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# Mathematical Model for the Transmission Dynamics of Bovine Tuberculosis in Human and Livestock with Control Strategies

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#### **Abstract**

A deterministic mathematical model for bovine tuberculosis (bovine TB) in humans and livestock is formulated and used to assess the effectiveness of dairy products inspection, human treatment and quarantine of infected livestock as the control of the disease transmission. The computed effective reproduction number shows that the disease can be cleared from the population if Re is less than unity and it persists if Re is greater than unity. It means that if Re<1 the disease-free equilibrium is asymptotically stable which means the disease can be cleared from the population and endemic when Re>1 which implies the disease persists in the population. Numerical analysis was carried out to investigate how the controls can help to minimize the spread of the disease.

**Key Words:** Bovine tuberculosis, Dairy products, Mathematical model, Livestock.

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#### 1. INTRODUCTION

Tuberculosis (TB) is a bacterial disease which caused by mycobacterium tuberculosis. The disease is listed among the diseases that kills large number of people around the world. Based on the global TB report of 2018, it is estimated that around 1.3 million people die annually due to TB of which 1.2-1.4 million people are HIV negative while 300,000 people are HIV positive [1]. Globally, it is estimated that about 10 million which range between 9.0-11.1 million people are exposed to TB where, 5.8 millions are men, 3.2 millions are women and 1.0 million are children [1]. Africa is leading by having large percentage of individuals who develops into active TB, followed by India, China, Indonesia, the Philippines, Pakistan, Nigeria and Bangladesh: (27%), (9%), (8%), (6%), (5%), (4%) and (3%) respectively [1]. Although TB seems to be very important health problem worldwide, extrapulmonary TB is not given attention and this makes difficult to attain the goals of ending TB around 2020. This attracts many researchers to find out about TB especially the extra pulmonary TB especially bovine tuberculosis.

Bovine tuberculosis (bovine TB) is a bacterial disease which have great impacts in agricultural sector [2]. It is animal's disease which affects both wild and domesticated animals such as cattle, goats, sheep, pig and pet animals [3]. Bovine TB is transmitted through inhalation of aerosols, sharing of grazing areas, water sources, and several contacts between susceptible and infected animals [3]. Though bovine TB plays very important role in animal's health and human's health it has given very little attention [4]. The disease have great negative impacts on economy of many countries since agriculture sector play big role in the growth of economy [5], [6]. However, many people loses their job especially livestock keepers and those who works in beef sectors due to the outbreak of the disease [4], [5], [7]. Since bovine TB causes economic, social and health problems worldwide it attracts much attention for many scholars to control the transmission of the disease [8]–[10].

It is not easy to diagnose animals with Bovine TB at once because an animal can be in latent state for long time and sometimes it can be diagnosed in old age or during the stress [11]. Some associated symptoms of bovine TB includes reducing productivity, gradual loose of weight, coughing, weakness and some animal's lymph nodes enlarge which may end up bursting [5], [11], [12]. Bovine TB is endemic in many developing countries while in developed countries they have controlled the disease in large extent

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[13]. In non-industrialized countries the disease seems to be endemic due to hygienic situation, economy and their living style [13].

Bovine TB is transmitted to human being through consumption of infected or contaminated dairy product, inhalation of aerosols and through scratches when someone contact infected dairy products [3]. However, it is difficult to control the spread of the disease especially in developing countries when the disease enters wild animals because many livestock keepers graze their animals in the bush where they interact with wild animals [14]. The fact that both domestic animals and wild animals share watering areas and grazing areas complicates the dynamics of the disease and consequently makes it difficult to control it in both animals and humans.

