not critical alternatives for WLM at MUWSA like in many WDS. The reasons are that the Authority is performing well to serve the customers and most of the water sources at MUWSA are springs hence water flow is by gravity. The Authority's 5 years corporate strategic plan is to avoid the use of boreholes sources which will lead to zero use of pumps and according to research data (MUWSA, 2019) the Authority will reduce the operating costs to about 2%, the budget set for pressure management in the WDS.

However, after solving the ILP equations, the following alternatives were selected: A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101}. Three alternatives, A_{21}, A_{31}, and A_{52} were eliminated from the list. The selected portfolio of alternatives cost 97% (TZS 235.71 million) of the total budget set for WLM by the Authority. Thus, the Authority will save 3% (TZS 7.29 million) if this portfolio of alternatives is implemented. These alternatives have maximum sum values of Z = 0.836 52 and 0.824 56, respectively for the models formulated from COPRAS and SAW methods.

The similarities of the results for the two models indicate that the two methods COPRAS and SAW are appropriate and efficient for decision-making and in solving issues in water management and planning in the situation similar to that of MUWSA.

4.4 Performance of sensitivity analysis

4.4.1 Determination of the most critical criterion

(i) The COPRAS method

By considering the results in Table 2 and Table 7, and equation 43 and equation 44 the values of change in criteria weights ($\delta_{k,i,j}$) were obtained as shown below:

$$\delta_{1,11,12} > \frac{P_{12}-P_{11}}{a_{1,12}-a_{1,11}} > \frac{0.058\ 143-0.059\ 045}{0.044\ 78-0.059\ 70} \tag{47}$$

$$\delta_{1,11,12} > 0.060\ 456$$

And

$$\frac{P_{12}-P_{11}}{a_{1,12}-a_{1,11}} \leq w_1 \Rightarrow 0.060\ 456 \leq 0.3704 \tag{48}$$

.

$$\delta_{1,11,16} > \frac{P_{16}-P_{11}}{a_{1,16}-a_{1,11}} > \frac{0.057\ 011-0.059\ 045}{0.05970-0.059\ 70} \tag{49}$$

$$\delta_{1,11,16} > \infty$$

It should be noted that all the values that do not satisfy the condition given in equation 44 are termed as infeasible (IF). That means it is difficult to change the rank of the alternative with any weight modification.

The values for other combinations of criteria and pairs of alternatives are shown in Appendix 6 and the values of per cent change in criteria weights $\left(\delta'_{k,i,j}\right)$ given in Appendix 7 were obtained using equation 45.

By considering Appendix 6, the negative changes indicate the increase of new weights(w_k^*), while positive changes indicate the decrease of new weights when applying equation 46. The new weights for each criterion were found using the minimum absolute change (highlighted in yellow colour) as per the following procedure:

$$\text{C}_1\text{: } \mathrm{w}_1{}^* = \mathrm{w}_1 - \min\delta_{1,13,63} \tag{50}$$

$$= 0.3704 - (-0.0166) = 0.387$$

Doing the same for all criteria, the following was found: $w_2^* = 0.2442$, $w_3^* = 0.1029$, $w_4^* = 0.1042$, $w_5^* = 0.0662$, $w_6^* = 0.0366$, and $w_7^* = 0.0135$ for criteria C_4, C_7, C_3, C_2, C_6, and C_5 respectively.

From Appendix 7, the minimum values of the percentage change in criteria weights $(min\delta'_{k,i,j})$ in each criterion are: $\delta'_{1,13,63} = -4.484\,34$, $\delta'_{5,14,91} = 9.115\,39$, $\delta'_{4,14,91} = 3.931\,80$, $\delta'_{2,13,63} = -7.297\,89$, $\delta'_{7,14,91} = 34.073\,53$, $\delta'_{6,14,91} = 17.160\,63$, and $\delta'_{3,11,12} = 34.090\,33$ corresponding to C_1, C_2, C_3, C_4, C_5, C_6, and C_7 correspondingly.

By referring to equations 40 and 41, the criticality degree and sensitivity coefficient of each criterion is given below. The criticality degree of the criteria are:

$$D_1' = min|\delta'_{1,13,63}| = |-4.484| = 4.484 \tag{51}$$

$$D_2' = min|\delta'_{5,14,91}| = 9.115 \tag{52}$$

$$D_3' = min|\delta'_{4,14,91}| = 3.932 \tag{53}$$

$$D_4' = min|\delta'_{2,13,63}| = |-7.298| = 7.298 \tag{54}$$

$$D_5' = min|\delta'_{7,14,91}| = 34.074 \tag{55}$$

$$D_6' = min|\delta'_{6,14,91}| = 17.161 \tag{56}$$

$$D_7' = min|\delta'_{3,13,12}| = 34.090 \tag{57}$$

The sensitivity coefficients of the criteria are:

$$Sens(C_1) = \frac{1}{D_1'} = \frac{1}{4.484} = 0.223 \tag{58}$$

$$Sens(C_2) = \frac{1}{D_2'} = \frac{1}{9.115} = 0.110 \tag{59}$$

$$Sens(C_3) = \frac{1}{D_3'} = \frac{1}{3.932} = 0.254 \tag{60}$$

$$Sens(C_4) = \frac{1}{D_4'} = \frac{1}{7.298} = 0.137 \tag{61}$$

$$Sens(C_5) = \frac{1}{D_5'} = \frac{1}{34.074} = 0.029 \tag{62}$$

$$Sens(C_6) = \frac{1}{D_6'} = \frac{1}{17.161} = 0.058 \tag{63}$$

$$Sens(C_7) = \frac{1}{D'_7} = \frac{1}{34.090} = 0.029 \quad (64)$$

This analysis established the new most critical decision criterion as $C_3 \succ C_1 \succ C_4 \succ C_2 \succ C_6 \succ C_5 \succ C_7$. This means that: $C_3 \rightarrow W_1$, $C_1 \rightarrow W_2$, $C_4 \rightarrow W_3$, $C_2 \rightarrow W_4$, $C_6 \rightarrow W_5$, $C_5 \rightarrow W_6$, $C_7 \rightarrow W_7$.

(ii) The SAW method

This method considered the results of Table 3 and Table 8 for normalized values and the ranking of alternatives respectively. Then the same procedures were followed as applied to the COPRAS method to find the new ranking of decision criteria.

The minimum percentage values ($\min\delta'_{k,i,j}$) in each criterion (column) were: $\delta'_{1,11,12} = -2.3524$, $\delta'_{5,11,12} = 1.942$, $\delta'_{4,11,12} = 2.5957$, $\delta'_{2,11,12} = -3.8283$, $\delta'_{7,51,61} = -43.1416$, $\delta'_{6,11,91} = -30.4121$, and $\delta'_{3,11,12} = 4.9133$ corresponding to C_1, C_2, C_3, C_4, C_5, C_6, and C_7 correspondingly.

