

**MODELING THE DYNAMICS OF *SPODOPTERA FRUGIPERDA*
INFESTATION IN MAIZE PRODUCTION WITH CONTROL
STRATEGIES**

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ABSTRACT

The ongoing demand for maize is driven by its nutritional value, ability to meet the food requirements of a increasing world population, its impact on ensuring a stable food supply, and the growing international investments in ethanol as a renewable fuel source. Nevertheless, the invasion and extensive spread of the *Spodoptera frugiperda* pest cause substantial losses in maize yields, resulting in a diminished standard of living and an economic downturn for those involved in maize production. This study formulates the systems of differential equations to simulate the dynamics of a model encompassing both *Spodoptera frugiperda* and maize biomass. The model considers the presence of predators and the application of best farming practices. The model demonstrates six equilibrium points, and they are locally asymptotically stable provided that the essential conditions are satisfied. Global stability for models' equilibria are established by Lyapunov functions. Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficient (PRCC) multivariate analysis were utilized to pinpoint the sensitive parameters that influence the pest. The predators attack rate and best farming practices was the most sensitive parameter, when increases maize biomass increases while pest population significantly decreases. Numerical simulations indicate that, during the initial phases, combining natural enemies with best farming practices emerges as a successful intervention by directly decreasing the pest population and fostering sustainable pest control. The optimal control strategy which combines farming education campaign and predator selection awareness on eggs and larval reduced pest infestation in early stages. Thus, to control the pest, we recommend that more efforts be directed to reduce eggs and larval populations and improving farming methods through education campaign.

DECLARATION

I, Yusuph Lucas Reuben, hereby declare to the Senate of Nelson Mandela African Institution of Science and Technology that this thesis is my original work. I confirm that it has not been submitted for a degree award at any other institution, nor is it currently under consideration elsewhere.

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CERTIFICATION

The undersigned hereby certify that they have read the dissertation and have found it acceptable to the Nelson Mandela African Institution of Science and Technology.

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DEDICATION

I dedicate this work to my entire family, with a special focus on my parents, Reuben Nkwaya and Grace Nanai. I also extend this dedication to my beloved wife, Obeja Idili, and my daughter, Caren. Their unwavering love, patience, encouragement, prayers, as well as their moral and spiritual support have been invaluable throughout this journey.

TABLE OF CONTENTS

ABSTRACT	i
DECLARATION	ii
COPYRIGHT	iii
CERTIFICATION	iv
ACKNOWLEDGEMENTS	v
DEDICATION	vi
TABLE OF CONTENTS	vii
LIST OF TABLES	x
LIST OF FIGURES	xi
ABRREVIATIONS	xii
CHAPTER ONE	1
INTRODUCTION	1
1.1 Background of the Problem	1
1.1.1 Life Cycle of the SF	3
1.1.2 Damage to Plant	4
1.1.3 Spodoptera frugiperda Control Strategies and Associated Challenges	4
1.1.4 Challenges for Control Strategies	6
1.2 Statement of the Problem	7
1.3 Research Objectives	7
1.3.1 General Objective	7
1.3.2 Specific Objectives	7
1.4 Research Questions	7
1.5 Rationale of the Study	8
1.6 Significance of the Study	8

1.7 Delineation of the Study	8
CHAPTER TWO	10
LITERATURE REVIEW	10
2.1 Introduction	10
2.2 Mathematical Models of Plant-Pest Interactions	10
CHAPTER THREE	13
MATERIALS AND METHODS	13
3.1 Introduction	13
3.2 SF-maize Interaction Model with Best Farming Practices and Eggs, Larvae Pre- dation	13
3.2.1 Assumptions to the model formulated	14
3.3 Basic Model Properties	16
3.3.1 Positivity of the Solution	16
3.3.2 System Boundedness	17
3.3.3 System of Equilibrium Points	18
3.3.4 Local Stability Analysis	20
3.3.5 Analysis of Global Stability	25
3.4 Development of the Optimal Control Problem	27
3.4.1 Description of the Optimal Control	28
CHAPTER FOUR	31
RESULTS AND DISCUSSION	31
4.1 Introduction	31
4.2 Analysis of Global Sensitivity	31
4.3 Impact of Modifying the Most Sensitive Parameters	32
4.3.1 Impact of Modifying the Parameters for Predator Attack Rate	32

4.4	Optimal Control Numerical Simulations for SF-Maize Biomass	33
4.4.1	When all Control Measures are Implemented	36
4.4.2	Farming Education Campaign and Predator selections awereness on Larval	37
4.4.3	Farming Education Campaign and Predator selections awereness on Eggs	37
4.4.4	Predator selections awereness on Eggs and Larval	37
CHAPTER FIVE		42
CONCLUSION AND RECOMMENDATIONS		42
5.1	Conclusion	42
5.2	Recomendations	43
REFERENCES		44
APPENDICES		48
RESEARCH OUTPUTS		63

LIST OF TABLES

Table 1:	Description of model parameters	15
Table 2:	Values of parameters employed in numerical simulation	34

LIST OF FIGURES

Figure 1:	Spodoptera frugiperda life cycle (Daudi <i>et al.</i> , 2021a)	4
Figure 2:	Maize biomass damage due to Spodoptera frugiperda infestations , (FAO, 2018)	5
Figure 3:	Model flow diagram	16
Figure 4:	PRCC values when assessed in relation to the escalating maize biomass population.	32
Figure 5:	PRCC values were assessed concerning the escalating populations of eggs, larvae, pupae, adult moths, egg predators, and larval predators.	33
Figure 6:	Increased predator attack rates on eggs significantly decrease SF populations: where $\delta_1 = 5 \times 10^{-6}$, $\delta_2 = 5 \times 10^{-5}$, $\delta_3 = 5 \times 10^{-4}$, $\delta_4 = 5 \times 10^{-3}$	35
Figure 7:	Increased predator attacks on larvae significantly decrease SF populations: where $\sigma_1 = 5 \times 10^{-6}$, $\sigma_2 = 5 \times 10^{-5}$, $\sigma_3 = 5 \times 10^{-4}$, $\sigma_4 = 5 \times 10^{-3}$	36
Figure 8:	Impact of farming education campaign and predator awereness both eggs and larval	38
Figure 9:	Impact of farming education campaign and predator selection awereness on larva	39
Figure 10:	Impact of farming education effort and predator selection awereness on eggs	40
Figure 11:	Impact of predator selection awereness on eggs and larval	41

LIST OF ABBREVIATIONS

FAO	Food and Agriculture Organization
IPM	Intergrated Pest Management
LHS	Latin Hypercube Sampling
PMP	Pontryagin's Maximum Principles
PPT	Push Pull Technology
PRCC	Partial Rank Correlation Coefficient
SDGs	Sustainable Development Goals
SF	Spodoptera Frugiperda
SSA	Sub-Saharan African

CHAPTER ONE

INTRODUCTION

1.1 Background of the Problem

Maize continues to hold a central position as a crucial dietary staple for millions in Sub-Saharan African (SSA) and various Asian nations (Mendesil *et al.*, 2023). It has evolved into a significant industrial commodity, playing key roles in enhancing the nutritional content of food, serving as animal feed, and serving as a source of income. Maize stands out among all food sources, constituting 19.5% of the worldwide consumption of calories (Waqas *et al.*, 2021; Prasanna *et al.*, 2021).

As defined by the Food and Agriculture Organization (FAO), food security is characterized by the consistent and sufficient availability of nutritious food that meet dietary requirements and individual choices, thereby ensuring the health and a physically active way of life for individuals (FAO, 2022). Looking forward, it is crucial to acknowledge that, global maize production will need to quadruple by 2050 to uphold global food security (Lai *et al.*, 2022).

On a global scale, maize stands as the second most versatile multi-purpose crop, just behind wheat (Mendesil *et al.*, 2023). The Americas serve as the predominant contributors to maize production, accounting for half of the global output, with Asia trailing behind at 32%. Europe and Africa contribute 11% and 7.4%, respectively (Singh *et al.*, 2022). Every year, the consumption of maize is estimated to surpass 116 million tonnes, with 30% of this consumption occurring globally and 21% specifically within SSA (Mendesil *et al.*, 2023).

In Africa, white maize constitutes the primary source of food, accounting for approximately 85% to 95% of total consumption in around 14 countries within SSA (FAO, 2018). However, the sustainable production of maize crops in these regions has been significantly hampered in recent times due to extensive infestation, damage, and destruction caused by the interaction between maize biomass and the invasive pest known as *Spodoptera frugiperda* (SF). The SF consumption of plant materials, often referred to as "biomass" in this context, exerts a substantial impact on maize plants (Mendesil *et al.*, 2023; Kandel & Poudel, 2020).

Spodoptera frugiperda, commonly known as the fall armyworm, is a highly polyphagous pest that presents a significant risk to global food security. This native agricultural pest has been detected in North and South America, and it is notorious for its swift and destructive spread (Prasanna *et al.*, 2022). *Spodoptera frugiperda* predominantly infests plants within the *Poaceae*

family, encompassing vital food crops like sorghum, rice, wheat, maize, and various types of pastures (Rukundo *et al.*, 2020). Previous research indicates that SF displays a pronounced preference for maize in comparison to other crops and pastures (De Groote *et al.*, 2020). It impact every stage of maize growth, beginning from germination and continuing through ear development, resulting in defoliation, whorl injury, and adverse effects on grain yields and quality (Sisay *et al.*, 2019; Day *et al.*, 2017).

According to the studies conducted by Chowdhury *et al.* (2019), plant biomass, including various components of maize such as kernels, leaves, stems, and other green parts, serves as the primary food source for SF. These pests have a strong preference for consuming maize biomass, with a particular focus on the foliage and whorls throughout all stages of maize growth (Rukundo *et al.*, 2020). They subsequently shift their feeding to the ear, cob, and kernels. This feeding pattern is particularly concerning as it leads to significant damage to the growth points and photosynthetic regions of maize plants. This damage impairs the plants' ability to generate their own sustenance, resulting in reduced grain yields. Such an effect is of critical concern for meeting the food requirements of a growing population (Bhusal *et al.*, 2020; Prasanna *et al.*, 2018).

Across the Americas, the presence of SF has led to substantial damage in maize crops, causing significant reductions in yields. This has been reported as 34% in Brazil, 72% in Argentina, and 40% in Honduras, as documented by Faithpraise *et al.* (2015); Murúa *et al.* (2006). In Africa, the situation is even more dire, with corn harvests suffering losses of up to 90% due to outbreaks originated in Nigeria and then extended to various countries across sub-Saharan Africa. The inaugural outbreak of SF in West Africa transpired in 2016, and it extended to East African nations in 2017. This infestation had repercussions in a total of 44 countries across the continent (De Groote *et al.*, 2020; Daudi *et al.*, 2021a; Pearce *et al.*, 2006; Singh *et al.*, 2022). On the farm level, SF infestations have been associated with significant yield losses, ranging from 22% to 67% in Ghana, 47% in Kenya, 9.4% in Zimbabwe and 30% to 41% in Tanzania (Sisay *et al.*, 2019; Niassy *et al.*, 2021).

According to Day *et al.* (2017), SF significantly contribute to maize-related losses in 12 African countries where maize is cultivated. Without any control measures in place, these losses could reach an estimated annual range of 2.5\$ to 6.2\$ billion, a figure sufficient to provide food for 40 to 100 million people, as emphasized in by FAO (2018). Similarly, Singh *et al.* (2022) estimated that the annual cost of SF in agriculture could soar as high as 9.4\$ billion, and potentially even more in the future. This is a matter of concern considering that the global population is expected

to increase by over 80%, reaching 9.3 billion people by 2050 (Kandel & Poudel, 2020; Bhusal *et al.*, 2020; Prasanna *et al.*, 2018; De Groote *et al.*, 2020).

1.1.1 Life Cycle of the SF

The life cycle of the SF involves four distinct stages including eggs, larval, pupa and adult moth as it undergoes complete metamorphosis. In the summer time, this life phases is typically finished in a period of 30 days. However, amid the winter and spring, the same cycle takes a longer period, ranging from 60 to 90 days to finish (FAO, 2018).

The egg exhibits a distinctive spherical form, featuring a compressed structure base and a curving, upward-pointing apex that ends in a rounded tip. While some females have been known to produce over 2000 eggs in their lifetime, the average number of eggs within an egg-mass can vary considerably, but it often falls within the range of 100 to 200 (Bista *et al.*, 2020; Sharanabasappa *et al.*, 2019). These eggs are typically laid in a single layer, adhering to vegetation, although sometimes they are deposited in multiple layers. To safeguard the eggs, the female envelops them with a coating of gray scales, creating an egg mass that appears somewhat fuzzy or moldy. During the summer, this egg stage is brief, lasting only 2-3 days (Prasanna *et al.*, 2018; FAO, 2018).

The life cycle of the larval consists of six developmental stages known as instars. By the third instar, the dorsal surface of their bodies begins to darken, taking on a brownish color, and white lines appear along their sides. It's during this third instar that the larvae significantly impact maize biomass at all growth stages, as discussed in Day *et al.* (2017). Furthermore, the larval stage typically endures for 14 days in the summer, while it extends to around 30 days in cooler weather conditions (FAO, 2018). Pupation typically occurs underground at a depth of 2 to 8 cm. The caterpillar employs silk to bind soil particles, forming an open oval cocoon that measures 20 to 30 mm in length. In cases where the soil is too compact, the larvae may weave leaves and other materials into a cocoon and build it on the surface of the earth. In summer, the pupal stage typically lasts for eight to nine days, while during winter, it extends to around twenty to thirty days (FAO, 2018).

Lastly, adult moths are most active during the hot and humid nighttime hours. Usually, the majority of a female's eggs are laid within the initial four to five days of her life, occurring during a preoviposition phase lasting three to four days. Nevertheless, there is the possibility of continued egg-laying for up to three weeks. The average lifespan of an adult moth is 10 days, with a range spanning from 7 to 21 days (Prasanna *et al.*, 2018).