Various mathematical studies such as Liu et al., [9], Augusto et al., [5], Hassan et al., [15] have been conducted to understand the dynamics of bovine TB and suggested ways of controlling the disease. Many of these studies recommended test and slaughter to be the best way of controlling the transmission. However, test and slaughter is almost impossible in developing countries because it needs farmers willingness and government readiness for compensation of farmers. More importantly, the fact that the environment can host mycobacterial bovis is not given much attention in most of these studies. Furthermore, international organizations such as, World Health organization (WHO), World Organization for Animal Health (OIE), Food and Agriculture Organization (FAO) discussed how reduce the spread of bovine TB up to 2030 [16]. They suggested different approaches like "one health approach, together we can save lives and secure live hoods" as strategies to control the spread of bovine TB but the problem is still there. However, very few of them highlighted the environment as a hindering factor for the control of the disease. This study uses mathematical model to assess the impacts of environment on the transmission of bovine TB to livestock and humans.

#### 1.1 Model Formulation

In this section, a mathematical model for the dynamics of bovine TB in human and animal populations is developed. Human population is divided into susceptible class  $S_h$  exposed class  $E_h$  and infected class  $I_h$ . Animals' population is divided into susceptible class  $S_a$ , exposed class  $E_a$  and infectious class  $I_a$ . Susceptible individuals S refers to the individuals who are likely to be infected with the disease. Exposed individuals or latent class E refers to the individuals who are exposed into the disease but they do not have any symptoms of the disease and they cannot spread the disease into others. Lastly, the infectious class these are individuals who are infected with the disease this is when they developed symptoms of the disease and at this stage an individual can spread the disease into others.

Susceptible class  $S_h$  increases due to birth and recovery at the rates  $\Lambda_h$  and  $\Pi_h$  respectively, and decreases when they acquire disease and become latent after come into contact with infectious humans and by consuming infectious dairy products from infectious animals at a rate  $X_h$ .

Exposed human class  $E_h$  increases when susceptible class acquire bovine TB and moves into the class at the rate  $X_h$ . However, they decreases by dying naturally at the rate  $\mu_h$  and when their TB status progress to active stages at the rate  $\gamma_h$  by either endogenous reactivation or exogenous reinfection

Infectious class  $I_h$  increases when individual from exposed class progress into infectious class at a rate  $\gamma_h$  and decreases due to natural mortality rate  $\mu_h$  and disease induced death rate of  $\alpha_h$ .

Susceptible animals  $S_a$  increases through birth and recovery at a rate  $\Lambda_a$  and  $\Pi_a$  respectively. They acquire bovine tuberculosis latent infection following contacts with infectious humans and animals, and after consuming infectious dairy products during breastfeeding at a rate  $X_a$ .

Exposed animals  $E_a$  increase following latent infection of susceptible animals  $S_a$  at a rate  $X_a$ . However, they decreases due to natural death  $\mu_a$  and by progressing into infectious class at a rate  $\gamma_a$ .

Infectious animals  $I_a$  increase at a rate  $\gamma_a$  and diminish following quarantine of infected animals  $\tau_a$ , natural and disease mortality at the rates  $\alpha_a$  ma respectively.

Dairy products are produced by infectious animals at a rate  $\rho$  where, susceptible human consumed the products at rate  $\beta_3$ , susceptible livestock consume dairy products at rate  $\beta_6$  through breastfeeding from infectious livestock and the remaining products leak at rate  $\omega$ . However the rate of producing dairy products decreases following quarantine of infected animals and inspection of dairy products at the rates  $\tau_a$ , and  $\epsilon$  respectively.

### 1.2 Model Flow Diagram

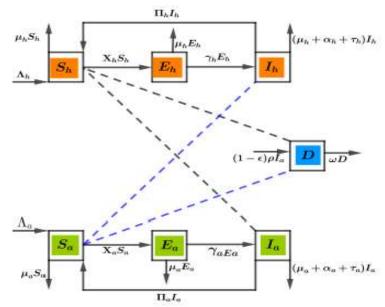


Figure 1: Model Flow Diagram

From the model flow diagram we have the following system of differential equations;