By referring to equations 40 and 41, the criticality degree and sensitivity coefficient of each criterion is given as follows:

$$D'_1 = min|\delta'_{1,11,12}| = |-2.352| = 2.352 \quad (65)$$

$$D'_2 = min|\delta'_{5,11,12}| = 1.924 \quad (66)$$

$$D'_3 = min|\delta'_{4,11,12}| = 2.596 \quad (67)$$

$$D'_4 = min|\delta'_{2,11,12}| = |-3.828| = 3.828 \quad (68)$$

$$D'_5 = min|\delta'_{7,51,61}| = |-43.142| = 43.142 \quad (69)$$

$$D'_6 = min|\delta'_{6,11,91}| = |-30.412| = 30.412 \quad (70)$$

$$D'_7 = min|\delta'_{3,13,12}| = |-4.913| = 4.913 \quad (71)$$

And the sensitivity coefficients of the criteria are:

$$Sens(C_1) = \frac{1}{D'_1} = \frac{1}{2.352} = 0.425 \quad (72)$$

$$Sens(C_2) = \frac{1}{D'_2} = \frac{1}{1.942} = 0.515 \quad (73)$$

$$Sens(C_3) = \frac{1}{D_3'} = \frac{1}{2.596} = 0.385 \tag{74}$$

$$Sens(C_4) = \frac{1}{D_4'} = \frac{1}{3.828} = 0.261 \tag{75}$$

$$Sens(C_5) = \frac{1}{D_5'} = \frac{1}{43.142} = 0.023 \tag{76}$$

$$Sens(C_6) = \frac{1}{D_6'} = \frac{1}{30.412} = 0.033 \tag{77}$$

$$Sens(C_7) = \frac{1}{D_7'} = \frac{1}{4.913} = 0.204 \tag{78}$$

This analysis found out that the new most critical decision criterion is $C_2 \succ C_1 \succ C_3 \succ C_4 \succ C_7 \succ C_6 \succ C_5$. This means that: $C_2 \rightarrow W_1$, $C_1 \rightarrow W_2$, $C_3 \rightarrow W_3$, $C_4 \rightarrow W_4$, $C_7 \rightarrow W_5$, $C_6 \rightarrow W_6$, $C_5 \rightarrow W_7$.

4.4.2 Ranking of alternatives

Using the new ranking of the decision criteria and the weights generated by the SMARTER method, the new weighted normalised values were calculated and ranking of the alternatives was done by the same procedures as in section (3.4.2) (iii). The results are shown in Table 9 to Table 13.

(i) The COPRAS method

Table 9: New weighted normalized matrix: Criteria vs Alternatives – COPRAS method

	Criteria						
	C_1	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**
Alternative	**0.2276**	**0.1085**	**0.3704**	**0.1561**	**0.0442**	**0.0728**	**0.0204**
A_{11}	0.013 59	0.007 75	0.027 44	0.009 32	0.003 01	0.004 55	0.001 38
A_{12}	0.010 19	0.002 58	0.013 72	0.006 99	0.003 01	0.004 55	0.001 04
A_{13}	0.016 99	0.005 17	0.027 44	0.011 65	0.003 01	0.004 55	0.001 38
A_{14}	0.013 59	0.005 17	0.027 44	0.009 32	0.003 01	0.004 55	0.001 38
A_{21}	0.013 59	0.007 75	0.027 44	0.009 32	0.002 01	0.004 55	0.001 04
A_{31}	0.013 59	0.007 75	0.027 44	0.009 32	0.002 01	0.003 03	0.001 04
A_{41}	0.013 59	0.005 17	0.013 72	0.009 32	0.002 01	0.004 55	0.001 04
A_{51}	0.016 99	0.007 75	0.027 44	0.011 65	0.001 00	0.003 03	0.001 38
A_{52}	0.016 99	0.010 33	0.027 44	0.011 65	0.001 00	0.003 03	0.001 38
A_{61}	0.013 59	0.005 17	0.013 72	0.009 32	0.003 01	0.004 55	0.001 38
A_{62}	0.013 59	0.002 58	0.013 72	0.009 32	0.003 01	0.004 55	0.001 38
A_{63}	0.013 59	0.005 17	0.013 72	0.009 32	0.003 01	0.004 55	0.001 38
A_{71}	0.016 99	0.010 33	0.027 44	0.011 65	0.004 02	0.006 07	0.001 38
A_{81}	0.013 59	0.010 33	0.027 44	0.009 32	0.004 02	0.006 07	0.001 38
A_{91}	0.013 59	0.007 75	0.041 16	0.009 32	0.004 02	0.006 07	0.001 38
A_{101}	0.013 59	0.007 75	0.013 72	0.009 32	0.003 01	0.004 55	0.001 04
Benefit/Cost	Benefit	Cost	Cost	Benefit	Benefit	Benefit	Benefit

Table 10: New sums of the weighted normalised values – COPRAS method

Alternative	S_{+i}	**Value**	S_{-i}	**Value**
A_{11}	S_{+11}	0.031 85	S_{-11}	0.035 19
A_{12}	S_{+12}	0.025 78	S_{-12}	0.016 30
A_{13}	S_{+13}	0.037 58	S_{-13}	0.032 60
A_{14}	S_{+14}	0.031 85	S_{-14}	0.032 60
A_{21}	S_{+21}	0.030 50	S_{-21}	0.035 19
A_{31}	S_{+31}	0.028 99	S_{-31}	0.035 19
A_{41}	S_{+41}	0.030 50	S_{-41}	0.018 89
A_{51}	S_{+51}	0.034 06	S_{-51}	0.035 19
A_{52}	S_{+52}	0.034 06	S_{-52}	0.037 77
A_{61}	S_{+61}	0.031 85	S_{-61}	0.018 89
A_{62}	S_{+62}	0.031 85	S_{-62}	0.016 30
A_{63}	S_{+63}	0.031 85	S_{-63}	0.018 89
A_{71}	S_{+71}	0.040 10	S_{-71}	0.037 77
A_{81}	S_{+81}	0.034 38	S_{-81}	0.037 77
A_{91}	S_{+91}	0.034 38	S_{-91}	0.048 91
A_{101}	S_{+101}	0.031 51	S_{-101}	0.021 47

Table 11: New priorities, utility, and rank of alternatives – COPRAS method

Rank	**Alt**	**P_i**	**U_i (%)**	**Rank**	**Alt**	**P_i**	**U_i (%)**
1.	A_{63}	0.082 316	100.000 00	9.	A_{14}	0.056 947	69.180 96
2.	A_{62}	0.079 274	96.304 49	10.	A_{51}	0.056 728	68.914 91
3.	A_{12}	0.073 151	88.866 08	11.	A_{91}	0.055 844	67.841 00
4.	A_{71}	0.072 301	87.833 47	12.	A_{11}	0.054 424	66.115 94
5.	A_{41}	0.069 620	84.576 51	13.	A_{52}	0.054 122	65.749 06
6.	A_{61}	0.068 723	83.486 81	14.	A_{21}	0.053 282	64.728 61
7.	A_{81}	0.064 997	78.960 35	15.	A_{101}	0.052 977	64.358 08
8.	A_{13}	0.063 123	76.683 76	16.	A_{31}	0.050 933	61.874 97

The new ranking of alternatives is $A_{63} \succ A_{62} \succ A_{12} \succ A_{71} \succ A_{41} \succ A_{61} \succ A_{81} \succ A_{13} \succ A_{14} \succ A_{51} \succ A_{91} \succ A_{11} \succ A_{52} \succ A_{21} \succ A_{101} \succ A_{31}$. This relation shows that A_{63} (report from the community on detection of physical or apparent losses) is the most preferable alternative having 100 % utility and A_{31} (network zoning and DMA) is the least preferable alternative having 61.874 97 %.