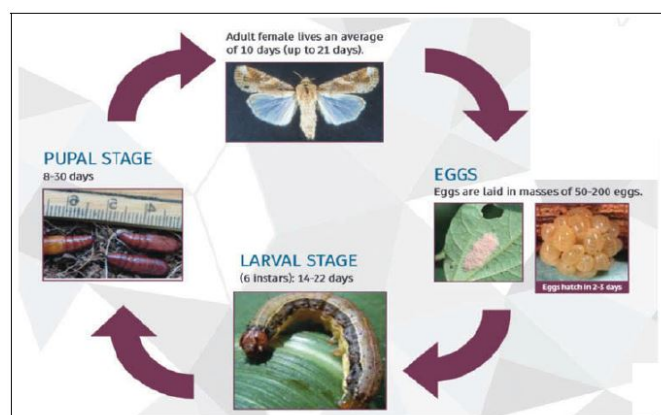


Figure 1: *Spodoptera frugiperda* life cycle (Daudi *et al.*, 2021a)

1.1.2 Damage to Plant

The initial whorl stage exhibited the lowest level of sensitivity to damage, according to Daudi *et al.* (2021a), followed by the middle and late whorl stages, which were all rather sensitive. They indicated that refining the writing style could lead to better communication. Moreover, they pointed out that when the average larval concentrations per plant range from 0.2 to 0.8 during the late whorl stage, there is a potential reduction in crop productivity by 5% to 20%. Larvae can also cut leaves or burrow into plant growth places, which stops the growth of plants. The larvae occasionally consume the ear of maize (FAO, 2018). The impact of SF on maize is said to start as soon as immature larvae use plant material as a food source and move to the growth sites, leaving semi-transparent windows and developing kernels inside the cobs of older plants that are shielded from the sun (Chapman *et al.*, 2000). The tiny larvae emerge during the night to feed on the biomass, taking shelter in crevices between the stem and biomass of maize plants. These SF larvae have voracious appetites; when they target leaves or whorls, they defoliate the host, and when they attack the stem and growth point, they harm young plants. Furthermore, they cause significant damage to mature plants by feeding on both the vegetative and reproductive systems. When there is a higher larval population in a field, they begin to spread throughout the corn field, consuming every plant in their path. However, in times of food scarcity, the older larvae resort to consuming the younger ones, this process is known as cannibalism (Assefa & Ayalew, 2019; Bhusal *et al.*, 2020).

1.1.3 *Spodoptera frugiperda* Control Strategies and Associated Challenges

A range of strategies can be employed to control SF when they provide a threat to a field (Matova *et al.*, 2020). This strategies are categorised into three groups: cultural farming practises, biological control practises, and synthetic and natural pesticide control practises.



Figure 2: Maize biomass damage due to *Spodoptera frugiperda* infestations , (FAO, 2018)

- (i) Synthetic and natural pesticide control practises. This calls for the use of both synthetic and natural insecticides to stop the spread of SF. This approach is thought to be the priciest when compared to other control systems (Matova *et al.*, 2020).
- (ii) Cultural farming practices, among the best agricultural techniques used include weeding, crop rotation, hand picking, sanitation, soil management, intercropping and host plant resistance. Previous studies on the subject of maize plants have shown that SF oviposition can be reduced by intercropping two or more crops or by incorporating plants from non-host crops in the field (Matova *et al.*, 2020). This strategy is regarded to be best course of action for managing this invasive pest, however, implementing regarding regional strategy is much prefered (Kalyebi *et al.*, 2023; Kumar *et al.*, 2022; Akeme *et al.*, 2021; Ndakidemi *et al.*, 2021).
- (iii) Approaches to biological controls including predators, parasitoids, and diseases serve as natural adversaries of the SF, actively regulating its population levels. In additional scenarios, intercropping fosters the development and growth of a natural enemy population significant enough to effectively regulate SF (FAO, 2018). Owing to this phenomenon, "push-pull technology" (PPT) has become more popular, and is currently recommended for SF management (Matova *et al.*, 2020). As per Midega *et al.* (2018), brachiaria 'Mulato II' is employed to envelop the greenleaf desmodium intercrop and maize.

In addition to repelling (pushing) the moths, desmodium also safeguards the maize by producing semiochemicals that draw (pull) the insects. These semiochemicals are released by the border crop. It has been stated that, in a field where the approach is being employed, SF infestations can be decreased by at least 80% (Midega *et al.*, 2018; Hailu *et al.*, 2018). Integrated Pest Management (IPM) strategy is to manage a pest utilising a

range of techniques with the least amount of negative impact on people, animals, and the environment (Matova *et al.*, 2020; Mendesil *et al.*, 2023; Prasanna *et al.*, 2022). The IPM strategies work to control current pest infestations while averting the emergence of new ones. Consistently conducting routine scouting to detect infestations and promptly applying appropriate IPM techniques for pest suppression is strongly recommended (Prasanna *et al.*, 2018).

1.1.4 Challenges for Control Strategies

Farmers' choices regarding SF control measures are influenced by various factors. These include the accessibility of control methods, their effectiveness, ease of use, available resources, and gender considerations (Matova *et al.*, 2020). The feeding and sheltering habits of SF on the host plant have made it increasingly challenging to control in Africa, particularly for smallholder farmers who bear the brunt of the harm the pest does to maize.

Common management options for SF in industrialised countries and later in Africa included the use of pesticides, biological control, and genetically modified crops (Bt maize). Regrettably, a significant number of African countries grapple with financial constraints, hindering their ability to effectively manage this pest on the continent (Prasanna *et al.*, 2022; Sisay *et al.*, 2019; De Groote *et al.*, 2020). This is because genetically engineered crops and extensive chemical insecticide spraying programmes are expensive. Furthermore, overuse of chemical pesticides has been connected to harmful effects on the environment and may lead to the rise of pest resistance (Matova *et al.*, 2020). In addition, about 10 000 deaths and 400 000 acute illnesses are reported daily in poor countries as a result of excessive pesticide use (Onstad & Knolhoff, 2023).

However, the juvenile larval hide during the day in the corn funnel and emerge only at night to feast on the biomass of the maize (Day *et al.*, 2017). Because young and old larvae prefer to stay inside the corn funnel, where they are protected from spray application by the foliage, using insecticides is therefore ineffective. Furthermore, farmers are exposed to hazardous chemical pollution when using pesticides to manage SF, and the ecology is disrupted by killing non-target species (Matova *et al.*, 2020). As a result, developing sustainable and ecologically compatible control methods for SF is crucial. Furthermore, because of its short life cycle, tendency to spread swiftly over large geographic areas, and wide host range, controlling this pest poses a formidable challenge (Prasanna *et al.*, 2018).

1.2 Statement of the Problem

Our food security at all stages of life has been threatened by SF and other pests, which has resulted in significant financial losses. For decades scientists have been describing the dynamics of plant-pest interactions with mathematical models. Even though SF infestations in maize fields pose major challenges to food security, and still few mathematical models available to help understand the implications of SF-maize interaction. As far as the author is aware, no deterministic model have been developed to assess how applying predators affects pest population density over time and the few that exists are not pest specific. Mostly of them were general and those few didn't capture other SF biological factors on their dynamics, including egg stage.

1.3 Research Objectives

1.3.1 General Objective

The main objective of the study is to develop and analyze mathematical model for the dynamics of *spodoptera frugiperda* infestation in maize production with control strategies.

1.3.2 Specific Objectives

The specific objectives of the study were:

- (i) To formulate and analyze deterministic model for the *spodoptera frugiperda* dynamics in maize with best farming practices and predators interventions.
- (ii) To determine parameters that drive *spodoptera frugiperda* infestations.
- (iii) To determine the optimal control strategy for the *spodoptera frugiperda* in maize biomass.

1.4 Research Questions

This research was guided by the following questions:

- (i) How is it possible to create and analyse a deterministic model for the dynamics and control of *Spodoptera frangeperda*?
- (ii) How can parameters that drive *spodoptera frangeperda* be determined?
- (iii) Are there any optimal control options that can be considered to minimize the infestation of *spodoptera frangeperda* in maize?

1.5 Rationale of the Study

Maize holds the position of the second most crucial cereal crop globally, following wheat. It is a major source of income, food, feed, and industrial grain, and it contributes significantly to the total production of cereal grains in the world economy. Because of its superior nutritional qualities, ability to feed the growing global population, and role in promoting food security, maize is becoming more and more in demand. However, the SF invasion and extensive infestation have resulted in large losses in maize output, endangering the crop's productivity and, ultimately, reducing the growers' means of subsistence. In mitigating the effects of SF in the field, smallholder farmers in Sub-Saharan African nations must have a comprehensive understanding of the symptoms, impacts, and the spread of SF on maize biomass. The research contributes to agricultural management and policymaking by assisting stakeholders in selecting and implementing effective control strategies to curb the spread of pests. Additionally, the chosen topic serves as a valuable resource for the literature review.

1.6 Significance of the Study

- (i) To educate farmers and locals the need of implementing an efficient IPM management plan.
- (ii) The findings will serve as a foundation for future investigation for academics who wish to investigate the modelling of the dynamics of SF infestation.
- (iii) To provide guidance on the most effective ways to increase maize output while preserving the environment to the government, policymakers, and agriculture sector.

1.7 Delineation of the Study

This study aims to formulate and analyze mathematical models that capture the dynamics and control of SF. The research work is structured into five chapters. Chapter One is devoted to general introduction of the research topic, describing in detail how SF infestation on maize. In Chapter Two deals with the literature review on mathematical and non-mathematical studies for the SF-maize dynamics and control. In Chapter Three, a deterministic model for the dynamics of SF formulations has been formulated and evaluated. Furthermore, the optimal management has been carried out. Chapter Four deals with numerical simulation of the previously established mathematical models whereby the results are presented graphically and in tabular form, followed by their detail discussions. Chapter Five provides conclusion remarks and recommendations following the results obtained in Chapters Three and Four. However, our study is

limited to the dynamics of SF in areas where there is no migration of SF. Due to lack of SF data from the Ministries of Health and online sources especially for developing countries, parameter values have been obtained from literature. To offer deeper insights into pest dynamics and evaluate the effectiveness of different control strategies.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Mathematical modeling goes beyond basic arithmetic, applying mathematics to real-world issues for advantageous societal outcomes. It enables a sophisticated understanding, analysis, and resolution of complex challenges, contributing to societal well-being and progress. In many real-world scenarios, using mathematical models to investigate and guide policy formulation has been shown to be beneficial (Tang *et al.*, 2010; Faithpraise *et al.*, 2015; Daudi *et al.*, 2021a). The current status of modelling dynamics related to interactions between SF and maize is referenced in this chapter.

2.2 Mathematical Models of Plant-Pest Interactions

The study conducted by Faithpraise *et al.* (2015) formulated a deterministic model to explore the potential utilization of naturally occurring beneficial insects for controlling infestations of African armyworm (AAW) in cereal agricultural regions. Predators were one of several forms of control employed. The research findings illustrated the usefulness of the proposed approach in providing potential means of limiting AAW outbreaks through the employment of natural enemies. But because pests are not universal, the study is broad.

Páez Chávez *et al.* (2017) proposed and analysed mathematical models ranging from ordinary differential equations with reformulation of impulses to better understand the effects of combining chemical, biological, and cultural approaches to control pests. In summary, the proposed model examined how Integrated Pest Management (IPM) can lower insect populations in agricultural areas. Many noteworthy discoveries were made, including periodic doubling cascades that cause disordered behaviour, hysteric effects, and fold and flip bifurcations in limit cycles.

Abraha *et al.* (2021) created a deterministic mathematical model of the relationship between plants and pests to investigate the effects of delays and the most effective way to apply biopesticides while accounting for vegetable matter and the effects of agricultural campaigns. When the authors of this study added a time delay aspect in their analysis, they took into consideration the amount of time that farmers needed to learn about management techniques or take action following a pest invasion. Their analysis's findings suggested that developing solutions for managing plant-pest interactions would benefit from optimal control. The limitation of this study, did not consider biological control agents.

Daudi *et al.* (2021a) created a fractional-order model to examine the role of farming awareness and natural enemies. The SF-maize interaction dynamics were studied through simulation using the established model. In their model simulations, they evaluated various control methods, including biological control and agricultural awareness. According to the study, in order to completely eradicate SF, non-time-dependent farming awareness efforts should target close to 100% of farmers. Furthermore, the simulation findings showed that farming awareness efforts had a critical role in lowering SF infestation. Nevertheless, the influence of two or more predators was not caught, nor were the dynamics of the egg and larvae phases captured by the study. Furthermore, in order to identify the most sensitive drive parameter in pest control, the study did not undertake global sensitivity.

In different study, Ogunfunmilayo *et al.* (2021) created a statistical model to investigate how parastoids and predators affect maize biomass in order to control SF. Natural enemies include *Trombidium* sp. (Acari.: Trombidiidae), *Telenomus remus* (Hymenoptera: Platygasteridae), and *Euplectrus laphygmae* (Hymenoptera: Eulophidae). As a result, the natural enemies decreased the SF's ability to feed, develop, and migrate, as well as to die especially in the first instar. Additionally, the study advised an advocacy effort to incorporate their use into IPM, which attracts and fosters the growth of natural enemies for the management of superficities, as it is a more practical and environmentally safe alternative to the synthetic pesticides now advocated.

In similar vein, Ntare *et al.* (2022) examined the impact of Uganda's present smallholder agricultural production methods on SF infestation. These findings lay the groundwork for understanding the important interactions between intercropping systems and the reduction of SF infestations. The limitation of this studies, the mathematical model used to examine the effects of best farming practises, like intercropping systems, was not taken into account in this study, though. Furthermore, optimal control was not used in the study to evaluate how well best agricultural practises were applied.

Moreover, Abbas *et al.* (2022), the lack of funds to purchase chemicals or expensive pure seeds, in addition to the harmful effects of pesticides on the environment, human health, and the emergence of insect pest resistance, have led to a heightened search for a low-risk, target-specific alternative strategy. Focusing on their effectiveness against SF, the research investigated the use of biological control across various developmental stages. This encompassed predators, parasitoids, entomopathogenic fungi, viruses, nematodes, bacteria, and biopesticides. The results suggested that biological control be used to enhance invasive pest management initiatives for SF. Likewise, Senay *et al.* (2022), the results indicate that a substantial proportion of African

maize cultivation is at risk of seasonal SF infestations, with nearly 92% of maize areas contending with the presence of the pest year-round. The pest risk assessment emphasizes that in over 95% of African maize regions suitable for SF, the survival of three or more pests is sustained throughout the year. Addressing these multi-peril risks for African maize producers requires the adoption of sophisticated strategies in crop breeding, biological, chemical, and other management approaches.