$$\frac{dS_h}{dt} = \Lambda_h + \Pi_h - \left(\frac{(1 - \tau_h)\beta_1 I_h + (1 - \tau_a)\beta_2 I_a + (1 - \epsilon)\beta_3 D}{N_h}\right) S_h - \mu_h S_h$$
 (1.1a)

$$\frac{dE_h}{dt} = \left(\frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\epsilon)\beta_3 D}{N_h}\right) S_h - (\gamma_h + \mu_h) E_h$$
 (1.1b)

$$\frac{dI_h}{dt} = \gamma_h E_h - (\alpha_h + \mu_h + \tau_h) I_h$$
 (1.1c)

$$\frac{dS_a}{dt} = \Lambda_a + \Pi_a - \left(\frac{(1-\tau_h)\beta_4 I_h + (1-\tau_a)\beta_5 I_a + (1-\epsilon)\beta_6 D}{N_a}\right) S_a - \mu_a S_a \tag{1.1d}$$

$$\frac{dE_a}{dt} = \left(\frac{(1 - \tau_h)\beta_4 I_h + (1 - \tau_a)\beta_5 I_a + (1 - \epsilon)\beta_6 D}{N_a}\right) S_a - (\gamma_a + \mu_a) E_a$$
 (1.1e)

$$\frac{dI_a}{dt} = \gamma_a E_a - (\alpha_a + \mu_a + \tau_a) I_a \tag{1.1f}$$

$$\frac{dD}{dt} = (1 - \epsilon)\rho I_a - (\omega + \theta)D \tag{1.1g}$$

Subject to their initial conditions;

$$S_h(0) > 0, E_h(0) > 0, I_h(0) > 0, S_a(0) > 0, S_a(0) > 0, E_a(0) > 0, I_a(0) > 0 & D(0) > 0$$

Where  $X_h = \frac{(1-\tau_h)\beta_1 I_h + (1-\tau_a)\beta_2 I_a + (1-\epsilon)\beta_3 D}{N_h}$ , and  $X_a = \frac{(1-\tau_h)\beta_4 I_h + (1-\tau_a)\beta_5 I_a + (1-\epsilon)\beta_6 D}{N_h}$  given  $\tau_h$  is humans treatment rate,  $\tau_a$  is quarantine of infected animals and  $\varepsilon$  is dairy products inspection rate.

**Table1: Model Variable descriptions** 

Symbol	Descriptions
$S_h$	Number of susceptible human at time t.
$E_h$	Number of exposed human at time t.
$I_h$	Number of infected human beings at time t.
$S_a$	Number of susceptible animals at time t.
$E_a$	Number of Exposed animals at time t.
$I_a$	Number of infected animals at time t.
D	Dairy products at time t

Parameter	Descriptions		
$\Lambda_h$	Human recruitment rate.		
$\mu_h$	Human natural death.		
$\mu_a$	Animals' natural death.		
$\gamma_h$	Progression rate from $E_h$ to $I_h$		
$\gamma_a$	Progression rate from $E_a$ to $I_a$		
$\propto_h$	Humans mortality rate due to disease		
$\propto_a$	Animals mortality rate due to disease		
ρ	Rate of producing infectious dairy products		
ε	Rate of inspecting dairy products		
$ au_h$	Infected humans treatment rate		
$ au_a$	Quarantine rate of Infected animals		
$\Pi_h$	Human recovery rate		
$\Pi_a$	Animals recovery rate		
$\beta_1, \beta_2 \& \beta_3$	Humans infection rates from infected humans, animals and dairy products respectively		
$\beta_4$ , $\beta_5 \& \beta_6$	Animals infection rates from infected humans, animals and dairy products respectively		
ω	Decay rate of unused dairy products		
$N_h$	Human total populations		
$N_a$	Animals total populations		

# 1.3 Analysis of the Model

# 1.4 Invariant Region

Invariant region shows the feasibility of the model solutions. To find the invariant region, we denote humans and livestock populations by  $N_h$  and  $N_h$  respectively.