(ii) The SAW method

Table 12: New weighted normalised matrix: Criteria versus Alternatives – SAW method

	Criteria						
	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	$\mathbf{c_4}$	$\mathbf{c_5}$	$\mathbf{c_6}$	$\mathbf{c_7}$
Alter	**0.2276**	**0.3704**	**0.1561**	**0.1085**	**0.0204**	**0.0442**	**0.0728**
A_{11}	0.013 59	0.017 03	0.007 20	0.006 48	0.001 39	0.002 76	0.004 94
A_{12}	0.010 19	0.051 09	0.014 41	0.004 86	0.001 39	0.002 76	0.003 70
A_{13}	0.016 99	0.025 54	0.007 20	0.008 10	0.001 39	0.002 76	0.004 94
A_{14}	0.013 59	0.025 54	0.007 20	0.006 48	0.001 39	0.002 76	0.004 94
A_{21}	0.013 59	0.017 03	0.007 20	0.006 48	0.000 93	0.002 76	0.003 70
A_{31}	0.013 59	0.017 03	0.007 20	0.006 48	0.000 93	0.001 84	0.003 70
A_{41}	0.013 59	0.025 54	0.014 41	0.006 48	0.000 93	0.002 76	0.003 70
A_{51}	0.016 99	0.017 03	0.007 20	0.008 10	0.000 46	0.001 84	0.004 94
A_{52}	0.016 99	0.012 77	0.007 20	0.008 10	0.000 46	0.001 84	0.004 94
A_{61}	0.013 59	0.025 54	0.014 41	0.006 48	0.001 39	0.002 76	0.004 94
A_{62}	0.013 59	0.051 09	0.014 41	0.006 48	0.001 39	0.002 76	0.004 94
A_{63}	0.013 59	0.025 54	0.014 41	0.006 48	0.001 39	0.002 76	0.004 94
A_{71}	0.016 99	0.012 77	0.007 20	0.008 10	0.001 85	0.003 68	0.004 94
A_{81}	0.013 59	0.012 77	0.007 20	0.006 48	0.001 85	0.003 68	0.004 94
A_{91}	0.013 59	0.017 03	0.004 80	0.006 48	0.001 85	0.003 68	0.004 94
A_{101}	0.013 59	0.017 03	0.014 41	0.006 48	0.001 39	0.002 76	0.003 70
Ben/Cost	Benefit	Cost	Cost	Benefit	Benefit	Benefit	Benefit

Table 13: New value of S_i and the rank of alternatives – SAW method

Rank	Alternative	Value (S_i)	Rank	Alternative	Value (S_i)
1.	A_{62}	0.094 65	8.	A_{51}	0.056 56
2.	A_{12}	0.088 40	9.	A_{71}	0.055 53
3.	A_{61}	0.069 11	10.	A_{11}	0.053 39
3.	A_{63}	0.069 11	11.	A_{91}	0.052 37
4.	A_{41}	0.067 41	12.	A_{52}	0.052 30
5.	A_{13}	0.066 92	13.	A_{21}	0.051 69
6.	A_{14}	0.061 90	14.	A_{31}	0.050 77
8.	A_{101}	0.059 36	15.	A_{81}	0.050 52

The new ranking of alternatives is $A_{62} \succ A_{12} \succ A_{61}$, $A_{63} \succ A_{41} \succ A_{13} \succ A_{14} \succ A_{101} \succ A_{51} \succ A_{71} \succ A_{11} \succ A_{91} \succ A_{52} \succ A_{21} \succ A_{31} \succ A_{81}$ with A_{62} (comparison between the bulk meter and customers' meter on detection of physical or apparent losses) indicating the best alternative and A_{81} (installation of quality pipes) indicating is the worst alternative.

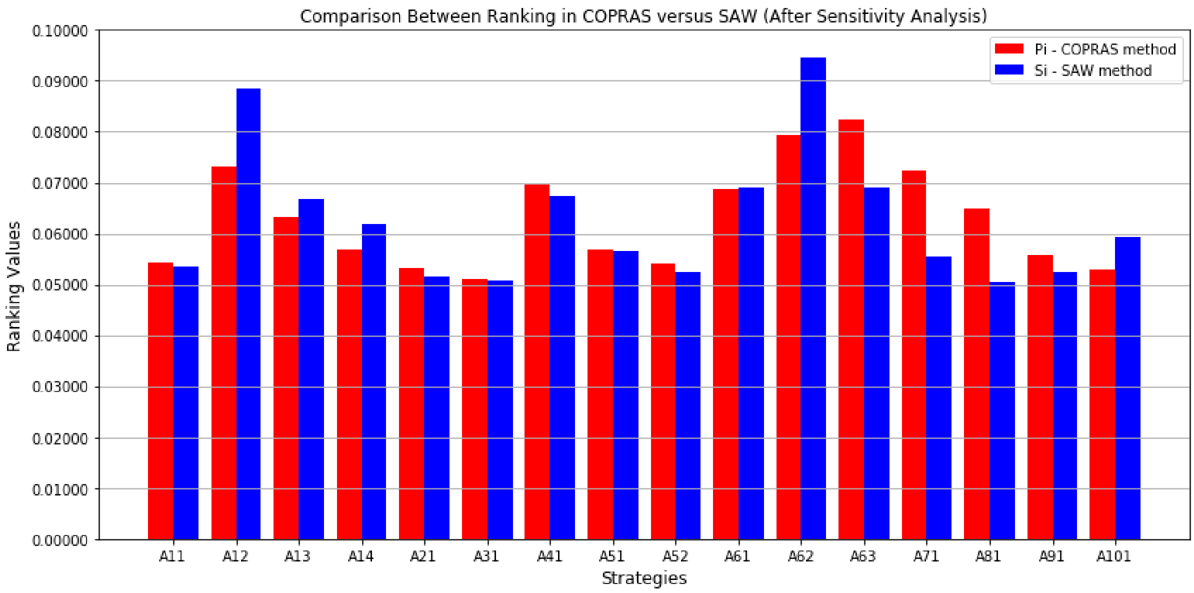

Figure 6: Ranking comparison between COPRAS and SAW methods (After sensitivity analysis)

4.4.3 The ILP models

The same procedures were used in the ILP model formulation and constraints as in section (3.4.3). The differences are the coefficients of the objective functions.

The new ILP models are:

(i) The COPRAS method

Objective function

$$Max\ Z = 0.054\,424A_{11} + 0.073\,151A_{12} + 0.063\,123A_{13} + 0.056\,947A_{14} + 0.053\,282A_{21} + 0.050\,933A_{31} + 0.069\,620A_{41} + 0.057\,280A_{51} + 0.054\,122A_{52} + 0.068\,723A_{61} + 0.079\,274A_{62} + 0.082\,316A_{63} + 0.072\,301A_{71} + 0.064\,997A_{81} + 0.055\,844A_{91} + 0.052\,977A_{101} \quad (79)$$

(ii) The SAW method

Objective function

$$Max\ \ Z = 0.053\,394A_{11} + 0.088\,40A_{12} + 0.066\,92A_{13} + 0.061\,90A_{14} + 0.051\,69A_{21} + 0.050\,77A_{31} + 0.067\,41A_{41} + 0.056\,56A_{51} + 0.052\,30A_{52} + 0.069\,11A_{61} + 0.094\,65A_{62} + 0.069\,11A_{63} + 0.055\,53A_{71} + 0.050\,52A_{81} + 0.052\,37A_{91} + 0.059\,36A_{101} \quad (80)$$

Subject to constraints

$$14.58A_{11} + 14.58A_{12} + 14.58A_{13} + 14.58A_{14} + 31.59A_{21} + 29.16A_{31} + 2.43A_{41} + 75.33A_{51} + 75.33A_{52} + 4.86A_{61} + 4.86A_{62} + 4.86A_{63} + 29.16A_{71} + 51.03A_{91} + 4.86A_{101} \leq 243 \quad (81)$$

$$A_{11} + A_{12} + A_{13} + A_{14} \geq 1 \quad (82)$$

$$A_{51} + A_{52} \geq 1 \quad (83)$$

$$A_{61} + A_{62} + A_{63} \geq 1 \quad (84)$$

$$A_{11} + A_{12} + A_{13} + A_{14} + A_{21} + A_{31} + A_{41} + A_{51} + A_{52} + A_{61} + A_{62} + A_{63} + A_{71} + A_{81} + A_{91} + A_{101} \leq 16 \quad (85)$$

$$A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{31}, A_{41}, A_{51}, A_{52}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, A_{101} = 0 \text{ or } 1 \quad (86)$$