Chisonga *et al.* (2023), employed structural equation models to investigate both the direct impact of leaf damage on yield and the indirect effects through plant height. Through a statistical approach, the research predicted yield losses. The models developed played a pivotal role in crafting decision-support tools for Integrated Pest Management (IPM). The study suggests that leaf damage inflicted by SF has a discernible yet modest impact on yield during specific plant developmental stages. The results highlight the significance of IPM strategies that prioritize interventions to improve plant vigor and acknowledge the role of natural enemies. Implementing such strategies is likely to yield more significant gains in crop yield at a lower cost compared to a singular focus on SF control.

Our knowledge of plant-pest interactions has advanced both qualitatively and quantitatively thanks to the aforementioned studies and others highlighted in them. Even yet, there are still a number of issues in mathematically simulating plant-pest interactions. Firstly, most of the previously stated research were broad in nature rather than pest-specific, suggesting that their findings were also broad. rather, they follow distinct biological development cycles. Certain types of pests, like tarnished plant bugs, have the ability to diapause, but others, like SF, do not. Secondly, most of studies didnt capture other SF stages, for example eggs population which is also crutial part on the dynamics of SF infestations. Moreover, many studies considered statistical model, and few considered fractional order differential equations. This dissertation offers mathematical models for the interaction between SF and maize in order to fill this gap. When compared to insecticides, bio-control methods (eggs and larvae predation) are less damaging to the ecosystem. These and other benefits have led to an increase in their usage in pest management over the last ten years. Specifically, the mathematical model sought to ascertain the impact of intervention, calculate variations in pest population density over time, and calculate the amount of maize biomass resulting from the use of farming practices and biocontrol measures during an outbreak.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Introduction

In this chapter, we outline and scrutinize a deterministic model for understanding how SF infestation in maize biomass evolves. This model is established using non-linear ordinary differential equations, which, in turn, yield dynamic systems that govern the dynamics of SF infestation in maize. The model incorporated various interventions, such as predators and best farming practices include weeding, crop rotation, hand picking, sanitation, soil management, intercropping and host plant resistance, to assess their effectiveness in controlling the pest. To validate our model, we tested it to ensure that its solutions remain positive and bounded.

Additionally, we computed the equilibrium points and conducted an analysis of their stability. To assess the local stability of these equilibrium points, we employed the linearization method along with the characteristic polynomial. Furthermore, for the global stability of interior equilibrium points, we used Lyapunov functions in our analysis. LHS and PRCC multivariate analysis were utilized to identify the crucial parameters that significantly impact maize production. Furthermore, the Pontryagin's Maximum Principle (PMP) was applied to establish the most optimal pest control strategy, while factors such as farming education campaign and predator selection awareness were investigated to determine their influence on pest control.

3.2 SF-maize Interaction Model with Best Farming Practices and Eggs, Larvae Predation

To evaluate the impact of adopting best farming practices on eggs, larvae, and the ultimate maize biomass, we have put forth a nonlinear model. This model integrates biological control strategies to effectively manage the population of SF (Faithpraise *et al.*, 2015; Abbas *et al.*, 2022). Our model encompasses three distinct populations: SF, predators, and maize biomass. Within the SF population, there are four classes: egg (E), larva (L), pupa (P), and adult moth (A). The predators targeting the egg and larval stages are denoted as (R) and (Q), respectively. Furthermore, we represent maize biomass as (M).

Concerning maize biomass, our hypothesis involves a logistic growth model incorporating a carrying capacity denoted as K_M and a gross growth rate represented by r . The efficiency of biomass conversion is expressed as e , while the consumption rate of SF larvae is indicated by

β . The dynamics of the SF egg and larva populations are anticipated to follow a logistic growth model in the presence of maize. The carrying capacities for the SF egg and larva populations are denoted as K_E and K_L , respectively. The net growth rates for the SF egg and larva populations are represented by a and α_E , respectively. The transition from egg to larva is expected to occur after $\frac{1}{\alpha_E}$ days. The natural mortality rates for the SF egg and larva are indicated by μ_E and μ_L , respectively. Additionally, the population decrease resulting from deaths or removals due to farmers adopting best farming practices is represented by h .

The eggs and larvae are vulnerable to a variety of parasitoids and predators, including birds, rodents, beetles, spiders, and earwigs (Daudi *et al.*, 2021a; Assefa & Ayalew, 2019; Kenis *et al.*, 2022). Let ε represent the conversion efficiency, and σ denote the frequency of predation on larvae. Predators are expected to have an average lifespan of $\frac{1}{\eta}$ days. Similarly, for the impact of egg predation, designate ψ as the conversion efficiency, and δ as the rate at which predators attack the eggs. Egg predators are estimated to have an average lifespan of $\frac{1}{\gamma}$ days.

3.2.1 Assumptions to the model formulated

- (i) Factors such as weather conditions, environmental factors, and the specific method used to plant maize seeds were believed to promote seed germination and the subsequent growth of maize biomass, assuming there is no natural mortality rate before harvest (Daudi *et al.*, 2021a).
- (ii) At the beginning of the growing season, it was assumed that maize seeds were all planted in a single day and grew at a continuous and constant rate.
- (iii) It was assumed that once SF moths had migrated into a plot, no migration of these moths out of the plot occurred before the maize harvest (Daudi *et al.*, 2021a).
- (iv) The growth of maize biomass was presumed to adhere to a logistic pattern.
- (v) The number of maize plants in a plot was not expected to exceed a certain limit, denoted as K_M , as time progressed toward the maturity stage.
- (vi) In the absence of maize, it was assumed that larvae would experience exponential mortality since maize was their sole source of food.
- (vii) At time $t = 0$ the initial population of maize plants, denoted as $M(0)$ was assumed to be equal to the maximum number of maize plants that the plot could support, represented as K_M .

(viii) The dynamics of SF egg and larval populations were expected to align with a logistic growth model in the presence of maize.

Table 1: Description of model parameters

Parameter	Descriptions
a	Number of eggs laid per day per female moth
v	Proportion of adult female moth
α_E^{-1}	Eggs transition rate
α_L^{-1}	Larva transition rate
α_P^{-1}	Pupa transition rate
r	Growth rate of maize biomass
K_M	Maximum carrying capacity of the maize biomass
K_E	Egg environmental carrying capacity
K_L	larvae environmental carrying capacity
μ_E^{-1}	Inherent mortality rate rate of eggs
μ_L^{-1}	Inherent mortality rate of larval
μ_P^{-1}	Inherent mortality rate of pupa
μ_A^{-1}	Inherent mortality rate of adult moth
β	Larva attack rate
e	Efficiency of conversion biomass by larvae
h	Death or removal rate due to interventions
η	Larva's predator life span
γ	Egg's predator life span
δ	Attack rate of predator on eggs
ε	Efficiency of predator conversion on larvae
ψ	Efficiency of predator conversion on eggs
σ	Attack rate of predators on larvae

The intricate interactions among SF, maize biomass, and predators are visually represented in the flow diagram illustrated in Fig. 3. The governing dynamics are succinctly encapsulated by a set of differential Equation (3.1).

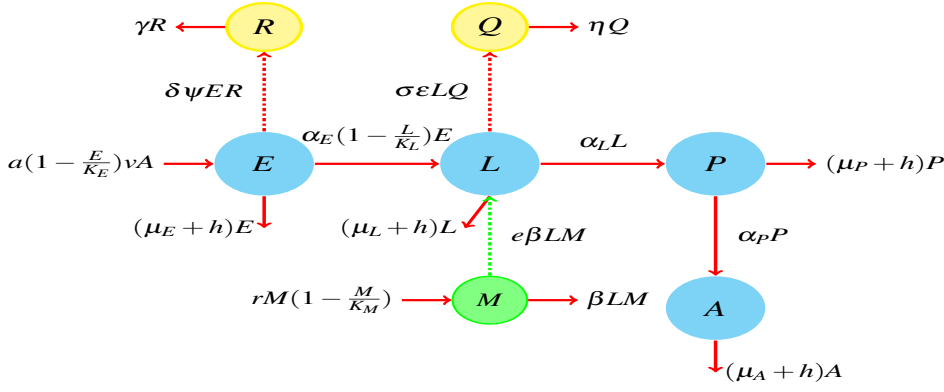


Figure 3: Model flow diagram

$$\left\{ \begin{array}{l} \frac{dM}{dt} = rM \left(1 - \frac{M}{K_M} \right) - \beta LM \\ \frac{dE}{dt} = a \left(1 - \frac{E}{K_E} \right) vA - (\alpha_E + \mu_E + h)E - \delta \psi RE \\ \frac{dL}{dt} = \alpha_E \left(1 - \frac{L}{K_L} \right) E + e\beta ML - (\alpha_L + \mu_L + h)L - \sigma \epsilon QL \\ \frac{dP}{dt} = \alpha_L L - (\mu_P + \alpha_P + h)P \\ \frac{dA}{dt} = \alpha_P P - (\mu_A + h)A \\ \frac{dR}{dt} = \delta \psi ER - \gamma R \\ \frac{dQ}{dt} = \sigma \epsilon LQ - \eta Q \end{array} \right. \quad (3.1)$$

where $M(0) \geq 0, E(0) \geq 0, L(0) \geq 0, P(0) \geq 0, A(0) \geq 0, R(0) \geq 0, Q(0) \geq 0$.

3.3 Basic Model Properties

To determine whether the model has mathematical and biological importance, two model features are examined: positivity and the boundness of the solutions.

3.3.1 Positivity of the Solution

Theorem 3.1

Let $M(0) \geq 0, E(0) \geq 0, L(0) \geq 0, P(0) \geq 0, A(0) \geq 0, R(0) \geq 0$ and $Q(0) \geq 0$. Then, all solutions of the model system Eq. (3.1) above are non-negative in $\mathbb{R}_+^7$ for all $t \geq 0$.

Proof. Consider the equation for the maize population,

$$\begin{aligned}\frac{dM}{dt} &= rM \left(1 - \frac{M}{K_M}\right) - \beta LM \\ &\leq rM \left(1 - \frac{M}{K_M}\right).\end{aligned}\tag{3.2}$$

Whose solution is,

$$M \leq \frac{K_M}{1 + Ce^{-rt}},\tag{3.3}$$

where C is the constant. At $t = 0$, $M = M(0)$,

$$M \leq \frac{K_M M(0)}{M(0) + (K_M - M(0))e^{-rt}}.\tag{3.4}$$

This implies that, the maize population lies in the interval of $0 \leq M(t) \leq K_M$, $\forall t \geq 0$. Using the same approach, it was shown that $E(t) \geq 0$, $L(t) \geq 0$, $P(t) \geq 0$, $A(t) \geq 0$, $R(t) \geq 0$ and $Q(t) \geq 0$, $\forall t \geq 0$. Hence, the model is well-posed and its sufficient to check its dynamics. $\square$

3.3.2 System Boundedness

Theorem 3.2

All the solutions of the model system Eq. (3.1), which initiate in $\mathbb{R}_+^6$, are uniformly bounded. That is there exists a constant $G > 0$ such that $E(t) \leq G$, $L(t) \leq G$, $P(t) \leq G$, $A(t) \leq G$, $R(t) \leq G$ and $Q(t) \leq G$ for each solution of the model system with all t large enough.

Proof. Considering pest population equations. Consider $E(t)$, $L(t)$, $P(t)$, $A(t)$ with favorable initial conditions:

$$N = E + L + P + A.\tag{3.5}$$

Thus,

$$\frac{dN}{dt} = \frac{dE}{dt} + \frac{dL}{dt} + \frac{dP}{dt} + \frac{dA}{dt}.\tag{3.6}$$

Therefore,

$$\frac{dN}{dt} \leq (av - \mu_A - h)A - (\mu_E + h)E - (\mu_L + h)L - (\mu_P + h)P,\tag{3.7}$$

implies,

$$\frac{dN}{dt} \leq -\theta^*(E + L + P + A) + 2(av - \mu_A - h)A.\tag{3.8}$$

where $\theta^* = \min\{(\mu_E + h), (\mu_L + h), (\mu_P + h), (av - (\mu_A + h))\} > 0$, provided $av > (\mu_A + h)$.

Therefore,

$$\frac{dN}{dt} + \theta^* N \leq G_0,\tag{3.9}$$

where $G_0 = (av - \mu_A - h)^2 > 0$.