Beginning with human population we have:

$$N_h = S_h + E_h + I_h,$$

$$\frac{dN_h}{dt} \le \Lambda_h - \mu_h S_h$$
(1.2)

Solving inequality (1.2), we have

$$\begin{split} N_h &\leq \frac{\Lambda_h}{\mu_h} + \left( \mathbf{N}_h(0) - \frac{\Lambda_h}{\mu_h} \right) e^{-\mu_h t} \leq \frac{\Lambda_h}{\mu_h}, \\ \lim_{t \to \infty} \left( \mathbf{N}_h(0) - \frac{\Lambda_h}{\mu_h} \right) e^{-\mu_h t} &\to 0, \\ 0 &\leq N_h \leq \frac{\Lambda_h}{\mu_h}. \end{split} \tag{1.3}$$

Then,

The total animals population is given by;

$$N_a = S_a + E_a + I_a,$$

$$\frac{dN_a}{dt} \le \Lambda_a - \mu_a S_a$$
(1.4)

Solving equation (1.4) we obtain,

$$\begin{split} N_a & \leq \frac{\Lambda_a}{\mu_a} + \left( \mathbf{N}_a(0) - \frac{\Lambda_a}{\mu_a} \right) e^{-\mu_a t} \leq \frac{\Lambda_a}{\mu_a}, \\ \lim_{t \to \infty} \left( \mathbf{N}_a(0) - \frac{\Lambda_a}{\mu_a} \right) e^{-\mu_a t} & \to 0, \end{split}$$

Then, 
$$0 \le N_a \le \frac{\Lambda_a}{\mu_a}. \tag{1.5}$$

For dairy products we have;

$$\frac{dD}{dt} \le (1 - \varepsilon)\rho I_a - (\omega + \theta)D,$$

Since

$$I_a \leq \frac{\Lambda_a}{\mu_a}$$

Then,

$$\frac{dD}{dt} \le (1 - \varepsilon) \frac{\Lambda_a}{\mu_a} \rho - (\omega + \theta) D, \tag{1.6}$$

From (1.6) we have

$$D(0) \leq \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho + \left(D(0) - \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho\right) e^{-(\omega+\theta)t},$$

$$\lim D(t)_{t\to\infty} \le \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho e^{-(\omega+\theta)t} \to \frac{\Lambda_a}{\mu_a} \left(\frac{1-\varepsilon}{\omega+\theta}\right) \rho, \tag{1.7}$$

Therefore the model (1.1) is positive invariant in the region;

$$Z = \left\{ (S_h, E_h, I_h, S_a, E_a, I_a, D) \in R_+^7 : 0 \le N_h \le \frac{\Lambda_h}{\mu_h}; 0 \le N_a \le \frac{\Lambda_a}{\mu_a}; 0 \le D \le \left(\frac{1 - \varepsilon}{\omega + \theta}\right) \rho \right\}$$
(1.8)

#### 1.5 Positivity of solution

Theorem 1: Let the initial values for the state variables for the model (1.1) be  $S_h(0) \ge 0$ ,  $E_h(0) \ge 0$ ,  $E_h$ 

Proof: Let's consider the equations (1.1a) of the model system (1.1) which is

$$\frac{dS_h}{dt} = \Lambda_h + \Pi_h - \left(\frac{(1-\tau_h)\beta_1I_h + (1-\tau_a)\beta_2I_a + (1-\epsilon)\beta_3D}{N_h}\right)S_h - \mu_hS_h,$$

$$\frac{dS_{h}}{dt} \ge -\left(\frac{(1-\tau_{h})\beta_{1}I_{h} + (1-\tau_{a})\beta_{2}I_{a} + (1-\epsilon)\beta_{3}D}{N_{h}} + \mu_{h}\right)S_{h},\tag{1.9}$$

Solving differential equation (1.9) and apply initial conditions we get;

$$S_h(t) \ge S_h(0)e^{\int_0^t -\left(\frac{(1-\tau_h)\beta_1 I_h(s) + (1-\tau_a)\beta_2 I_a(s) + (1-\epsilon)\beta_3 D(s)}{N_h(s)} + \mu_h\right)ds},$$
(1.10)

Then,

$$S_h(t) \ge t$$
,  $\forall t \ge 0$ .