4.4.4 The ILP solutions

After solving the models formulated in section 4.4.3, equations 79 to 86 using LINDO version 6.1 software whose codes are given in Appendices 2 and 4, the computation yielded the following results:

(i) The COPRAS method

OBJECTIVE FUNCTION VALUE: 0.850 425

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
A11	1	-0.054 424	A52	0	-0.054 122
A12	1	-0.073 151	A61	1	-0.068 723
A13	1	-0.063 123	A62	1	-0.079 274
A14	1	-0.056 947	A63	1	-0.082 316
A21	0	-0.053 282	A71	1	-0.072 301
A31	0	-0.050 933	A81	1	-0.064 997
A41	1	-0.069 620	A91	1	-0.055 844
A51	1	-0.056 728	A101	1	-0.052 977

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	7.289 999	0.000 000

(ii) The SAW method

OBJECTIVE FUNCTION VALUE 0.845 23

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
A11	1	-0.053 390	A52	0	-0.052 300
A12	1	-0.088 400	A61	1	-0.069 110
A13	1	-0.066 920	A62	1	-0.094 650
A14	1	-0.061 900	A63	1	-0.069 110
A21	0	-0.051 690	A71	1	-0.055 530
A31	0	-0.050 770	A81	1	-0.050 520
A41	1	-0.067 410	A91	1	-0.052 370
A51	1	-0.056 560	A101	1	-0.059 360

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	7.289 999	0.000 000

The solution shows that the selected portfolio has the same alternatives ($i.e\ A_{11}$, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101}), implementation cost (TZS 235.71 million) as the former selection. The only difference is the sum of values of alternatives in which the new sums are, respectively, Z = 0.850 425 and Z = 0.845 23 for COPRAS and SAW methods. This proves the robustness of the ILP models in selecting the portfolio of alternatives used in water loss management regardless of the ranking of alternatives and the change of weights of evaluation criteria.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The research aimed to optimise the water loss management strategies used in the water distribution system by using MCDM and Numerical Optimisation approaches to prioritise and optimise the strategies used in water loss management in the WDS. The study revealed that the combinations of the MAVT-SMARTER-COPRAS, MAVT – SMARTER – SAW the MCDM methods and ILP a numerical optimisation technique are the appropriate approaches in decision-making in water resource management.

The strategies used at MUWSA to manage water losses in their WDS were investigated and evaluated. Sixteen main alternatives were identified and evaluated by seven decision criteria using MCDM methods of MAVT - SMARTER - COPRAS, and MAVT - SMARTER – SAW.

The COPRAS method ranked the alternative of replacement of deteriorated pipes as the best alternative and the alternative of network zoning by establishing DMA as the least alternative. On the other hand, the SAW method ranks the alternative of comparison between bulk meter with customers' meter to detect physical and apparent losses as the best alternative and also ranks network zoning by establishing DMA as the least alternative.

Furthermore, the two optimisation decision models on selecting the best alternatives for WLM among those used at MUWSA were developed and analysed through the integration of MCDM and ILP methods and they were solved using the LINDO version 6.1 software. Each model selected thirteen strategies/alternatives: advertising campaigns, education campaign in schools, ward meeting with the society, meeting with local leaders, indicators for quantifying the losses, calibration of meters, visual inspection, comparison between the bulk meter and customers' meter, the reports from the community, Replacement of deteriorated (decay) pipes, installation of quality pipes, timely repair of pipe and fitting leaks, and pressure management. The models eliminated three alternatives: illegal control, network zoning by establishing District Metering Areas, and replacement of defect meters. It was established that the selected alternatives cost 97% (TZS 235.71 million) of the total budgets set for water loss management by MUWSA.

The authority will save 3% (TZS 7.3 million) of its budget in which can allocate in other operational activities.

Additionally, the comparison of the two ranking methods, COPRAS and SAW in prioritising and selecting the best alternatives were made. Both methods selected the same portfolio of the best alternatives and the ranking values of alternatives were more than 50% when compared with the best alternative, meaning that the alternatives used for water loss management at MUWSA are all important.

Finally, the sensitivity of the COPRAS and SAW methods was analysed by changing the weights of the decision criteria. The two models selected the same number of alternatives as those selected before changing the weights of decision criteria. This implies that the two methods are robust for selecting the best alternatives applicable in WLM in the WDS.

It is thus concluded that the MCDM methods and the ILP technique are the appropriate tools for evaluating and selecting the alternatives used in organisation planning in UWSSAs. Furthermore, the findings of this study are the best because of the methodology used went further not only on prioritising the alternatives as most of the studies did but also included the ILP technique for selecting the portfolio of the best alternatives, Nevertheless, the study employed a new MCDM ranking method, COPRAS, in its evaluation and compared the results with the SAW method in prioritising and selecting the alternative to validate the methods with other studies done.

5.2 Recommendations

The techniques developed by this study were applied to investigate the sixteen alternatives used by MUWSA administration for WLM in the WDS. The optimal number of alternatives has been identified, if used they can reduce by 3% the cost incurred for WLM by MUWSA.

The findings of this study could assist the water distribution authorities in a setting similar to MUWSA to select cost-effective alternatives.

It is therefore recommended that the same study should be done in large WDS with large coverage areas and more connected customers than that of MUWSA to see how the model could improve the saved revenue.

Moreover, a user-friendly application could be designed which can be used by managers to enter their WLM alternatives and get outputs of selected alternatives that are cost-effective ones.

Furthermore, this study has used MAVT, SMARTER, COPRAS, and SAW MCDM methods, and the ILP technique. It is recommended to extend the study to other MCDM methods such as TOPSIS, ELECTRE and PROMETHEE and optimisation techniques such as MILP, NLP, and MINLP to gain computational simplicity, visualisation, and validation of the methods.

Finally, further studies should be done to investigate the balance between WLM strategies and water supply-demand strategies to see whether the NRW is caused by a low supply of water with high demand or not.

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APPENDICES

Appendix 1: The LINDO software codes before sensitivity analysis – COPRAS method

```
Max 0.05909A11 + 0.06428A12 + 0.06816A13 + 0.06086A14 +
0.05389A21 + 0.05258A31 + 0.06145A41 + 0.05526A51 + 0.005375A52
+ 0.06586A61 + 0.07221A62 + 0.07278A63 + 0.07715A71 + 0.06919A81
+ 0.06533A91 + 0.05889A101
St
14.58A11 + 14.58A12 + 14.58A13 + 14.58A14 + 31.59A21 + 29.16A31
+ 2.43A41 + 75.33A51 + 75.33A52 + 4.86A61 + 4.86A62 + 4.86A63 +
29.16A71 + 51.03A91 + 4.86A101 <=243
A11 + A12 + A13 + A14 >=1
A51 + A52 >=1
A61 + A62 + A63 >=1
A11 + A12 + A13 + A14 + A21 + A31 + A41 + A51 + A52 + A61 + A62
+ A63 + A71 + A81 + A91 + A101 <= 16
END
INT A11
INT A12
INT A13
INT A14
INT A21
INT A31
INT A41
INT A51
INT A52
INT A61
INT A62
INT A63
INT A71
INT A81
INT A91
INT A101
```