Applying the theory of differential inequality, we obtain,

$$N(t) \leq \frac{G_0}{\theta^*}(1 - e^{-\theta^*t}) + N(0)e^{-\theta^*t}. \quad (3.10)$$

For $t \rightarrow \infty$, we get $N(t) \leq \frac{G_0}{\theta^*}$. So $N(t)$ is ultimately bounded. Therefore, there exist a positive constant $G = \frac{G_0}{\theta^*}$ such that each solution of the model system Eq. (3.1), $E(t) \leq G$, $L(t) \leq G$, $P(t) \leq G$ and $A(t) \leq G$ when t is sufficient large. Again, considering predator population,

$$\frac{dN_1}{dt} \leq (\gamma - \delta\psi G)R - (\eta - \sigma\varepsilon G)Q, \quad (3.11)$$

$$\frac{dN_1}{dt} \leq -\pi(R + Q). \quad (3.12)$$

where $\pi = \min\{(\gamma - \delta\psi G), (\eta - \sigma\varepsilon G)\} > 0$, $N_1 = R + Q$, provided $\gamma > \delta\psi G$ and $\eta > \sigma\varepsilon G$. Therefore,

$$N_1(t) \leq \frac{G_0^*}{\pi}(1 - e^{-\pi t}). \quad (3.13)$$

Using the same approach, solution of maize and predators are bounded as

$0 \leq M_m \leq K_M$ and $N_1(t) \leq \frac{G_0^*}{\pi} = B$ as $t \rightarrow \infty$ respectively. $\square$

3.3.3 System of Equilibrium Points

The system described (3.1) possesses six equilibrium points, enumerated as follows:

- (i) The trivial equilibrium point $\xi_1 : (0, 0, 0, 0, 0, 0, 0)$ is consistently present. Biologically, at this equilibrium point ξ_1 , the species described in (3.1) are entirely absent, either due to natural losses or as a result of conservation measures.
- (ii) The equilibrium point associated with the extinction of the pest is represented as $\xi_2 : (k_M, 0, 0, 0, 0, 0, 0)$. This signifies that, with the absence of the SF population, a robust maize biomass is anticipated at the season's end.
- (iii) The maize-extinction equilibrium point $\xi_3 : (0, E_3, L_3, P_3, A_3, R_3, Q_3)$, where:

$$\begin{cases} E_3 = \frac{\gamma}{\delta\psi}; L_3 = \frac{\eta}{\sigma\varepsilon}; P_3 = \frac{\eta\alpha_L}{\sigma\varepsilon m_3}; A_3 = \frac{\eta\alpha_L\alpha_P}{\sigma\varepsilon m_3 m_4}, \\ R_3 = \frac{(\sigma\varepsilon m_1 m_3 m_4 K_E + a\eta v \alpha_L \alpha_P)(\mathcal{R}_1 - 1)}{\delta\varepsilon\psi\sigma K_E m_3 m_4}, \\ Q_3 = \frac{(\delta\psi m_2 K_L + \gamma\alpha_E)(\mathcal{R}_2 - 1)}{\delta\psi\varepsilon\sigma K_L}, \end{cases} \quad (3.14)$$

with $\mathcal{R}_1 = \frac{a\alpha_L\alpha_P v\psi\eta\delta K_E}{\gamma(\sigma\epsilon m_1 m_3 m_4 K_E + a\eta v\alpha_L\alpha_P)}$, $\mathcal{R}_2 = \frac{\sigma\gamma\epsilon\alpha_E K_L}{\eta(\delta\psi m_2 K_L + \gamma\alpha_E)}$, $m_1 = (\alpha_E + \mu_E + h)$, $m_2 = (\alpha_L + \mu_L + h)$, $m_3 = (\alpha_P + \mu_P + h)$ and $m_4 = (\mu_A + h)$. This implies that, existence of the equilibrium point ξ_3 depends on the satisfactions of the following conditions

$$\mathcal{R}_i > 1, \quad \forall i \in \{1, 2\} \quad (3.15)$$

(iv) The equilibrium between maize and predator extinction: $\xi_4 : (0, E_4, L_4, P_4, A_4, 0, 0)$, where:

$$\begin{cases} E_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{\alpha_E (av K_L \alpha_L \alpha_P + K_E m_1 m_3 m_4)}, & L_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{av \alpha_L \alpha_P (K_L m_2 + K_E \alpha_E)}, \\ P_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{av \alpha_P m_3 (K_E \alpha_E + K_L m_2)}, & A_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{av m_3 m_4 (K_E \alpha_E + K_L m_2)} \end{cases} \quad (3.16)$$

and $\mathcal{R} = \frac{av\alpha_E\alpha_L\alpha_P}{m_1 m_2 m_3 m_4}$. Thus the equilibrium point ξ_4 occurs and exist if and only if $\mathcal{R} > 1$. This implies that, the SF population exist if the threshold quantity $\mathcal{R}$ is greater than unity.

(v) The predator-extinction equilibrium point is: $\xi_5 : (M_5, E_5, L_5, P_5, A_5, 0, 0)$, where:

$$\begin{cases} M_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2ra\beta ev K_L \alpha_L \alpha_P}; & E_5 = \frac{K_E (\mathcal{R}_3 - 1) \sqrt{h_1}}{2a\beta^2 ev K_L^2 \alpha_L \alpha_P K_M}, \\ L_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2a\beta^2 ev K_L \alpha_L \alpha_P K_M}; & P_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2a\beta^2 ev K_L \alpha_P K_M m_3}, \\ A_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2a\beta^2 ev K_L K_M m_3 m_4}, \end{cases} \quad (3.17)$$

$$\text{and } \mathcal{R}_3 = \frac{h_2}{\sqrt{h_1}},$$

$$\begin{cases} h_1 = (((-\beta e K_M + m_2)^2 r + 4\alpha_E e \beta^2 K_E K_M) K_L^2 + 2\alpha_E r K_E (-\beta e K_M \\ + m_2) K_L + \alpha_E^2 r K_E^2) a^2 v^2 r \alpha_P^2 \alpha_L^2 - 2m_1 m_3 K_L \beta^2 a e v K_E K_M m_4 r \alpha_P \\ ((-\beta e K_M + m_2) K_L - \alpha_E K_E) \alpha_L + e^2 \beta^4 K_E^2 K_M^2 K_L^2 m_1^2 m_3^2 m_4^2, \\ h_2 = ((\beta e K_M + m_2) v r a K_L + a \alpha_E r v K_E) \alpha_P \alpha_L + \beta^2 e K_E K_L K_M m_1 m_3 m_4. \end{cases} \quad (3.18)$$

As a result, the equilibrium point ξ_5 is real and has biological significance if $h_1 > 0$ and $\mathcal{R}_3 > 1$.

(vi) The interior equilibrium point: $\xi^* : (M^*, E^*, L^*, P^*, A^*, R^*, Q^*)$, where:

$$\begin{cases} M^* = \frac{\beta\eta K_M(\mathcal{R}_4 - 1)}{r\sigma\varepsilon}; & E^* = \frac{\gamma}{\delta\psi}; & L^* = \frac{\eta}{\varepsilon\sigma}; & P^* = \frac{\eta\alpha_L}{\sigma\varepsilon m_3}; & A^* = \frac{\alpha_L\alpha_P\eta}{\sigma\varepsilon m_3 m_4}, \\ R^* = \frac{(\sigma\varepsilon m_1 m_3 m_4 K_E + a\alpha_L\alpha_P\eta)(\mathcal{R}_5 - 1)}{\delta\varepsilon\psi\sigma K_E m_3 m_4}, \\ Q^* = \frac{\beta^2\delta\varepsilon\eta\psi K_L K_M + \delta\varepsilon\psi r\sigma K_L m_2 + \varepsilon\gamma r\sigma\alpha_E}{\psi\delta\varepsilon^2\sigma^2 r K_L}(\mathcal{R}_6 - 1), \end{cases} \quad (3.19)$$

with $\mathcal{R}_4 = \frac{\sigma r \varepsilon}{\eta \beta}$, $\mathcal{R}_5 = \frac{a\alpha_L\alpha_P\eta\psi\delta K_E}{\sigma\varepsilon m_1 m_3 m_4 K_E \gamma + a\alpha_L\alpha_P\eta\gamma}$ and $\mathcal{R}_6 = \frac{\varepsilon r \sigma K_L (\varepsilon \gamma \sigma \alpha_E + \beta \delta \varepsilon \eta \psi K_M)}{\beta^2 \delta \varepsilon \eta^2 \psi K_L K_M + \delta \varepsilon \eta \psi r \sigma K_L m_2 + \varepsilon \eta \gamma r \sigma \alpha_E}$. Therefore, the equilibrium point ξ^* exists and has ecological significance if $\mathcal{R}_i > 1 \quad \forall i \in \{4, 5, 6\}$.

3.3.4 Local Stability Analysis

In this section, local stability behavior of the six equilibrium points computed earlier using the Jacobian matrix and the Routh-Hurwitz Criteria were examined and discussed. The Jacobian matrix corresponding to the model system (3.1) is expressed as:

$$J = \begin{pmatrix} -\Theta_1 & 0 & -\beta M & 0 & 0 & 0 & 0 \\ 0 & -\Theta_2 & 0 & 0 & \Theta_4 & -\delta\psi E & 0 \\ e\beta L & \Theta_5 & -\Theta_3 & 0 & 0 & 0 & -\sigma\varepsilon L \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & R\delta\psi & 0 & 0 & 0 & \Theta_6 & 0 \\ 0 & 0 & Q\sigma\varepsilon & 0 & 0 & 0 & \Theta_7 \end{pmatrix}, \quad (3.20)$$

with $\Theta_1 = -r\left(1 - \frac{M}{K_M}\right) + \frac{rM}{K_M} + \beta L$, $\Theta_2 = \frac{avA}{K_E} + m_1 + R\delta\psi$, $\Theta_3 = \frac{\alpha_E E}{K_L} - e\beta M + m_2 + Q\sigma\varepsilon$, $\Theta_4 = av\left(1 - \frac{E}{K_E}\right)$, $\Theta_5 = \alpha_E\left(1 - \frac{L}{K_L}\right)$, $\Theta_6 = E\delta\psi - \gamma$ and $\Theta_7 = L\varepsilon\sigma - \eta$. Criteria conditions of Lemma (3.1) and (3.20) are used to ascertain the local stability of the system.

Lemma 3.1

Given the polynomial

$$K(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n, \quad (3.21)$$

where the coefficients a_i are real constants ($i = 1, \dots, n$), the Hurwitz matrices $H_1, H_2, \dots, H_n$

can be defined as follows:

$$H_1 = [a_1], H_2 = \begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix}, H_3 = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 \end{bmatrix}, \quad (3.22)$$

and

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & \dots & 0 \\ a_3 & a_2 & a_1 & 1 & \dots & 0 \\ a_5 & a_4 & a_3 & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & a_n \end{bmatrix}. \quad (3.23)$$

where $a_j = 0$ if $j > n$. All roots of the polynomial $K(\lambda)$ are negative or have a negative real portion if and only if all determinants of Hurwitz matrices are positive,

$$\det(H_j) > 0, \quad j = 1, 2, \dots, n. \quad (3.24)$$

The Routh-Hurwitz criteria for $n = 2, 3$, and 4 are as follows:

$$\begin{aligned} (C1) \quad n = 2 : & \quad a_1 > 0 \text{ and } a_2 > 0, \\ (C2) \quad n = 3 : & \quad a_1 > 0, a_3 > 0, \text{ and } a_1 a_2 > a_3, \\ (C3) \quad n = 4 : & \quad a_1 > 0, a_3 > 0, a_4 > 0, \text{ and } a_1 a_2 a_3 > a_3^2 + a_1^2 a_4, \\ (C4) \quad n = 5 : & \quad \tilde{H}_1 : a_1 > 0, a_1 a_2 > a_3, \tilde{H}_2 : a_1 a_2 a_3 + a_1 a_5 - a_1^2 a_4 - a_3^2 > 0, \\ & \quad \tilde{H}_3 : (a_3 a_4 - a_2 a_5)(a_1 a_2 - a_3) - (a_1 a_4 - a_4)^2 > 0 \text{ and } a_4 \tilde{H}_3 > 0. \end{aligned} \quad (3.25)$$

These criteria set the parameters for assessing stability, relying on the coefficients of the distinguishing polynomial of a system.

The Jacobian matrix of system Eq. (3.1) evaluated at ξ_1 is,

$$J(\xi_1) = \begin{pmatrix} r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_1 & 0 & 0 & av & 0 & 0 \\ 0 & \alpha_E & -m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta \end{pmatrix}.$$

Clearly, one of the eigenvalues of matrix $J(\xi_1)$ is $\lambda_1 = r > 0$, suggesting that the trivial point ξ_1 is locally asymptotically unstable.

Similarly, system (3.1) evaluated at ξ_2 is,

$$J(\xi_2) = \begin{pmatrix} -r & 0 & -\beta K_M & 0 & 0 & 0 & 0 \\ 0 & -m_1 & 0 & 0 & av & 0 & 0 \\ 0 & \alpha_E & \hat{p} & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta \end{pmatrix},$$

with $\hat{p} = -\beta e K_M + m_2$. Eigenvalues for the matrix $J(\xi_2)$ are $\lambda_1 = -r$, $\lambda_2 = -\gamma$ and $\lambda_3 = -\eta$ and the reduced matrix is used, with the following characteristic equation:

$$\lambda^4 + s_1^* \lambda^3 + s_2^* \lambda^2 + s_3^* \lambda + s_4^*, \quad (3.26)$$

with,

$$\begin{cases} s_1^* = m_4 + m_3 + \hat{p} + m_1, \\ s_2^* = \hat{p}(m_1 + m_3 + m_4) + m_1 m_3 + m_1 m_4 + m_3 m_4, \\ s_3^* = \hat{p}(m_1 m_3 + m_1 m_4 + m_3 m_4) + m_1 m_3 m_4, \\ s_4^* = \hat{p} m_1 m_3 m_4 - \alpha_P \alpha_E \alpha_L av. \end{cases} \quad (3.27)$$

Following condition (C_3) of lemma 3.1, it can be observed that ξ_2 is locally asymptotically stable.