By following the same procedure to the remaining equations of the model system (1.1) we obtain the following;

$$E_h(t) \ge E_h(0)e^{-(\gamma_h + \mu_h)t}$$
 (1.11)

$$I_h(t) \ge I_h(0)e^{-(\alpha_h + \mu_h + \tau_h)t}$$
 (1.12)

$$S_a(t) \ge S_a(0) e^{\int_0^t -\left(\frac{(1-\tau_h)\beta_4 I_h(s) + (1-\tau_a)\beta_5 I_a(s) + (1-\epsilon)\beta_6 D(s)}{N_a(s)} + \mu_a\right) ds}. \tag{1.13}$$

$$D(t) \ge D(0)e^{-(\omega+\theta)t} \tag{1.14}$$

Therefore solutions of the model satisfies the conditions hence a model is positive and bounded since;

$$S_h(0) \geq 0, E_h(0) \geq 0, I_h(0) \geq 0, S_a(0) \geq 0, S_a(0) \geq 0, E_a(0) \geq 0, I_a(0) \geq 0 \ \& \ D(0) \geq 0, \forall t \geq 0 \leq 0, I_a(0) \geq 0, \forall t \geq 0 \leq 0, I_a(0) \geq 0, \forall t \geq 0 \leq 0, I_a(0) \geq 0, I_a(0) \geq 0, \forall t \geq 0, I_a(0) \geq 0, I$$

#### 1.6 Disease free equilibrium

The disease free equilibrium point is the state when there is no disease in the population. When there is no bovine TB in human and animal populations, the disease free equilibrium is given by:

$$DF^{0} = (S_{h}, E_{h}, I_{h}, S_{a}, E_{a}, I_{a}, D) = \left(\frac{\Lambda_{h}}{\mu_{h}}, 0, 0, \frac{\Lambda_{a}}{\mu_{a}}, 0, 0, 0\right)$$
(1.15)

## 1.7 Endemic Equilibrium of Model With Control

Stability of an endemic equilibrium of model with control is analyzed using force of infection due to complexity of the model equations. By equating the right side of the model system (1.1) equals to zero and solve simultaneously we obtain;

$$S_{h}^{*} = \frac{\Lambda_{h}(\gamma_{h} + \mu_{h})(\alpha_{h} + \mu_{h} + \tau_{h})}{(\gamma_{h} + \mu_{h})(\alpha_{h} + \mu_{h} + \tau_{h})(X_{h}^{*} + \mu_{h}) - \Pi_{h} X_{h}^{*} \gamma_{h}},$$
(1.16)

$$E_h^* = \frac{(\alpha_h + \mu_h + \tau_h)\Lambda_h X_h^*}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(X_h^* + \mu_h) - \Pi_h X_h^* \gamma_h},$$
(1.17)

$$I_h^* = \frac{\Lambda_h \gamma_h X_h^*}{(\gamma_h + \mu_h)(\alpha_h + \mu_h + \tau_h)(X_h^* + \mu_h) - \Pi_h X_h^* \gamma_h},\tag{1.18}$$

$$S_a^* = \frac{\Lambda_a(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a},$$
(1.19)

$$E_a^* = \frac{(\alpha_a + \mu_a + \tau_a)\Lambda_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a'}$$
(1.20)

$$I_a^* = \frac{\Lambda_a \gamma_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - \Pi_a X_a^* \gamma_a}$$
 and (1.21)

$$D^* = \frac{(1 - \varepsilon)\gamma_a \Lambda_a X_a^*}{(\gamma_a + \mu_a)(\alpha_a + \mu_a + \tau_a)(X_a^* + \mu_a) - (\omega + \theta)\Pi_a X_a^* \gamma_a}$$
(1.22)