Appendix 2: The LINDO software codes after sensitivity analysis – COPRAS method

```
Max 0.054424A11 + 0.073151A12 + 0.063123A13 + 0.056947A14 +
0.053282A21 + 0.050933A31 + 0.069620A41 + 0.056728A51 +
0.054122A52 + 0.068723A61 + 0.079274A62 + 0.082316A63 +
0.072301A71 + 0.064997A81 + 0.055844A91 + 0.052977A101
St

14.58A11 + 14.58A12 + 14.58A13 + 14.58A14 + 31.59A21 + 29.16A31
+ 2.43A41 + 75.33A51 + 75.33A52 + 4.86A61 + 4.86A62 + 4.86A63 +
29.16A71 + 51.03A91 + 4.86A101 <=243
A11 + A12 + A13 + A14 >=1
A51 + A52 >=1
A61 + A62 + A63 >=1
A11 + A12 + A13 + A14 + A21 + A31 + A41 + A51 + A52 + A61 + A62
+ A63 + A71 + A81 + A91 + A101 <= 16
END

INT A11
INT A12
INT A13
INT A14
INT A21
INT A31
INT A41
INT A51
INT A52
INT A61
INT A62
INT A63
INT A71
INT A81
INT A91
INT A101
```

Appendix 3: The LINDO software codes before sensitivity analysis – SAW method

```
Maximize 0.05879A11 + 0.05892A12 + 0.06939A13 + 0.06047A14 +
0.05568A21 + 0.05476A31 + 0.06236A41 + 0.06587A51 + 0.06503A52
+ 0.06547A61 + 0.07049A62 + 0.06547A63 + 0.06827A71 + 0.05934A81
+ 0.05857A91 + 0.06115A101
St
14.58A11 + 14.58A12 + 14.58A13 + 14.58A14 + 31.59A21 +
29.16A31 + 2.43A41 + 75.33A51 + 75.33A52 + 4.86A61 + 4.86A62 +
4.86A63 + 29.16A71 + 51.03A91 + 4.86A101 <=243
A11 + A12 + A13 + A14 >=1
A51 + A52 >=1
A61 + A62 + A63 >=1
A11 + A12 + A13 + A14 + A21 + A31 + A41 + A51 + A52 + A61 +
A62 + A63 + A71 + A81 + A91 + A101 <= 16
END
INT A11
INT A12
INT A13
INT A14
INT A21
INT A31
INT A41
INT A51
INT A52
INT A61
INT A62
INT A63
INT A71
INT A81
INT A91
INT A101
```

Appendix 4: The LINDO software codes before sensitivity analysis – SAW method

```
Maximize 0.05339A11 + 0.08840A12 + 0.06692A13 + 0.06190A14 +
0.05169A21 + 0.05077A31 + 0.06741A41 + 0.05656A51 + 0.05230A52
+ 0.06911A61 + 0.09465A62 + 0.06911A63 + 0.05553A71 +
0.05052A81 + 0.05237A91 + 0.05936A101
St
14.58A11 + 14.58A12 + 14.58A13 + 14.58A14 + 31.59A21 +
29.16A31 + 2.43A41 + 75.33A51 + 75.33A52 + 4.86A61 + 4.86A62 +
4.86A63 + 29.16A71 + 51.03A91 + 4.86A101 <=243
A11 + A12 + A13 + A14 >=1
A51 + A52 >=1
A61 + A62 + A63 >=1
A11 + A12 + A13 + A14 + A21 + A31 + A41 + A51 + A52 + A61 +
A62 + A63 + A71 + A81 + A91 + A101 <= 16
END
INT A11
INT A12
INT A13
INT A14
INT A21
INT A31
INT A41
INT A51
INT A52
INT A61
INT A62
INT A63
INT A71
INT A81
INT A91
INT A10
```

Appendix 5: Questionnaire for water loss management at MUWSA

A. Survey of strategies and Evaluation Criteria

This questionnaire aims to gather information on water loss management strategies used at MUWSA. The strategies will be evaluated by weighted evaluation criteria to prioritise and select the portfolios of best strategies used in water loss management.

- *Survey of strategies*

1. Which strategies do you apply as a preventive action for water loss in WDS?

    i)……………………………………………………………………………………..
    ii)…………………………………………………………………………………….
    iii)…………………………………………………………………………………....
    iv)…………………………………………………………………………………...,
    v)…………………………………………………………………………………….
    vi)……………………………………………………………………………………
    vii)…………………………………………………………………………………...
    viii)………………………………………………………………………………….
    ix)……………………………………………………………………………………
    x)…………………………………………………………………………………….

2. Which equipment, instruments, and apparatus do you use to detect leaks in WDS?

    a)……………………………………………………………………………………
    b)……………………………………………………………………………………
    c)……………………………………………………………………………………
    d)……………………………………………………………………………………
    e)……………………………………………………………………………………

3. How do you educate the society on the good use and save of water inside and outside the home and encourage people to report the visible leaks to expedite the maintenance process and avoid excessive water loss?

    a)……………………………………………………………………………………
    b)……………………………………………………………………………………
    c)……………………………………………………………………………………
    d)……………………………………………………………………………………

4. How do you do a fraud Audit for illegal use of water for end-users? (eg illegal connection, reversal, and violation of the water meter)

   ………………………………………………………………………………………

   ………………………………………………………………………………………

5. Is your WDS segmented/or installed with a pressure reducing valves? YES/NO

   If Yes, how many zones/segments…………………….. and how many pressure reducing

   valves…………………………………………………………………………...............

6. a) How do you continuously monitoring the working ability of the WDS?

   ………………………………………………………………………………………

   ………………………………………………………………………………………

   b) Have you installed equipment like data loggers, Tubo de pilot, thermographic camera system and automation system in the WDS? ……………………………………..

   ....

   How do you quantify the water loss in the system?

   a)………………………………………………………………………………………

   b)………………………………………………………………………………………

   c)………………………………………………………………………………………

7. How do you detect the apparent- physical losses?

   a)………………………………………………………………………………………

   b)………………………………………………………………………………………

   c)………………………………………………………………………………………

   d)………………………………………………………………………………………

   e)………………………………………………………………………………………

8. How do you control the water losses caused by the inaccuracy of meter?

   a)………………………………………………………………………………………

   b)………………………………………………………………………………………

   c)………………………………………………………………………………………

   d)………………………………………………………………………………………

   e)………………………………………………………………………………………

- *Evaluation Criteria for water loss control in the WDS*

1. In your own opinion, which criteria do you think will be used to evaluate the strategies for water loss management in your WDS?

………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

2. How can you rank the importance of criteria in evaluating the decision-making strategies?
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

B. Evaluation of the alternatives

You are given the recommended alternatives used in water loss management and evaluation criteria; use this information to fill the evaluation matrix below by indicating the scores of each alternative basing on the evaluation criteria.

- *The surveyed alternatives for WLM are:*

1. **Alternative (A_1): Education** on the effective use of water which may facilitate saving water inside and outside the home, and encourage people to reports visible leaks and faults which may speed up the maintenance process and avoid excessive water loss,
   A_{11} – Advertising campaigns
   A_{12} – Educational campaigns in schools
   A_{13} - Ward meetings with the society
   A_{14} – Meeting with local leaders
2. **Alternative (A_2)**: A_{21} **- Illegal use control -** A measure aimed at losses that occur with the illegal use of water from end-users (illegal connection, a reversal of the meter, and violation of the infrastructure).
3. **Alternative (A_{31})**: **A_{31} - Network zoning (Establishment of District Metering Areas -DMA)**
4. **Alternative (A_4): Using a bulk meter to quantify the losses -** important information for the planning of action needs to be taken to control losses.
   A_{41} –24 hours Zone Measuring (HZM)

5. **Alternative (A_5)**: **Strategies used to control inaccuracy meter**.