The Jacobian matrix of system (3.1) evaluated at ξ_3 is,

$$J(\xi_3) = \begin{pmatrix} r - \beta L_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta_1 & 0 & 0 & \theta_4 & -\theta_5 & 0 \\ e\beta L_3 & \theta_3 & -\theta_2 & 0 & 0 & 0 & -\theta_6 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & \delta\psi R_3 & 0 & 0 & 0 & -\theta_7 & 0 \\ 0 & 0 & \sigma\epsilon Q_3 & 0 & 0 & 0 & -\theta_8 \end{pmatrix},$$

with, $\theta_1 = \frac{avA_3}{K_E} + m_1 + R_3\delta\psi$, $\theta_2 = \frac{\alpha_E E_3}{K_L} + m_2 + Q_3\sigma\epsilon$, $\theta_3 = \alpha_E \left(1 - \frac{L_3}{K_L}\right)$, $\theta_4 = av \left(1 - \frac{E_3}{K_E}\right)$, $\theta_5 = \delta\psi E_3$, $\theta_6 = \sigma\epsilon L_3$, $\theta_7 = -E_3\delta\psi + \gamma$ and $\theta_8 = -L_3\epsilon\sigma + \eta$. Eigenvalues of matrix $J(\xi_3)$ are $\lambda_1 = r - \frac{\beta\eta}{\sigma\epsilon}$, $\lambda_2 = -L_3\epsilon\sigma + \eta$. and the reduced matrix is used to find

the remaining eigenvalues, with the following characteristic equation:

$$\lambda^5 + \hat{s}_1 \lambda^4 + \hat{s}_2 \lambda^3 + \hat{s}_3 \lambda^2 + \hat{s}_4 \lambda + \hat{s}_5, \quad (3.28)$$

with,

$$\begin{cases} \hat{s}_1 = \theta_6 + m_4 + m_3 + \theta_1 + \theta_2, \\ \hat{s}_2 = (m_4 + \theta_1 + \theta_2 + \theta_6)m_3 + (\theta_8 + \theta_1 + \theta_2 + \theta_6)m_4 + z_1, \\ \hat{s}_3 = ((\theta_8 + \theta_1 + \theta_2 + \theta_6)m_4 + z_1)m_3 + z_1m_4 + z_2, \\ \hat{s}_4 = (h_1m_4 + z_2)m_3 + z_2m_4 - \alpha_P \alpha_L \theta_3 \theta_4, \\ \hat{s}_5 = h_z m_4 m_3 - \theta_3 \theta_4 \theta_6 \alpha_L \alpha_P. \end{cases} \quad (3.29)$$

$$\begin{cases} z_1 = (\theta_1 + \theta_2)\theta_8 + Q\theta_6\sigma\varepsilon + \theta_1\theta_2, \\ z_2 = (\theta_8 + \theta_1 + \theta_2)\theta_7 + R\theta_5\delta\psi, \\ z_3 = (\theta_5R\delta\psi + \theta_1\theta_2)\theta_8 + Q\sigma\varepsilon\theta_1\theta_6 + R\theta_5\theta_1\delta\psi, \\ z_4 = (\theta_5R\delta\psi + \theta_1\theta_7)(\theta_6Q\varepsilon\sigma + \theta_2\theta_8). \end{cases} \quad (3.30)$$

Thus, if $\min\left(\frac{r}{\beta}, L_3\right) < \frac{\eta}{\varepsilon\sigma}$ and condition (C_4) of lemma 3.1 holds, then ξ_3 is locally asymptotically stable, otherwise it is unstable.

Following the same procedures, we have:

$$J(\xi_4) = \begin{pmatrix} r - \beta L_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varpi_1 & 0 & 0 & \varpi_4 & -\delta\psi E_4 & 0 \\ e\beta L_4 & \varpi_3 & -\varpi_2 & 0 & 0 & 0 & -\sigma\varepsilon L_4 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_4\delta\psi - \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_4\varepsilon\sigma - \eta \end{pmatrix},$$

with, $\varpi_1 = \frac{avA_4}{K_E} + m_1$, $\varpi_2 = \frac{\alpha_E E_4}{K_L} + m_2$, $\varpi_3 = \alpha_E \left(1 - \frac{L_4}{K_L}\right)$ and $\varpi_4 = av \left(1 - \frac{E_4}{K_E}\right)$. Eigenvalues of matrix $J(\xi_4)$ are $\lambda_1 = r - \beta L_4$, $\lambda_2 = E_4\delta\psi - \gamma$, $\lambda_3 = L_4\varepsilon\sigma - \eta$ and the reduced matrix is used to find the remaining eigenvalues, with the characteristic equation:

$$\lambda^4 + \bar{s}_1 \lambda^3 + \bar{s}_2 \lambda^2 + \bar{s}_3 \lambda + \bar{s}_4 \quad (3.31)$$

with,

$$\begin{cases} \bar{s}_1 = m_4 + m_3 + \bar{\omega}_2 + \bar{\omega}_1, \\ \bar{s}_2 = \bar{\omega}_1 \bar{\omega}_2 + \bar{\omega}_1 m_3 + \bar{\omega}_1 m_4 + \bar{\omega}_2 m_3 + \bar{\omega}_2 m_4 + m_3 m_4, \\ \bar{s}_3 = \bar{\omega}_1 \bar{\omega}_2 m_3 + \bar{\omega}_1 \bar{\omega}_2 m_4 + \bar{\omega}_1 m_3 m_4 + \bar{\omega}_2 m_3 m_4, \\ \bar{s}_4 = \bar{\omega}_1 \bar{\omega}_2 m_3 m_4 - \alpha_P \alpha_L \bar{\omega}_3 \bar{\omega}_4. \end{cases} \quad (3.32)$$

In this regards, if $\frac{r}{\beta} < L_4 < \frac{\eta}{\varepsilon \sigma}$, $E_4 < \frac{\gamma}{\delta \psi}$ and condition (C_3) of lemma 3.1 holds, then ξ_4 is locally asymptotically stable, otherwise it is unstable.

On another hand,

$$J(\xi_5) = \begin{pmatrix} -\bar{w}_5 & 0 & -\beta M_5 & 0 & 0 & 0 & 0 \\ 0 & -\bar{w}_1 & 0 & 0 & \bar{w}_4 & -\delta \psi E_5 & 0 \\ e\beta L_5 & \bar{w}_3 & -\bar{w}_2 & 0 & 0 & 0 & -\sigma \varepsilon L_5 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_5 \delta \psi - \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_5 \varepsilon \sigma - \eta \end{pmatrix},$$

with, $\bar{w}_1 = \frac{avA_5}{K_E} + m_1$, $\bar{w}_2 = \frac{\alpha_E E_5}{K_L} - e\beta M_5 + m_2$, $\bar{w}_3 = \alpha_E \left(1 - \frac{L_5}{K_L}\right)$, $\bar{w}_4 = av \left(1 - \frac{E_5}{K_E}\right)$ and $\bar{w}_5 = -r \left(1 - \frac{M_5}{K_E}\right) + \frac{rM_5}{K_M} + \beta L_5$. Eigenvalues of matrix $J(\xi_5)$ are $\lambda_1 = E_5 \delta \psi - \gamma$ and $\lambda_2 = L_5 \varepsilon \sigma - \eta$ the reduced matrix is used, and the characteristic equation is as follows:

$$\lambda^5 + \hat{c}_1 \lambda^4 + \hat{c}_2 \lambda^3 + \hat{c}_3 \lambda^2 + \hat{c}_4 \lambda + \hat{c}_5 \quad (3.33)$$

with,

$$\begin{cases} \hat{c}_1 = m_3 + m_4 + \bar{w}_1 + \bar{w}_2 + \bar{w}_5, \\ \hat{c}_2 = (m_4 + \bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_3 + (\bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_4 + \bar{n}_1, \\ \hat{c}_3 = ((\bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_4 + \bar{n}_1)m_3 + \bar{n}_1 m_4 + \bar{n}_2, \\ \hat{c}_4 = ((\bar{n}_1 m_4 + \bar{n}_2)m_3 + \bar{n}_2 m_4 - \alpha_P \alpha_L \bar{w}_3 \bar{w}_4), \\ \hat{c}_5 = \bar{n}_2 \bar{w}_3 \bar{w}_4 - \alpha_P \alpha_L \bar{w}_3 \bar{w}_4 \bar{w}_5, \\ \bar{n}_1 = (\bar{w}_2 + \bar{w}_5)\bar{w}_1 + e\beta^2 LM + \bar{w}_2 \bar{w}_5, \\ \bar{n}_2 = \bar{w}_1 (LM\beta^2 e + \bar{w}_2 \bar{w}_5). \end{cases} \quad (3.34)$$

Thus, if $E_5 < \frac{\gamma}{\delta \psi}$, $L_5 < \frac{\eta}{\varepsilon \sigma}$ and condition (C_4) of lemma 3.1, holds, then ξ_5 is locally asymptotically stable, otherwise it is unstable.

Since the interior equilibrium point has non-zero values for every variable, it follows that matrix (3.20) is the Jacobian matrix. We observe, that $\lambda_1 = E^* \delta \psi - \gamma$, $\lambda_2 = L^* \varepsilon \sigma - \eta$ and the reduced matrix is used to find the remaining eigenvalues,

$$\begin{pmatrix} -\Theta_1 & 0 & -\beta M^* & 0 & 0 \\ 0 & -\Theta_2 & 0 & 0 & \Theta_4 \\ e\beta L^* & \Theta_5 & -\Theta_3 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 \end{pmatrix},$$

The characteristic equation is as follows:

$$\lambda^5 + s_1 \lambda^4 + s_2 \lambda^3 + s_3 \lambda^2 + s_4 \lambda + s_5 \quad (3.35)$$

with,

$$\begin{cases} s_1 = m_4 + m_3 + \Theta_2 + \Theta_1 + \Theta_3, \\ s_2 = (m_4 + \Theta_1 + \Theta_2 + \Theta_3)m_3 + (\Theta_3 + \Theta_2 + \Theta_1)m_4 + z_1^*, \\ s_3 = ((\Theta_3 + \Theta_2 + \Theta_1)m_4 + z_1^*)m_3 + z_1^*m_4 + z_2^*, \\ s_4 = (z_1^*m_4 + z_2^*)m_3 + z_2^*m_4 - \alpha_P \alpha_L \Theta_4 \Theta_5, \\ s_5 = z_2^*m_3m_4 - \alpha_P \alpha_L \Theta_5 \Theta_4 \Theta_1, \\ z_1^* = (\Theta_1 + \Theta_3)\Theta_2 + e\beta^2 L^* M^* + \Theta_1 \Theta_3, \\ z_2^* = \Theta_2(e\beta^2 L^* M^* + \Theta_1 \Theta_3). \end{cases} \quad (3.36)$$

Since $E^* < \frac{\gamma}{\delta \psi}$, $L^* < \frac{\eta}{\varepsilon \sigma}$, it follows that condition (C_4) of lemma 3.1 holds, then ξ^* is locally asymptotically stable, otherwise it is unstable.

3.3.5 Analysis of Global Stability

In this section, the interior equilibrium point (3.19) is elucidated.

Theorem 3.3

The interior equilibrium point ξ^* achieves global asymptotic stability under the following conditions:

$$(i) \quad (k_1 a v)^2 \left(\frac{G}{K_E} - 1 \right)^2 < B_1 m_4,$$

$$(ii) \quad (k_2 \alpha_E)^2 \left(\frac{L^*}{K_L} - 1 \right)^2 < B_1 B_2,$$

$$(iii) \quad \alpha_L^2 < m_3 B_2,$$

$$(iv) \quad \alpha_P^2 < m_3 m_4,$$

If not, they are deemed unstable.

Proof. To demonstrate the theorem, we examine the following positive definite function.

$$V = \left(M - M^* - M^* \ln \frac{M}{M^*} \right) + \frac{k_1}{2} (E - E^*)^2 + \frac{k_2}{2} (L - L^*)^2 + \frac{k_3}{2} (P - P^*)^2 + \frac{k_4}{2} (A - A^*)^2 + \frac{k_5}{2} (R - R^*)^2 + \frac{k_6}{2} (Q - Q^*)^2. \quad (3.37)$$

where $k_i > 0 \quad \forall i \in \{1, 2, \dots, 6\}$ need to be appropriately selected. Differentiating the aforementioned equation with respect to t along the solution system of (3.1), we obtain:

$$\begin{aligned} \frac{dV}{dt} = & \left(1 - \frac{M^*}{M} \right) M' + k_1 (E - E^*) E' + k_2 (L - L^*) L' + k_3 (P - P^*) P' \\ & + k_4 (A - A^*) A' + k_5 (R - R^*) R' + k_6 (Q - Q^*) Q'. \end{aligned} \quad (3.38)$$

Through direct calculation along the solution system (3.1), we derive the following outcome:

$$\begin{aligned} \frac{dV}{dt} = & - \left[\frac{B_1}{2} (E - E^*)^2 + k_1 a v \left(\frac{E}{K_E} - 1 \right) (E - E^*) (A - A^*) + \frac{m_4}{2} (A - A^*)^2 \right] \\ & - \left[\frac{B_2}{2} (L - L^*)^2 + k_2 \alpha_E \left(\frac{L^*}{K_L} - 1 \right) (L - L^*) (E - E^*) + \frac{B_1}{2} (E - E^*)^2 \right] \\ & - \left[\frac{m_3}{2} (P - P^*)^2 - \alpha_L (P - P^*) (L - L^*) + \frac{B_2}{2} (L - L^*)^2 \right] \\ & - \left[\frac{m_4}{2} (A - A^*)^2 - \alpha_P (A - A^*) (P - P^*) + \frac{m_3}{2} (P - P^*)^2 \right] \\ & - \left[\frac{r}{K_M} (M - M^*)^2 \right] - [(-\delta \psi E + \gamma) (R - R^*)^2] \\ & - [K_6 (-\sigma \varepsilon E + \eta) (Q - Q^*)^2]. \end{aligned} \quad (3.39)$$

We now choose $k_1 = \frac{R^*}{E}$, $k_2 = \frac{1}{eL}$, $k_i = 1 \quad \forall i \in \{3, 4, 5\}$, $k_6 = \frac{1}{eQ^*}$. Now $\frac{dV}{dt} \leq 0$, and achieves global stability if the following criteria are met:

$$(i) \quad (k_1 a v)^2 \left(\frac{G}{K_E} - 1 \right)^2 < B_1 m_4,$$

$$(ii) \quad (k_2 \alpha_E)^2 \left(\frac{L^*}{K_L} - 1 \right)^2 < B_1 B_2,$$

$$(iii) \quad \alpha_L^2 < m_3 B_2,$$

$$(iv) \quad \alpha_P^2 < m_3 m_4,$$

$$\text{otherwise, it is unstable, given that } G < \frac{\gamma}{\delta \psi}, \quad G < \frac{\eta}{\sigma \varepsilon}, \quad B_1 = k_1 \left(\frac{A^* \alpha \nu G}{K_E} + m_1 + \delta \psi R^* \right), \\ B_2 = k_2 \left(\frac{G \alpha_E}{K_L} + m_2 + \sigma \varepsilon Q^* - M^* \right).$$

□

3.4 Development of the Optimal Control Problem

In this section, we administer time-dependent control variables to determine if they can eradicate SF infestation. The selection of control variables is guided by the outcomes of the sensitivity analysis, which are also presented in Reuben *et al.* (2023). These results show that interventions focusing on reducing the population of SF infestation rate in maize are effective.

To optimally control SF, we incorporate three time-dependent control variables: $u_1(t)$, which measures the effect of farming education campaign in reducing the possibility of maize losses due to SF; $u_2(t)$ and $u_3(t)$, which represent the control for predator selection awareness for larvae and eggs, respectively.