# 1.8 Effective Reproduction Number $R_e$

To determine effective reproduction number when control parameters are administered, we use next generation approach by [14]. The control strategies are effective when the effective reproduction number  $R_e < 1$  and they are ineffective if reproduction number  $R_e > 1$ . If infectious and transfer terms are denoted by  $H_i$  and  $P_i$  respectively, then the effective reproduction number  $R_e$  is given as the maximum eigenvalue. That is;

$$R_e = \rho(HP)^{-1}$$

$$H = \frac{\partial H_i}{\partial X_i} (DF^0) \text{ and } P = \frac{\partial P_i}{\partial X_i} (DF^0)$$
(1.23)

Where,

From the model system (1.1) we therefore computed the effective reproduction number  $R_e$  and obtain the following;

$$R_{e} = \frac{1}{2} \left( \frac{(1 - \tau_{a})(\omega + \theta)\beta_{5}\gamma_{a} + (1 - \varepsilon)^{2}\beta_{6}\rho\gamma_{a}}{(\gamma_{a} + \mu_{a})(\alpha_{a} + \mu_{a} + \tau_{a})(\omega + \theta)} + \frac{(1 - \tau_{h})\beta_{6}\gamma_{h}}{(\gamma_{h} + \mu_{h})(\alpha_{h} + \mu_{h} + \tau_{h})} + \sqrt{\left( \frac{(1 - \tau_{a})(\omega + \theta)\beta_{5}\gamma_{a} + (1 - \varepsilon)^{2}\beta_{6}\rho\gamma_{a}}{(\gamma_{a} + \mu_{a})(\alpha_{a} + \mu_{a} + \tau_{a})(\omega + \theta)} - \frac{(1 - \tau_{h})\beta_{6}\gamma_{h}}{(\gamma_{h} + \mu_{h})(\alpha_{h} + \mu_{h} + \tau_{h})} \right)^{2} + 4rt} \right) (1.24)$$

Where;

$$rt = \frac{(1-\tau_a)(1-\tau_h)(\omega+\theta)\beta_2\beta_4\gamma_h\gamma_a + (1-\tau_h)(1-\epsilon)^2\rho\beta_3\beta_4\gamma_h\gamma_a}{(\gamma_h+\mu_h)(\alpha_h+\mu_h+\tau_h)(\gamma_a+\mu_a)(\alpha_a+\mu_a+\tau_a)(\omega+\theta)}$$

The effective reproduction number decreases as we increases human's treatment rate  $\tau_h$ , animals quarantine rates  $\tau_a$  and dairy products inspection  $\varepsilon$ . The transmission of the disease can be minimized if these control strategies are used effectively and this is when effective reproduction number  $R_e$  become less than a unit.

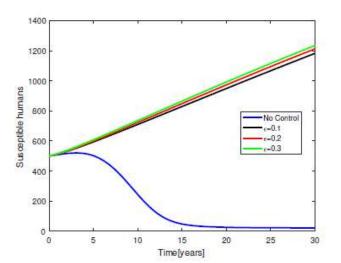
#### 2. Numerical Simulation

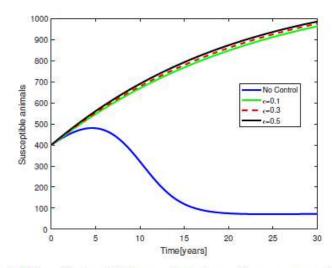
We performed a numerical simulation to discuss how bovine TB can be eliminated from the population using different strategies such as dairy products inspections, treatment of infected humans and quarantine of infected animals. In numerical simulation we used some parameters from the related studies and some are estimated. Some of the parameters are estimated because the disease is neglected and there is no surveillance data despite the negative impacts it brings to the health of people as well as to the economy.