    A_{51} - Calibration of the meter

    A_{52} - Replacement of the defect meters

6. **Alternative (A_6): Detection of apparent/physical losses**

    A_{61} – Visual inspection of the WDS

    A_{62} – comparison between the bulk water meter and customer water meter readings

    A_{63} – Report from the community on the detected leak through a toll-free telephone

7. **Alternative (A_7): A_{71} -** Replacement of deteriorated pipes
8. **Alternative (A_8): A_{81} -** Installation of quality pipes
9. **Alternative (A_9): A_{91} -** Timely repair of pipe leaks (active leakage control)
10. **Alternative (A_{10}): A_{101} -** Pressure management

- *Evaluation Criteria*

The identified evaluation criteria are;

1. **Criteria (C_1): Revenue generation** – The ability of the alternative to improve revenue. The higher the score value, the most preferable the alternative is.
2. **Criteria (C_2)**: **Investment cost-** cost needed to implement the alternative. The lower the score value (cost) the most preferable the alternative is.
3. **Criteria (C_3)**: **Operation &Maintenance cost -** The cost related to the implementation of the alternative. The lower the score value (cost) the most preferable the alternative is.
4. **Criteria (C_4): Saving of Water** – The ability of the alternative to reduce water loss. The higher the score value, the most preferable the alternative is.
5. **Criteria (C_5): Quality of Water –** The ability of the alternative to retaining water quality. The higher the score value, the most preferable the alternative is.
6. **Criteria (C_6): Water Supply reliability –** The ability of the alternative to reduce supply disruptions. The fewer the frequency of disruptions (burst, leaks, and illegal uses) the most preferable the alternative is.
7. **Criteria (C_7)**: **Efficiency** of the alternative to reduce water losses. The higher the score value, the most preferable the alternative is.

- ***Evaluation Matrix***

CRITERIA (C)	Alternative **(A)**															
	Education				Illegal Contr	DMA	Indic. quant. losses	Cont. Inaccuracy Meter		App/real loss detection			Repl. pipes	Qual. pipes	Leak contr	Press. mgnt
	A_{11}	A_{12}	A_{13}	A_{14}	A_{21}	A_{31}	A_{41}	A_{51}	A_{52}	A_{61}	A_{62}	A_{63}	A_{71}	A_{81}	A_{91}	A_{101}
Revenue (C_1) – Benefit																
Investment cost (C_2) – Cost																
Opera.& maintan. cost (C_3) – Cost																
Water save (C_4) – Benefit																
Water quality (C_5) – Benefit																
Supply reliability (C_6) – Benefit																
The efficiency of methods (C_7) – Benefit																

Note: The evaluation score values for criteria are scaled as: ***Very high – 5; High – 4; Fair – 3; Low – 2; Very low – 1.***

Other information

1. What is the total cost for the implementation of the set of alternatives? ……………….
2. What is the total loss of water per year? ………………………….
3. What is the implementation period to achieve a water loss reduction target? …………

Appendix 6: Absolute change in criteria weights ($\delta_{k,i,j}$)

Pair of Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7
	0.3704	**0.0728**	**0.1085**	**0.2276**	**0.0204**	**0.0442**	**0.1561**
A_{11}-A_{12}	0.060 46	0.018 94	0.024 36	0.060 46	IF	IF	0.053 22
A_{11}-A_{13}	IF	-0.454 94	IF	IF	IF	IF	IF
A_{11}-A_{14}	IF	-0.077 19	IF	IF	IF	IF	F
A_{11}-A_{21}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{31}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{41}	IF	-0.130 41	-0.083 85	IF	-0.136 60	IF	-0.183 19
A_{11}-A_{51}	IF	IF	IF	IF	-0.164 55	-0.359 05	IF
A_{11}-A_{52}	IF	IF	IF	IF	-0.133 91	-0.292 18	IF
A_{11}-A_{61}	IF	-0.222 55	-0.143 10	IF	IF	IF	IF
A_{11}-A_{62}	IF	-0.237 86	-0.305 89	IF	IF	IF	IF
A_{11}-A_{63}	IF	-0.465 35	-0.299 22	IF	IF	IF	IF
A_{11}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{91}	IF	IF	0.053 89	IF	IF	IF	IF
A_{11}-A_{101}	IF	IF	0.054 93	IF	IF	IF	0.120 00
A_{12}-A_{13}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{14}	0.183 65	IF	0.073 99	0.183 65	IF	IF	IF
A_{12}-A_{21}	-0.12 10	-0.037 90	-0.048 74	-0.12098	IF	IF	IF
A_{12}-A_{31}	IF	-0.063 38	-0.081 50	-0.20228	IF	IF	IF
A_{12}-A_{41}	0.268 57	IF	IF	IF	-0.176 29	IF	IF
A_{12}-A_{51}	0.280 77	IF	IF	IF	-0.184 40	-0.402 35	IF
A_{12}-A_{52}	0.234 10	IF	IF	IF	-0.153 75	-0.335 48	IF
A_{12}-A_{61}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{62}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{91}	0.194 24	0.060 86	0.039 13	0.194 24	IF	IF	IF
A_{12}-A_{101}	-0.07 59	-0.023 77	IF	-0.07587	IF	IF	IF

A_{13}-A_{14}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{21}	IF	-0.568 63	IF	IF	IF	IF	IF
A_{13}-A_{31}	IF	-0.619 57	IF	IF	IF	IF	IF
A_{13}-A_{41}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{51}	IF	-0.140 82	IF	IF	IF	IF	IF
A_{13}-A_{52}	IF	-0.099 66	IF	IF	IF	IF	IF
A_{13}-A_{61}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{62}	-0.03316	-0.020 79	-0.013 37	-0.03316	IF	IF	IF
A_{13}-A_{63}	-0.01661	IF	-0.006 70	-0.01661	IF	IF	IF
A_{13}-A_{71}	IF	0.072 05	IF	IF	IF	IF	IF
A_{13}-A_{81}	IF	-0.129 88	IF	IF	-0.272 11	-0.296 93	IF
A_{13}-A_{91}	IF	-0.371 11	-0.238 55	IF	-0.388 74	-0.424 20	IF
A_{13}-A_{101}	IF	-0.540 36	IF	IF	IF	IF	IF
A_{14}-A_{21}	IF	-0.190 89	IF	IF	IF	IF	IF
A_{14}-A_{31}	IF	-0.241 83	IF	IF	IF	IF	IF
A_{14}-A_{41}	IF	IF	-0.034 22	IF	-0.055 74	IF	-0.074 75
A_{14}-A_{51}	IF	IF	IF	IF	-0.124 11	-0.270 81	IF
A_{14}-A_{52}	0.284 53	IF	IF	IF	-0.093 47	-0.203 94	IF
A_{14}-A_{61}	IF	IF	-0.093 47	IF	IF	IF	IF
A_{14}-A_{62}	IF	-0.398 53	-0.256 25	IF	IF	IF	IF
A_{14}-A_{63}	IF	IF	-0.249 58	IF	IF	IF	IF
A_{14}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{14}-A_{81}	IF	0.058 99	IF	IF	IF	IF	IF
A_{14}-A_{91}	IF	0.006 64	0.004 27	IF	0.006 95	0.007 59	IF
A_{14}-A_{101}	IF	-0.162 62	0.104 56	IF	IF	IF	IF
A_{21}-A_{31}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{41}	IF	-0.244 10	-0.156 95	IF	IF	IF	IF
A_{21}-A_{51}	IF	IF	IF	IF	-0.448 33	-0.489 01	IF
A_{21}-A_{52}	IF	IF	IF	IF	-0.387 02	-0.422 13	0.518 76
A_{21}-A_{61}	IF	-0.336 25	-0.216 20	IF	IF	IF	IF
A_{21}-A_{62}	IF	-0.294 71	-0.378 99	IF	IF	IF	IF
A_{21}-A_{63}	IF	-0.579 04	-0.372 32	IF	IF	IF	IF
A_{21}-A_{71}	IF	IF	IF	IF	IF	IF	IF