Thus, model (3.1) is transformed to:

$$\left\{ \begin{array}{l} \frac{dM}{dt} = rM \left(1 - \frac{M}{K_M} \right) - \beta LM, \\ \frac{dE}{dt} = a \left(1 - \frac{E}{K_E} \right) \nu A - (\alpha_E + \mu_E + u_1(t)h)E - (1 + u_3(t))\delta \psi RE, \\ \frac{dL}{dt} = \alpha_E \left(1 - \frac{L}{K_L} \right) E + e\beta ML - (\alpha_L + \mu_L + u_1(t)h)L - (1 + u_2(t))\sigma \varepsilon QL, \\ \frac{dP}{dt} = \alpha_L L - (\mu_P + \alpha_P + u_1(t)h)P, \\ \frac{dA}{dt} = \alpha_P P - (\mu_A + u_1(t)h)A, \\ \frac{dR}{dt} = (1 + u_3(t))\delta \psi ER - \gamma R, \\ \frac{dQ}{dt} = (1 + u_2(t))\sigma \varepsilon LQ - \eta Q. \end{array} \right. \quad (3.40)$$

We are interested in minimizing the number of eggs, larvae, pupae, and adult moths, as well as the cost of administering the controls. The objective function that minimizes the SF population and control costs is defined as follows:

$$\mathcal{J} = \min \int_0^T \left(D_1 E + D_2 L + D_3 P + D_4 A + \frac{1}{2} \sum_{i=1}^{i=3} W_i u_i^2 \right) dt. \quad (3.41)$$

subject to system of differential equations (3.40), where D_1 , D_1 , D_1 and D_1 are the constant for minimizing eggs, larval, pupa, and adult moth population respectively. The coefficients W_1 , W_2 , and W_3 represent relative cost weights for the control measures used to transform the integral into costs incurred over a period of T days, which is the duration for applying the controls (Mwasunda *et al.*, 2022). The control variables u_i^2 are designed to minimize the cost of implementing the controls. The inclusion of squared terms for the controls accounts for the non-linear nature of the cost function, while the half-term minimizes the impact of control applications. Initial values are set at 10, 500, 100, 100, 100, 50, and 50 for the M , E , L , P , A , R , and Q categories, respectively.

Thus, our aim is to identify the optimal control u_1^* , u_2^* and u_3^* such that:

$$\mathcal{J}(u_1^*, u_2^*, u_3^*) = \min_U \mathcal{J}(u_1, u_2, u_3) \quad (3.42)$$

where $U = \{u : u \text{ is quantifiable and } 0 \leq u_i(t) \leq 1 \text{ for } t \in [0, T]\}$ is the control set.

3.4.1 Description of the Optimal Control

We implement the PMP methodology, as elaborated in Mayengo *et al.* (2022); Mwasunda *et al.* (2022); Pontryagin *et al.* (1962). This framework delineates the fundamental prerequisites for an optimal control problem. By adhering to this principle, we convert the system of differential equations presented in 3.40. Equation (3.41) is subsequently reformulated into a point-wise Hamiltonian ($\mathcal{H}$) minimization problem pertaining to the control variables u_1 , u_2 , and u_3 .

In formulating the control problem, we introduce a Lagrangian denoted as $\mathcal{L}$, defined as follows:

$$\mathcal{L} = \left(D_1 E + D_2 L + D_3 P + D_4 A + \frac{1}{2} \sum_{i=1}^{i=3} W_i u_i^2 \right). \quad (3.43)$$

then $\mathcal{H}$ is given by:

$$\mathcal{H} = \mathcal{L} + \lambda_n^* g_n, \quad (3.44)$$

where g_n are the right hand sides of the model system (3.40) and λ_n^* for $n = 1, 2, \dots, 7$ are the adjoint variables linked to state variables M , E , L , P , A , R and Q respectively.

Thus, Hamiltonian is given as:

$$\left\{ \begin{aligned} \mathcal{H} &= D_1 E + D_2 L + D_3 P + D_4 A + \frac{1}{2} \sum_{i=1}^{i=3} W_i u_i^2 \\ &+ \lambda_1^* \left(rM \left(1 - \frac{M}{K_M} \right) - \beta LM \right) \\ &+ \lambda_2^* \left(\left(1 - \frac{E}{K_E} \right) vA - (\alpha_E + \mu_E + u_1(t)h)E - (1 + u_3(t))\delta\psi RE \right) \\ &+ \lambda_3^* \left(\alpha_E \left(1 - \frac{L}{K_L} \right) E + e\beta ML - (\alpha_L + \mu_L + u_1(t)h)L - (1 + u_2(t))\sigma\epsilon QL \right) \\ &+ \lambda_4^* (\alpha_L L - (\mu_P + \alpha_P + u_1(t)h)P) \\ &+ \lambda_5^* (\alpha_P P - (\mu_A + u_1(t)h)A) + \lambda_6^* ((1 + u_3(t))\delta\psi ER - \gamma R) \\ &+ \lambda_7^* ((1 + u_2(t))\sigma\epsilon LQ - \eta Q), \end{aligned} \right. \quad (3.45)$$

Theorem 3.4

Given an optimal control sequence (u_1^*, u_2^*, u_3^*) and the associated solutions of the corresponding state system (3.40), which minimize the performance index $\mathcal{J}$ over the control space U , it follows that there exist adjoint variables λ_n^* satisfying:

$$\frac{d\lambda_n^*}{dt} = -\frac{\partial \mathcal{H}}{\partial x_n}, \forall x_n \in \{M, E, L, P, A, R, Q\} \quad (3.46)$$

with transversality conditions:

$$\lambda_n^*(T) = 0. \quad (3.47)$$

Moreover, the optimal control variables (u_1^*, u_2^*, u_3^*) are expressed as:

$$\begin{aligned} u_1^* &= \max \left\{ 0, \min \left(1, \frac{h(E\lambda_2^* + L\lambda_3^* + P\lambda_4^* + A\lambda_5^*)}{W_1} \right) \right\}, \\ u_2^* &= \max \left\{ 0, \min \left(1, \frac{\sigma\epsilon QL(\lambda_3^* - \lambda_7^*)}{W_2} \right) \right\}, \\ u_3^* &= \max \left\{ 0, \min \left(1, \frac{\delta\psi RE(\lambda_2^* - \lambda_6^*)}{W_3} \right) \right\}. \end{aligned} \quad (3.48)$$

Proof. Let (u_1^*, u_2^*, u_3^*) be an optimal control and $M^*, E^*, L^*, P^*, A^*, R^*$ and Q^* be the associated state solutions. Utilizing Pontryagin's Maximum Principle, there exist adjoint variables, which

are given by:

$$\left\{ \begin{array}{l} \frac{d\lambda_1^*}{dt} = \left(\left(-r + \frac{2rM}{K_M} \right) + \beta L \right) \lambda_1^* - e\beta L \lambda_3^*, \\ \frac{d\lambda_2^*}{dt} = -D_1 + \left(\frac{vA}{K_E} + (\alpha_E + \mu_E + u_1(t)h) + (1 + u_3(t))\delta\psi R \right) \lambda_2^* \\ \quad - \left(\alpha_E - \frac{\alpha_E L}{K_L} \right) \lambda_3^* - (1 + u_3(t))\delta\psi R \lambda_6^*, \\ \frac{d\lambda_3^*}{dt} = -D_2 + \beta M \lambda_1^* + \left(\frac{\alpha_E E}{K_L} - e\beta M + (\alpha_L + \mu_L + u_1(t)h) + (1 + u_2(t))\sigma\epsilon Q \right) \lambda_3^* \\ \quad - \alpha_L \lambda_4^* - (1 + u_2(t))\sigma\epsilon Q \lambda_7^*, \\ \frac{d\lambda_4^*}{dt} = -D_3 + (\mu_P + \alpha_P + u_1(t)h) \lambda_4^* - \alpha_P \lambda_5^*, \\ \frac{d\lambda_5^*}{dt} = -D_4 - \left(1 - \frac{E}{K_E} \right) av \lambda_2^* + (\mu_A + u_1(t)h) \lambda_5^*, \\ \frac{d\lambda_6^*}{dt} = (1 + u_3(t))\delta\psi E \lambda_2^* - (\delta\psi E - \gamma) \lambda_6^*, \\ \frac{d\lambda_7^*}{dt} = (1 + u_2(t))\sigma\epsilon L \lambda_3^* - (\sigma\epsilon L - \eta) \lambda_7^*, \end{array} \right. \quad (3.49)$$

with transversality conditions.

$$\lambda_n^*(T) = 0; \forall n \in \{1, 2, \dots, 7\} \quad (3.50)$$

To find optimal conditions, we derive the Hamiltonian function (3.45) with respect to control variables and solve for zero derivatives.

$$\frac{\partial H}{\partial u} = 0 \quad (3.51)$$

Considering the optimal control problem's characterization is relevant specifically within the interior of the control variable u^* , we can assert:

$$\begin{aligned} u_1^* &= \max \left\{ 0, \min \left(1, \frac{h(E\lambda_2^* + L\lambda_3^* + P\lambda_4^* + A\lambda_5^*)}{W_1} \right) \right\}, \\ u_2^* &= \max \left\{ 0, \min \left(1, \frac{\sigma\epsilon QL(\lambda_3^* - \lambda_7^*)}{W_2} \right) \right\}, \\ u_3^* &= \max \left\{ 0, \min \left(1, \frac{\delta\psi RE(\lambda_2^* - \lambda_6^*)}{W_3} \right) \right\}. \end{aligned} \quad (3.52)$$

□

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

In this section, we used the MATLAB programming language to simulate the mathematical model of interactions between maize and SF as introduced in chapter three. This simulation was conducted to reinforce our analytical discoveries and illustrate the effectiveness and influence of various strategies for mitigation.

4.2 Analysis of Global Sensitivity

A comprehensive sensitivity analysis was carried out to understand the factors affecting the dynamics of SF infestation when predators and best farming practices are involved. The investigation focused on specific parameters, namely the predator attack rate (denoted as δ and σ), best farming practices, and transition rates for eggs, larvae, and pupae (denoted as α_E , α_L , and α_P , respectively). Nevertheless, certain parameters were excluded from consideration due to the difficulties associated with controlling them in real-world situations.

In isolating critical parameters, we employed both LHS and PRCC analyses. Our findings highlight that parameters h , δ , and σ exhibit the most notable positive correlations with maize biomass, suggesting that augmenting these parameter values could potentially enhance maize biomass. On the flip side, a rise in α_E has the most adverse impact on maize biomass. Nonetheless, the gray zone extends from -0.3 to 0.3, signifying that the related input parameters exert minimal or negligible influence on the output variable; see Fig.4. In the early stages, sensitivity is heightened, but after 100 days, the driving parameter diminishes.

In a parallel vein, the investigation demonstrated that the parameters h , δ , and σ play a substantial role in diminishing populations of egg, larval, pupal, and adult moths. This underscores the potential effectiveness of increasing predator attack rates and implementing optimal farming practices in reducing SF infestations, as shown in Fig. 5(a)-(d).

However, σ and α_E exhibit the most positive correlation, causing an increase in eggs predator populations whenever they rise. On the other hand, an increase in the parameters h and δ decreases larval predator populations. Similarly, egg predator populations are positively affected by δ and α_P but negatively affected by h , as illustrated in Fig. 5(e)-(f).

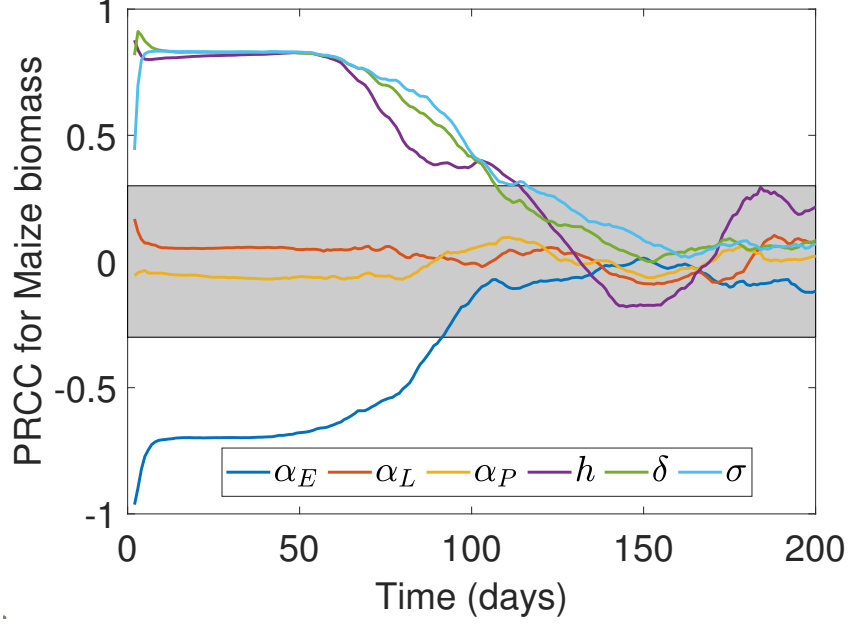


Figure 4: PRCC values when assessed in relation to the escalating maize biomass population.

4.3 Impact of Modifying the Most Sensitive Parameters

In this section, we initiate an exploration of the behavior of SF in the presence of natural enemies using numerical simulations. The model (3.1) is simulated with initial conditions $M(0) = 10$, $E(0) = 500$, $L(0) = 100$, $P(0) = 100$, $A(0) = 100$, $R(0) = 50$, and $Q(0) = 50$, utilizing parameter values drawn from the literature. Specific parameter values were carefully chosen to fall within plausible and acceptable range, ensuring the creation of reasonably realistic scenarios. For further details, please refer to Table 2.

4.3.1 Impact of Modifying the Parameters for Predator Attack Rate

We provide a numerical simulation demonstrating how predators can mitigate the impact of SF on maize biomass. When the attack rate of predators on eggs is increased, maize biomass progressively increases from its initial quantity to approach the anticipated maize biomass after 100 days (refer to Fig. 6(a)-(f)). Similarly, if the predator attack rate on larvae is higher, the impact of the SF population becomes minimal. This outcome can be attributed to various factors, including habitat manipulation, temperature, predation behavior, and conservation-oriented biological control. Additionally, the egg population does not decrease proportionally with time, as depicted in Figs. 7(a) and (b). These findings suggest that effective management of the SF impact is achievable when both egg and larval predator attack rates are elevated.