**Table 3: Summarized the parameter values** 

Parameter	Descriptions	Values/y <sup>-1</sup>	Source
$\gamma_a$	Progression rate from $E_a$ to $I_a$	0.18	[11]
$\mu_a$	Animals' natural death.	0.05	Estimated
$\gamma_h$	Progression rate from $E_h$ to $I_h$	0.18	Estimated
$\propto_h$	Humans mortality rate due to disease	0.139	[6]
$\propto_a$	Animals mortality rate due to disease	0.12	Estimated
$\mu_h$	Human natural death.	0.01	[6]
ρ	Rate of producing infectious dairy products	0.69	Estimated
ε	Rate of inspecting dairy products	0.5	Estimated
$ au_h$	Infected humans treatment rate	0.58	[6]

$ au_a$	Quarantine rate of Infected animals	0.65	Estimated
$\Pi_h$	Human recovery rate	0.01	[6]
$\Pi_a$	Animals recovery rate	0.013	[11]
$eta_1$	Humans infection rates from infected humans	0.35	Estimated
$eta_4$	Animals infection rates from infected humans	0.25	Estimated
ω	Decay rate of unused dairy products	0.4	Estimated
$eta_2$	Humans infection rates from infected animals	0.55	Estimated
$eta_5$	Animals infection rates from infected animals	0.6	Estimated
$\beta_3$	Humans infection rates from infected dairy products	0.999	Estimated
$eta_6$	Animals infection rates from infected dairy products	0.34	Estimated

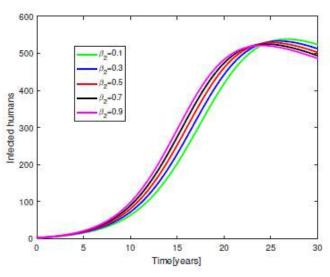


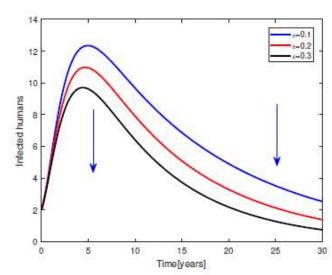


(a) The effects of dairy products inspection on susceptible (b) The effects of dairy products inspection on susceptible humans

Figure 2: The impacts of control parameters on susceptible humans and animals

Figure 2 shows the impacts of the inspection of dairy products on the transmission of bovine TB in humans and animals. The results shows how inspection of dairy products can helps to reduce the spread of the disease to both humans and animals since susceptible classes are increasing. The blue line graph indicates susceptible humans and animals before inspection of dairy products.



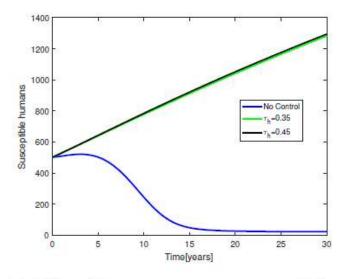


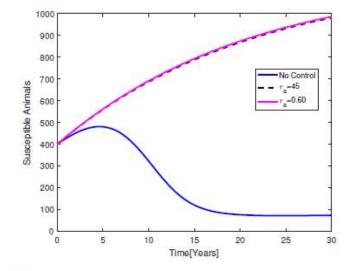
(a) Infected humans before controls

(b) Infected humans after dairy products inspection

Figure 3: Infected humans before and after controls

Infected humans class increases before any controls strategies as shown in Figure 3(a). However the class of infected humans decreases after inspection of dairy products as shown in Figure 3(b). Infected individual class decreases the more we inspect dairy products as we can see from 10% to 30%.

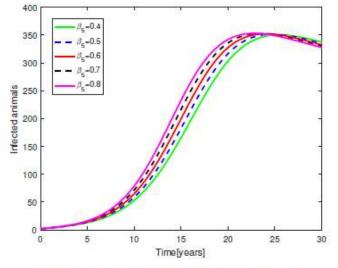


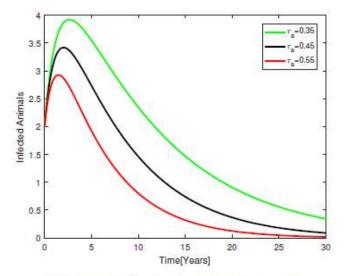


- (a) Effect of human treatment on susceptible humans
- (b) Effect of animals quarantine on susceptible animals

Figure 4: The effects of quarantine infected animals rate into susceptible humans and animals

Figure 4 shows that if infected humans and animals treated once they diagnosed having bovine TB the transmission of the disease into humans and animals decrease hence susceptible individual classes increasing.