A_{21}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{91}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{101}	IF	IF	-0.018 17	IF	IF	IF	IF
A_{31}-A_{41}	IF	-0.295 04	-0.189 71	IF	IF	IF	IF
A_{31}-A_{51}	IF	IF	IF	IF	-0.501 72	IF	IF
A_{31}-A_{52}	IF	IF	IF	IF	-0.440 41	IF	IF
A_{31}-A_{61}	IF	-0.387 19	-0.248 96	IF	IF	IF	IF
A_{31}-A_{62}	IF	-0.320 18	-0.411 75	IF	IF	IF	IF
A_{31}-A_{63}	IF	-0.629 99	-0.405 08	IF	IF	IF	IF
A_{31}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{91}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{101}	IF	IF	-0.050 93	IF	IF	IF	IF
A_{41}-A_{51}	0.292 97	IF	IF	IF	-0.192 52	-0.209 99	IF
A_{41}-A_{52}	0.199 67	0.062 60	0.080 50	0.199 67	-0.131 21	-0.143 11	IF
A_{41}-A_{61}	IF	IF	IF	IF	IF	IF	0.129 44
A_{41}-A_{62}	IF	-0.345 32	IF	IF	IF	IF	IF
A_{41}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{41}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{41}-A_{81}	IF	0.032 38	0.041 64	IF	IF	IF	0.090 97
A_{41}-A_{91}	IF	-0.046 58	-0.014 97	IF	-0.024 40	-0.053 24	-0.065 43
A_{41}-A_{101}	IF	-0.215 83	IF	IF	-0.226 09	IF	IF
A_{51}-A_{52}	IF	-0.058 51	IF	IF	IF	IF	IF
A_{51}-A_{61}	0.146 02	IF	0.058 87	0.146 02	-0.047 97	-0.104 66	IF
A_{51}-A_{62}	-0.25774	-0.080 81	-0.103 92	-0.25774	IF	IF	IF
A_{51}-A_{63}	-0.24119	-0.151 24	-0.097 25	-0.24119	IF	IF	IF
A_{51}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{51}-A_{81}	0.189 68	-0.118 94	IF	0.189 69	-0.041 54	-0.067 98	IF
A_{51}-A_{91}	0.367 25	IF	-0.148 03	IF	-0.080 42	-0.131 61	IF
A_{51}-A_{101}	IF	IF	IF	IF	-0.209 31	-0.456 70	IF
A_{52}-A_{61}	0.052 71	0.016 53	0.021 25	0.052 71	-0.017 32	-0.037 78	IF
A_{52}-A_{62}	-0.35104	-0.073 37	-0.141 53	-0.35104	IF	IF	IF
A_{52}-A_{63}	-0.33449	-0.104 87	-0.134 86	-0.33449	IF	IF	IF

A_{52}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{52}-A_{81}	0.096 38	IF	IF	0.096 38	-0.021 11	-0.034 54	IF
A_{52}-A_{91}	0.273 95	IF	-0.110 42	IF	-0.059 99	-0.098 18	IF
A_{52}-A_{101}	IF	IF	IF	IF	-0.178 66	-0.389 82	IF
A_{61}-A_{62}	IF	-0.253 17	IF	IF	IF	IF	IF
A_{61}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{61}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{61}-A_{81}	IF	-0.013 69	-0.017 61	IF	-0.028 69	-0.031 30	IF
A_{61}-A_{91}	IF	-0.138 72	-0.044 59	IF	-0.145 32	-0.158 57	IF
A_{61}-A_{101}	IF	-0.307 98	IF	IF	IF	IF	IF
A_{62}-A_{63}	IF	-0.010 37	IF	IF	IF	IF	IF
A_{62}-A_{71}	0.196 65	0.041 10	0.079 29	0.196 65	IF	IF	IF
A_{62}-A_{81}	IF	-0.093 52	-0.180 39	IF	-0.293 89	-0.320 69	IF
A_{62}-A_{91}	IF	-0.195 95	-0.125 98	IF	-0.410 52	-0.447 96	IF
A_{62}-A_{101}	IF	-0.280 58	IF	IF	IF	IF	IF
A_{63}-A_{71}	0.213 20	0.066 84	0.085 96	0.213 20	IF	IF	IF
A_{63}-A_{81}	IF	-0.135 09	-0.173 72	IF	-0.283 02	-0.308 83	IF
A_{63}-A_{91}	IF	-0.381 52	-0.122 64	IF	-0.399 65	-0.436 10	IF
A_{63}-A_{101}	IF	-0.550 78	IF	IF	IF	IF	IF
A_{71}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{71}-A_{91}	IF	IF	-0.331 18	IF	IF	IF	IF
A_{71}-A_{101}	IF	IF	IF	IF	IF	IF	IF
A_{81}-A_{91}	IF	IF	-0.071 57	IF	IF	IF	IF
A_{81}-A_{101}	IF	IF	IF	IF	IF	IF	IF
A_{91}-A_{101}	IF	IF	0.054 41	IF	IF	IF	IF

Appendix 7: Percent change in criteria weights ($\delta'_{k,i,j}$)

Pair of Alter	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_{11}-A_{12}	16.322	26.019	22.450	26.562	IF	IF	34.090
A_{11}-A_{13}	IF	-624.911	IF	IF	IF	IF	IF
A_{11}-A_{14}	IF	-106.036	IF	IF	IF	IF	IF
A_{11}-A_{21}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{31}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{41}	IF	-179.131	-77.282	IF	-669.626	IF	-117.351
A_{11}-A_{51}	-69.276	IF	IF	IF	-806.639	-812.329	IF
A_{11}-A_{52}	-96.652	IF	IF	IF	-656.399	-661.029	IF
A_{11}-A_{61}	IF	-305.705	-131.890	IF	IF	IF	IF
A_{11}-A_{62}	IF	-326.734	-281.924	IF	IF	IF	IF
A_{11}-A_{63}	IF	-639.218	-275.776	IF	IF	IF	IF
A_{11}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{11}-A_{91}	IF	IF	49.666	IF	IF	IF	IF
A_{11}-A_{101}	IF	IF	50.625	IF	IF	IF	76.874
A_{12}-A_{13}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{14}	49.580	IF	68.197	80.688	IF	IF	IF
A_{12}-A_{21}	-32.662	-52.066	-44.926	-53.154	IF	IF	IF
A_{12}-A_{31}	IF	-87.056	-75.117	-88.875	IF	IF	IF
A_{12}-A_{41}	72.507	IF	IF	IF	-864.151	IF	IF
A_{12}-A_{51}	75.802	IF	IF	IF	-903.924	-910.300	IF
A_{12}-A_{52}	63.203	IF	IF	IF	-753.683	-758.999	IF
A_{12}-A_{61}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{62}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{12}-A_{81}	IF	94.423	IF	IF	IF	IF	IF
A_{12}-A_{91}	52.440	83.594	36.060	85.341	IF	IF	IF
A_{12}-A_{101}	-20.484	-32.653	IF	-33.335	IF	IF	IF