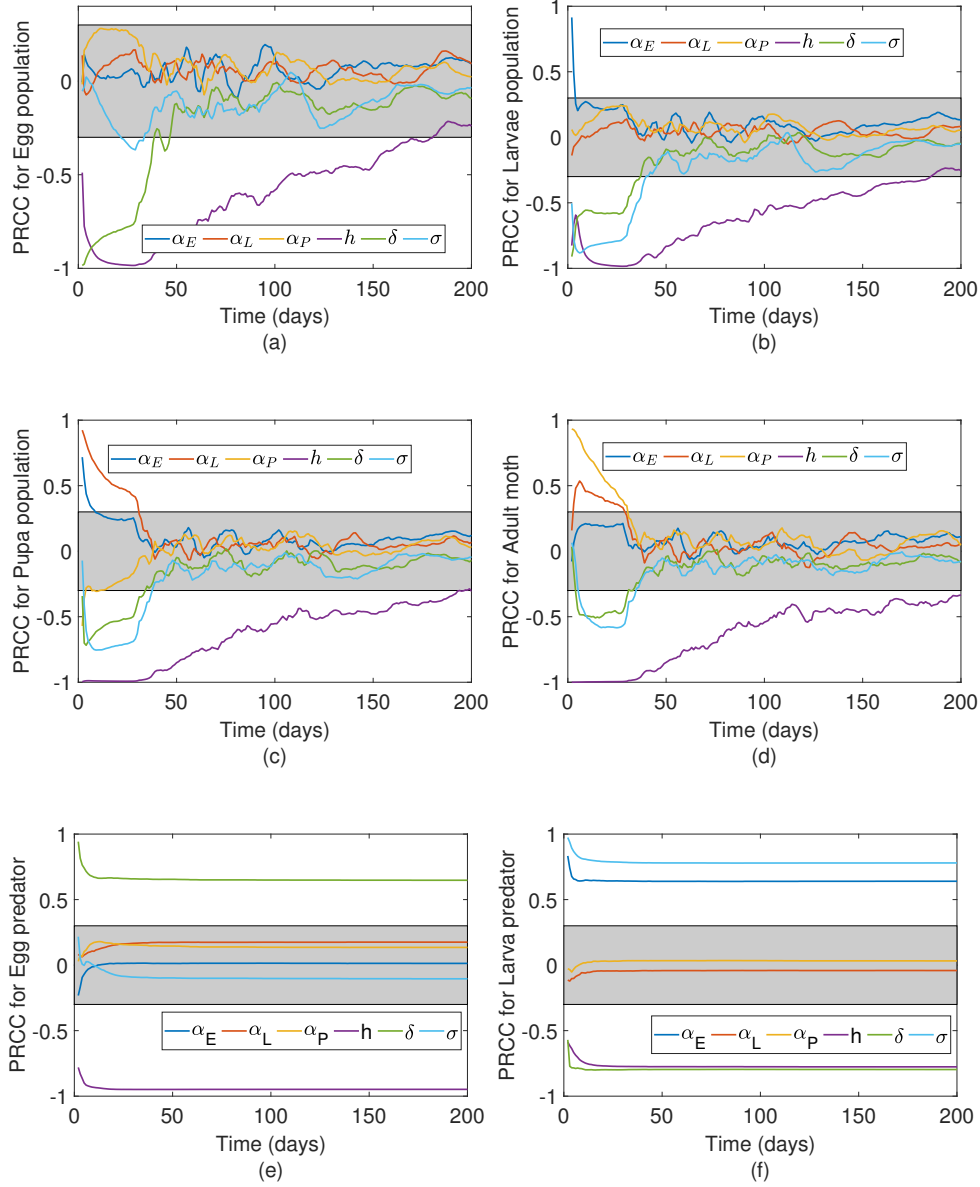


Figure 5: PRCC values were assessed concerning the escalating populations of eggs, larvae, pupae, adult moths, egg predators, and larval predators.

4.4 Optimal Control Numerical Simulations for SF-Maize Biomass

In this portion, we engage in numerical simulations to analyze the optimal control model associated with the dynamics of SF in maize biomass. The computational resolution of the optimal control problem involves applying the forward-backward sweep method to both the model system and the adjoint system using MATLAB. The parameters used for these simulations are detailed in the provided Table 2. The process initiates with the forward time integration of the model system, employing a fourth-order Runge-Kutta method. This integration relies on

Table 2: Values of parameters employed in numerical simulation

Parameters	Value	Source
a	125 eggs day^{-1}	Daudi <i>et al.</i> (2021b)
v	0.5	Daudi <i>et al.</i> (2021b); FAO (2018)
α_E^{-1}	3(2-5) days	Daudi <i>et al.</i> (2021b); FAO (2018)
α_L^{-1}	14(14-30) days	Daudi <i>et al.</i> (2021b); FAO (2018)
α_P^{-1}	9(8-30) days	Daudi <i>et al.</i> (2021b); FAO (2018)
r	0.05 day^{-1}	Daudi <i>et al.</i> (2021b)
K_M	50 Kg $plant^{-1}$	Daudi <i>et al.</i> (2021b)
K_E	10^8	Daudi <i>et al.</i> (2021b)
K_L	10^6	Daudi <i>et al.</i> (2021b)
μ_E^{-1}	0.09 day^{-1}	Faithpraise <i>et al.</i> (2015)
μ_L^{-1}	0.014 day^{-1}	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021a)
μ_P^{-1}	0.01 day^{-1}	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021a)
μ_A^{-1}	18(15-21) day^{-1}	Faithpraise <i>et al.</i> (2015)
β	$5 \times 10^{-8} day^{-1}$	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
e	0.2 day^{-1}	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
h	0.01 day^{-1}	Daudi <i>et al.</i> (2021b); Harrison <i>et al.</i> (2019)
η	100 days	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
γ	100 days	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
δ	$5 \times 10^{-5} day^{-1}$	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021a)
ε	0.1 day^{-1}	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
ψ	0.1 day^{-1}	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021b)
σ	$5 \times 10^{-5} day^{-1}$	Faithpraise <i>et al.</i> (2015); Daudi <i>et al.</i> (2021a)

the initial control values. Following this, a backward fourth-order Runge-Kutta method is executed, utilizing the computed state variable values and initial control values. This step is geared towards resolving the adjoint equations while adhering to the predetermined final conditions.

We evaluate the combined implementation of $u_1(t)$, $u_2(t)$, and $u_3(t)$ to manage pest issues in maize cultivation. These strategies are interrelated, and prior studies have emphasized the importance of IPM, particularly farming education campaign techniques such as intercropping and the use of repellent plants to mitigate infestations. Likewise, predators selection awareness relies on multiple factors, whether they are natural or artificial predators. These factors encom-

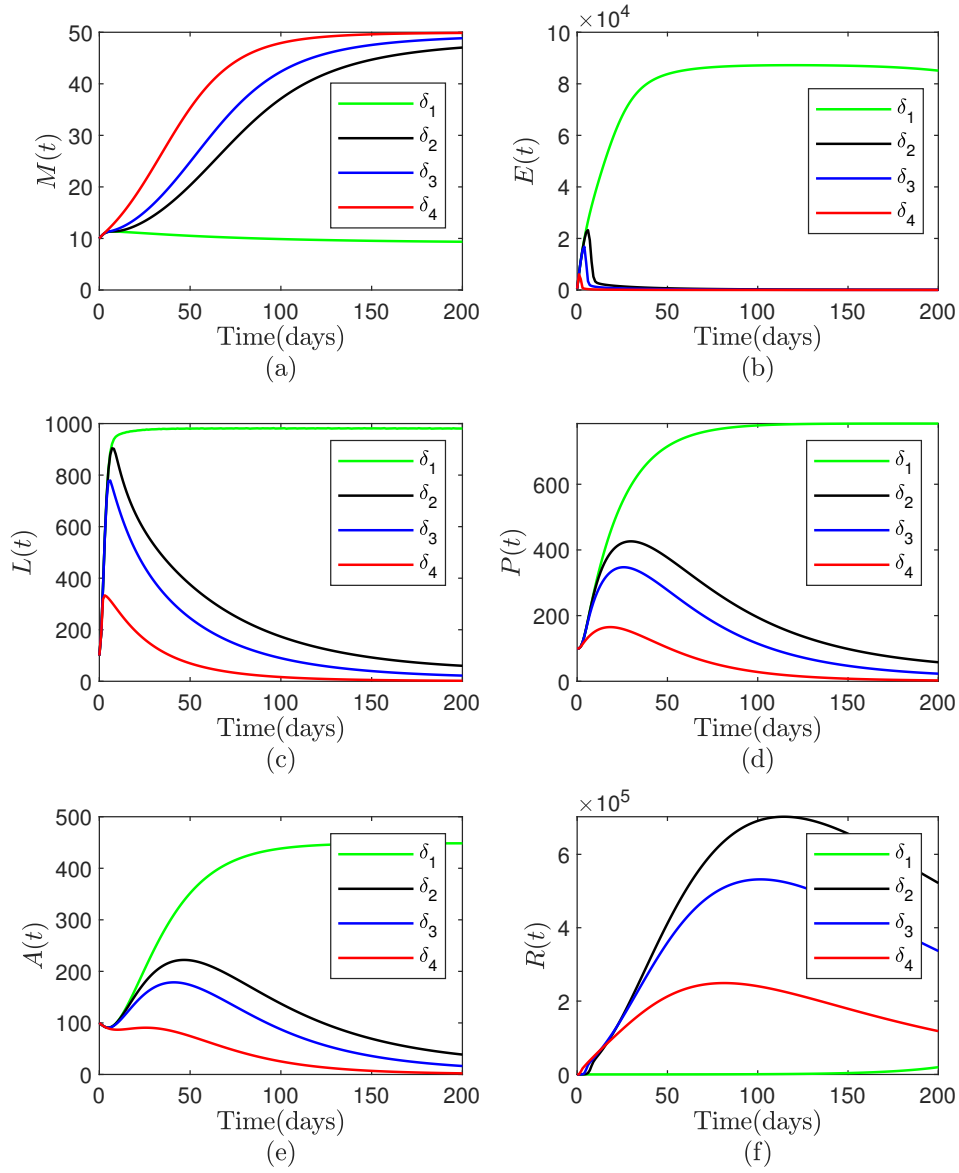


**Figure 6: Increased predator attack rates on eggs significantly decrease SF populations:
where $\delta_1 = 5 \times 10^{-6}$, $\delta_2 = 5 \times 10^{-5}$, $\delta_3 = 5 \times 10^{-4}$, $\delta_4 = 5 \times 10^{-3}$**

pass budget considerations, attack rates, and efficiency in reducing SF. The choice of predators is contingent on the farm's design and its capacity to support predator populations, ultimately impacting the predator's effectiveness in controlling pests. In this particular study, our primary focus is on the presence of predators influences.

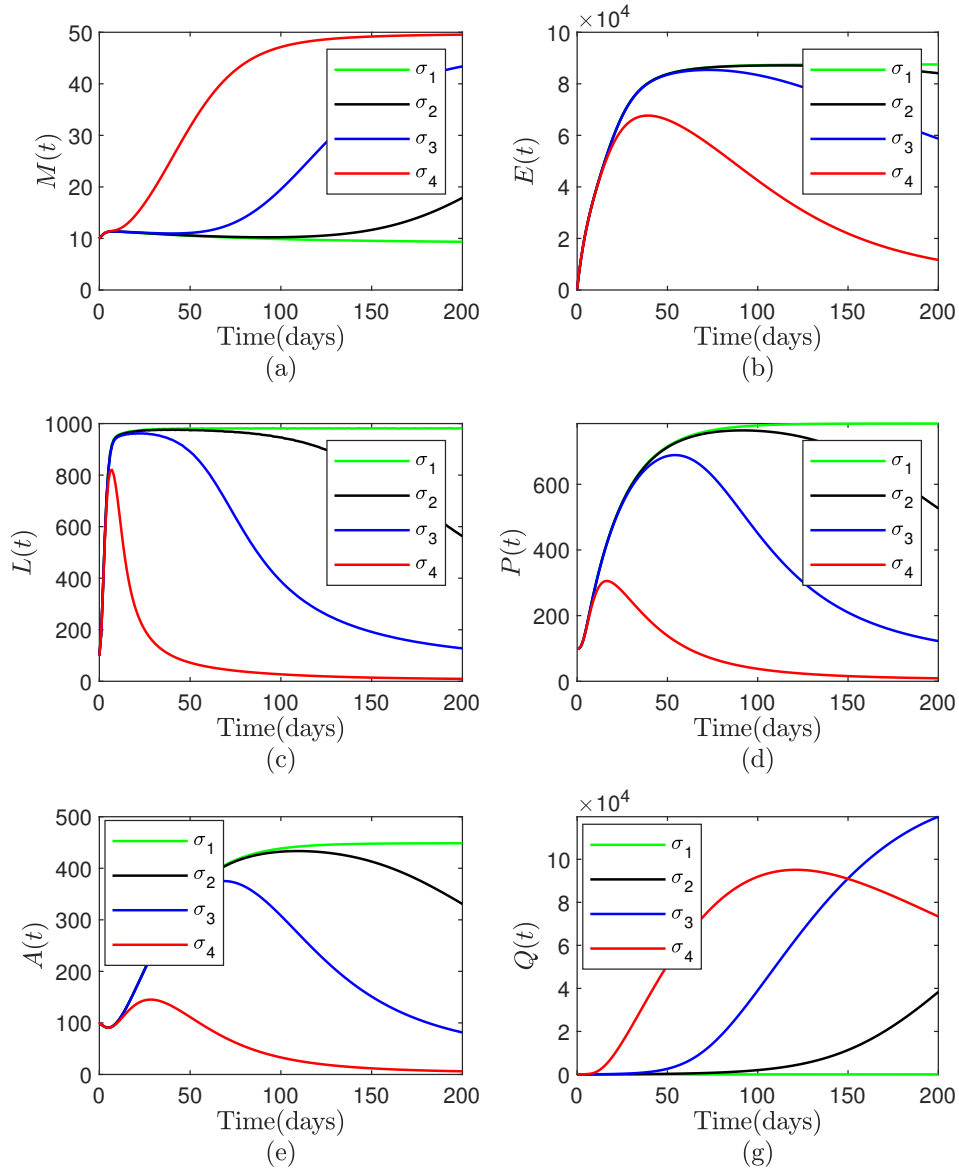


Figure 7: Increased predator attacks on larvae significantly decrease SF populations:

where $\sigma_1 = 5 \times 10^{-6}$, $\sigma_2 = 5 \times 10^{-5}$, $\sigma_3 = 5 \times 10^{-4}$, $\sigma_4 = 5 \times 10^{-3}$

4.4.1 When all Control Measures are Implemented

This strategy combines a farming education campaign with the selective promotion of predators for eggs and larvae. The results displayed in Fig. 8(a)-(h) indicate a substantial reduction in pest and predator populations when all time-dependent controls are put in place. Maize biomass reaches the expected level within 100 days. The control profiles in 8(h) illustrate that initially, the control variable $u_1(t)$ start at peaks and gradually decreases to zero over the course of 152

days. The control profiles for $u_2(t)$ and $u_3(t)$ start at their peaks and then eventually decrease to zero in 96 and 67 days, respectively. This implies that the farming education campaign must be carried effectively in the starting of the seasons. Likewise, predators must stay for some days to reduce the SF and boost maize production if the cost of implementation is low.