- (a) Infected animals before animals quarantine
- (b) Infected animals after animals quarantine

Figure 5: Infected animals before and after quarantine of infected animals

Before quarantine of infected individuals animal class were increasing as shown in Figure 5(a). After quarantine of infected individuals the infected individual class started decreasing as shown in Figure 5(b). This shows that the quarantine of infected individuals helps to reduce the spread of the disease.

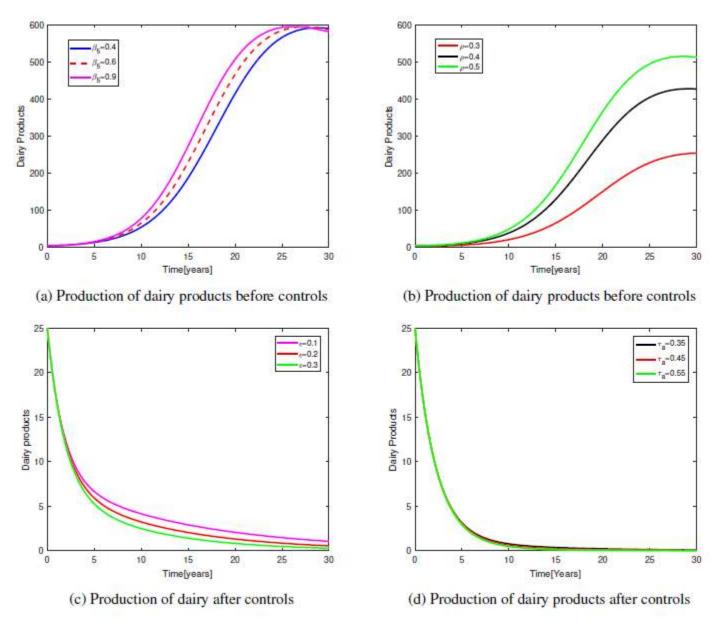


Figure 6: The impacts of control parameters on the production of dairy products

Figure 6(a) and (b) shows how production of infected dairy products was increasing before control, while Figure 6(b) and (c) shows that, after quarantine of infected animals and inspection of dairy products the production decreases. This helps to minimize the rate of consuming infected dairy products hence the spread of the disease also decreases.

## 3. CONCLUSION AND RECOMMENDATION

#### 3.1 Conclusion

A deterministic model of bovine TB for human and livestock is formulated to assess the impacts of control strategies on the spread of the disease. Control strategies that have been discussed includes human treatment, quarantine of infected animals as well as dairy products inspection. The results shows that, the treatment of infected humans and quarantine of infected animals helps at large to decreases the spread of the disease. Also the inspection of dairy products helps to reduce the production of infected dairy products. If these way are followed and implemented effectively we are not going to slaughter infected livestock anymore.

#### 3.2 Recommendations

This study recommends that, public health workers to conduct more research in order to have surveillance data since Tanzania is mentioned among the countries which do have enough information. Veterinary they have to inspect dairy products so that individuals to avoid the consumption of contaminated meat or milk, also screen of livestock several time to make sure that livestock and human are safe and free from the disease infection. Policy makers they have to form a policy that can be helpful for the country in terms of controlling disease transmission. Also further studies should be done to assess the transmission dynamics

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between wild animals, domestic animals and humans and determine ways of eradicating the disease. Also cost effective analysis should be carried out to determine the ways which have low cost and be affordable for all nations regardless the economy of the country.

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