A_{13}-A_{14}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{21}	IF	-781.081	IF	IF	IF	IF	IF
A_{13}-A_{31}	IF	-851.056	IF	IF	IF	IF	IF
A_{13}-A_{41}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{51}	IF	-193.438	IF	IF	IF	IF	IF
A_{13}-A_{52}	IF	-136.901	IF	IF	IF	IF	IF
A_{13}-A_{61}	IF	IF	IF	IF	IF	IF	IF
A_{13}-A_{62}	-8.951	-28.557	-12.320	-14.567	IF	IF	IF
A_{13}-A_{63}	-4.484	IF	-6.173	-7.298	IF	IF	IF
A_{13}-A_{71}	IF	98.969	IF	IF	IF	IF	IF
A_{13}-A_{81}	IF	-178.410	IF	IF	-1 333.860	-671.782	IF
A_{13}-A_{91}	IF	-509.759	-219.864	IF	-1 905.575	-959.719	IF
A_{13}-A_{101}	IF	-742.254	IF	IF	1F	IF	IF
A_{14}-A_{21}	IF	-262.206	IF	IF	IF	IF	IF
A_{14}-A_{31}	IF	-332.186	IF	IF	IF	IF	IF
A_{14}-A_{41}	IF	IF	-31.535	IF	-273.242	IF	-47.886
A_{14}-A_{51}	IF	IF	IF	IF	-608.404	-612.695	IF
A_{14}-A_{52}	76.816	IF	IF	IF	-458.163	-461.395	IF
A_{14}-A_{61}	IF	IF	-86.143	IF	IF	IF	IF
A_{14}-A_{62}	IF	-547.431	-236.177	IF	IF	IF	IF
A_{14}-A_{63}	IF	IF	-230.029	IF	IF	IF	IF
A_{14}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{14}-A_{81}	IF	81.027	IF	IF	IF	IF	IF
A_{14}-A_{91}	F	9.115	3.932	IF	34.074	17.161	IF
A_{14}-A_{101}	IF	-223.380	96.372	IF	IF	IF	IF
A_{21}-A_{31}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{41}	IF	-335.301	-144.658	IF	IF	IF	IF
A_{21}-A_{51}	IF	IF	IF	IF	-2197.684	-1106.349	IF
A_{21}-A_{52}	IF	IF	IF	IF	-1897.137	-955.048	332.326
A_{21}-A_{61}	IF	-461.875	-199.266	IF	IF	IF	IF
A_{21}-A_{62}	IF	-404.819	-349.300	IF	IF	IF	IF
A_{21}-A_{63}	IF	-795.388	-343.152	IF	IF	IF	IF
A_{21}-A_{71}	IF	IF	IF	IF	IF	IF	IF

A_{21}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{91}	IF	IF	IF	IF	IF	IF	IF
A_{21}-A_{101}	IF	IF	-16.751	IF	IF	IF	IF
A_{31}-A_{41}	IF	-405.280	-174.849	IF	IF	IF	IF
A_{31}-A_{51}	IF	IF	IF	IF	-2459.395	IF	IF
A_{31}-A_{52}	IF	IF	IF	IF	-2158.848	IF	IF
A_{31}-A_{61}	IF	-531.855	-229.456	IF	IF	IF	IF
A_{31}-A_{62}	IF	-439.809	-379.491	IF	IF	IF	IF
A_{31}-A_{63}	IF	-865.367	-373.343	IF	IF	IF	IF
A_{31}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{91}	IF	IF	IF	IF	IF	IF	IF
A_{31}-A_{101}	IF	IF	-46.942	IF	IF	IF	IF
A_{41}-A_{51}	79.095	IF	IF	IF	-943.714	-475.081	IF
A_{41}-A_{52}	53.905	85.989	74.196	87.726	-643.167	-323.780	IF
A_{41}-A_{61}	IF	IF	IF	IF	IF	IF	82.921
A_{41}-A_{62}	IF	-474.337	IF	IF	IF	IF	IF
A_{41}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{41}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{41}-A_{81}	IF	44.480	38.380	IF	IF	IF	58.279
A_{41}-A_{91}	IF	-63.979	-13.799	IF	-119.584	-120.453	-41.914
A_{41}-A_{101}	IF	-296.475	IF	IF	-1108.279	IF	IF
A_{51}-A_{52}	IF	-80.364	IF	IF	IF	IF	IF
A_{51}-A_{61}	39.421	IF	54.259	64.154	-235.122	-236.780	IF
A_{51}-A_{62}	-69.583	-110.998	-95.775	-113.240	IF	IF	IF
A_{51}-A_{63}	-65.117	-207.746	-89.627	-105.972	IF	IF	IF
A_{51}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{51}-A_{81}	51.211	-163.381	IF	83.341	-203.613	-153.798	IF
A_{51}-A_{91}	99.149	IF	-136.432	IF	-394.213	-297.767 1	IF
A_{51}-A_{101}	IF	IF	IF	IF	-1026.014	-1 033.251	IF
A_{52}-A_{61}	14.231	22.701	19.588	23.160	-84.881	-85.479	IF
A_{52}-A_{62}	-94.773	-100.786	-130.446	-154.234	IF	IF	IF
A_{52}-A_{63}	-90.306	-144.055	-124.298	-146.965	IF	IF	IF

A_{52}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{52}-A_{81}	26.021	IF	IF	42.347	-103.460	-78.148 25	IF
A_{52}-A_{91}	73.959	IF	-101.771	IF	-294.060	-222.117	IF
A_{52}-A_{101}	IF	IF	IF	IF	-875.774	-881.951	IF
A_{61}-A_{62}	IF	-347.762	IF	IF	IF	IF	IF
A_{61}-A_{63}	IF	IF	IF	IF	IF	IF	IF
A_{61}-A_{71}	IF	IF	IF	IF	IF	IF	IF
A_{61}-A_{81}	IF	-18.807	-16.228	IF	-140.611	-70.817	IF
A_{61}-A_{91}	IF	-190.554	-41.100	IF	-712.326	-358.754	IF
A_{61}-A_{101}	IF	-423.049	IF	IF	IF	IF	IF
A_{62}-A_{63}	IF	-14.250	IF	IF	IF	IF	IF
A_{62}-A_{71}	53.092	56.460	73.076	86.402	IF	IF	IF
A_{62}-A_{81}	IF	-128.459	-166.262	IF	-1 440.611	-725.546	IF
A_{62}-A_{91}	IF	-269.159	-116.106	IF	-2 012.327	-1013.483	IF
A_{62}-A_{101}	IF	-385.406	IF	IF	IF	IF	IF
A_{63}-A_{71}	57.558	91.815	79.223	93.671	IF	IF	IF
A_{63}-A_{81}	IF	-185.563	-160.114	IF	-1 387.343	-698.718	IF
A_{63}-A_{91}	IF	-524.066	-113.033	IF	-1 959.059	-986.656	IF
A_{63}-A_{101}	IF	-756.562	IF	IF	IF	IF	IF
A_{71}-A_{81}	IF	IF	IF	IF	IF	IF	IF
A_{71}-A_{91}	IF	IF	-305.237	IF	IF	IF	IF
A_{71}-A_{101}	IF	IF	IF	IF	IF	IF	IF
A_{81}-A_{91}	IF	IF	-65.964	IF	IF	IF	IF
A_{81}-A_{101}	IF	IF	IF	IF	IF	IF	IF
A_{91}-A_{101}	IF	IF	50.146	IF	IF	IF	IF