4.4.2 Farming Education Campaign and Predator selections awereness on Larval

In this approach, the control variables $u_1(t)$ and $u_2(t)$ are deployed to oversee the farming education campaign and enhance awareness of predator selections on larvae, thereby optimizing the specified objective function. Simultaneously, the adjustment of the control $u_3(t)$ for predator selection on the egg rate to zero is observed. The results presented in Fig. 9 clearly demonstrate a decline in the population of SF within the field. The control profiles outlined in Fig. 9(h) indicate a continuous full utilization of the control variable $u_1(t)$, while $u_2(t)$ is maximized in the initial 180 days, progressively tapering to zero over time.

4.4.3 Farming Education Campaign and Predator selections awereness on Eggs

In this approach, the strategy involves utilizing control variables $u_1(t)$ and $u_2(t)$ to optimize the objective function, where these variables correspond to a farming education campaign and awareness of predator selection on eggs. The results showcased in Figure 10 exhibit a significant reduction in the population of SF in the field. Analyzing the control profiles in Figure 10(h) unveils the consistent full utilization of the control variables $u_1(t)$ and the complete utilization of $u_2(t)$ in the initial 160 days, followed by a gradual decline to zero over time.

4.4.4 Predator selections awereness on Eggs and Larval

In implementing this strategy, optimizing the objective function requires fine-tuning the control variables $u_2(t)$ and $u_3(t)$ to intensify awareness of predator selection for larvae and eggs. Conversely, the control variable $u_1(t)$, governing the rate of farming education campaigns, is intentionally kept at zero. The results depicted in Fig. 11 underscore a noteworthy decrease in the population of SF in the field. Examining the control profiles in Fig. 11(h), it becomes apparent that the control variable $u_2(t)$ is fully utilized and starts decreasing after 195 days, while $u_3(t)$ is fully employed throughout the season. This implies that predation must be applied throughout the season to reduce SF populations.

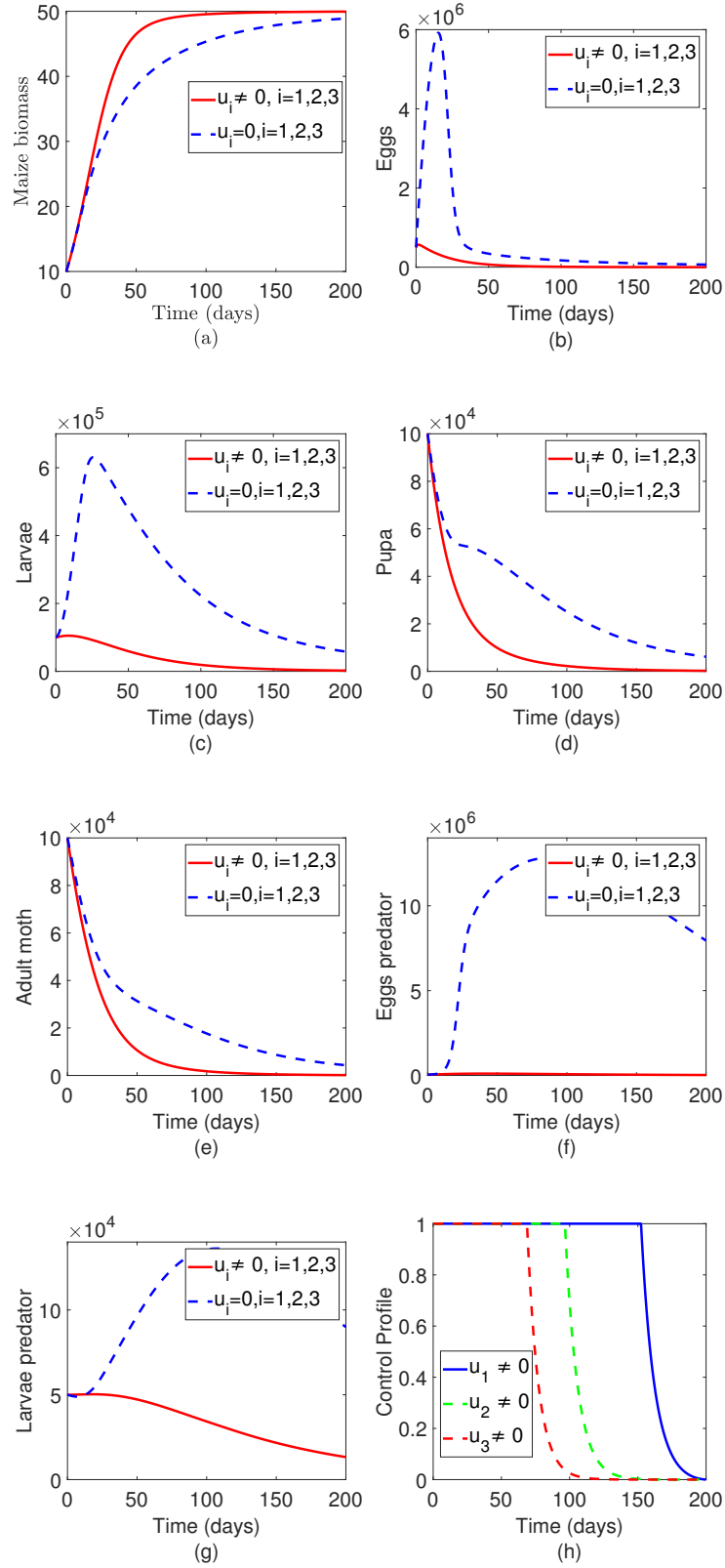


Figure 8: Impact of farming education campaign and predator awareness both eggs and larval

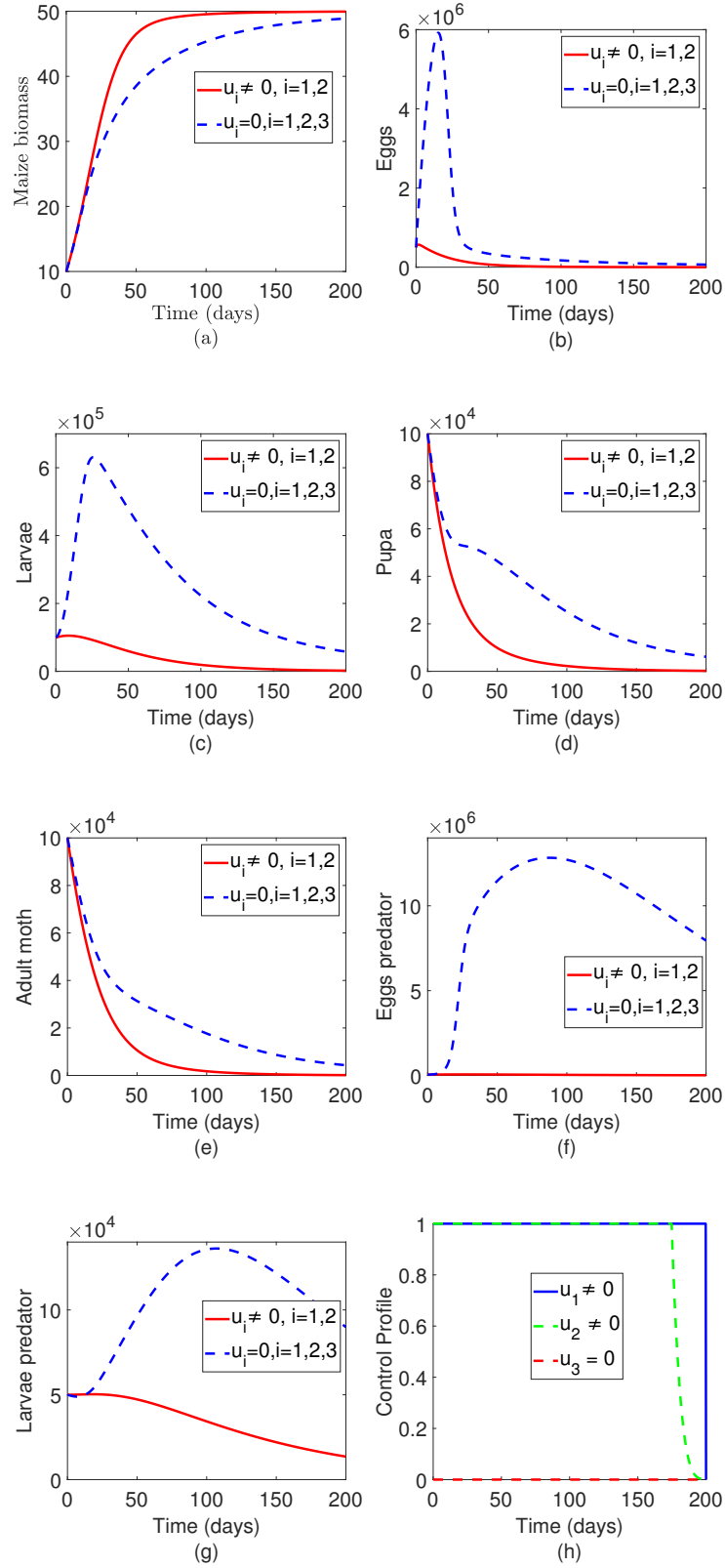


Figure 9: Impact of farming education campaign and predator selection awareness on larva

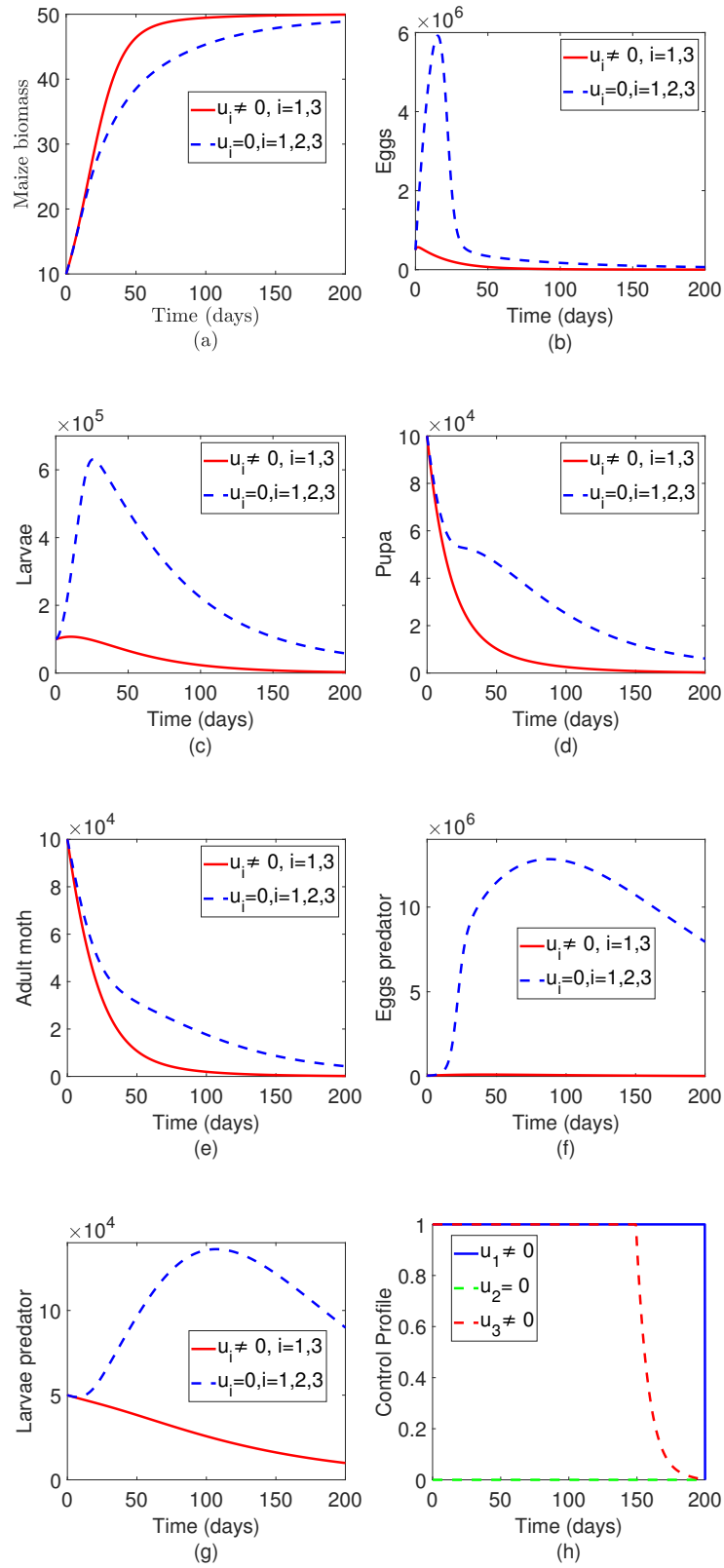


Figure 10: Impact of farming education effort and predator selection awareness on eggs

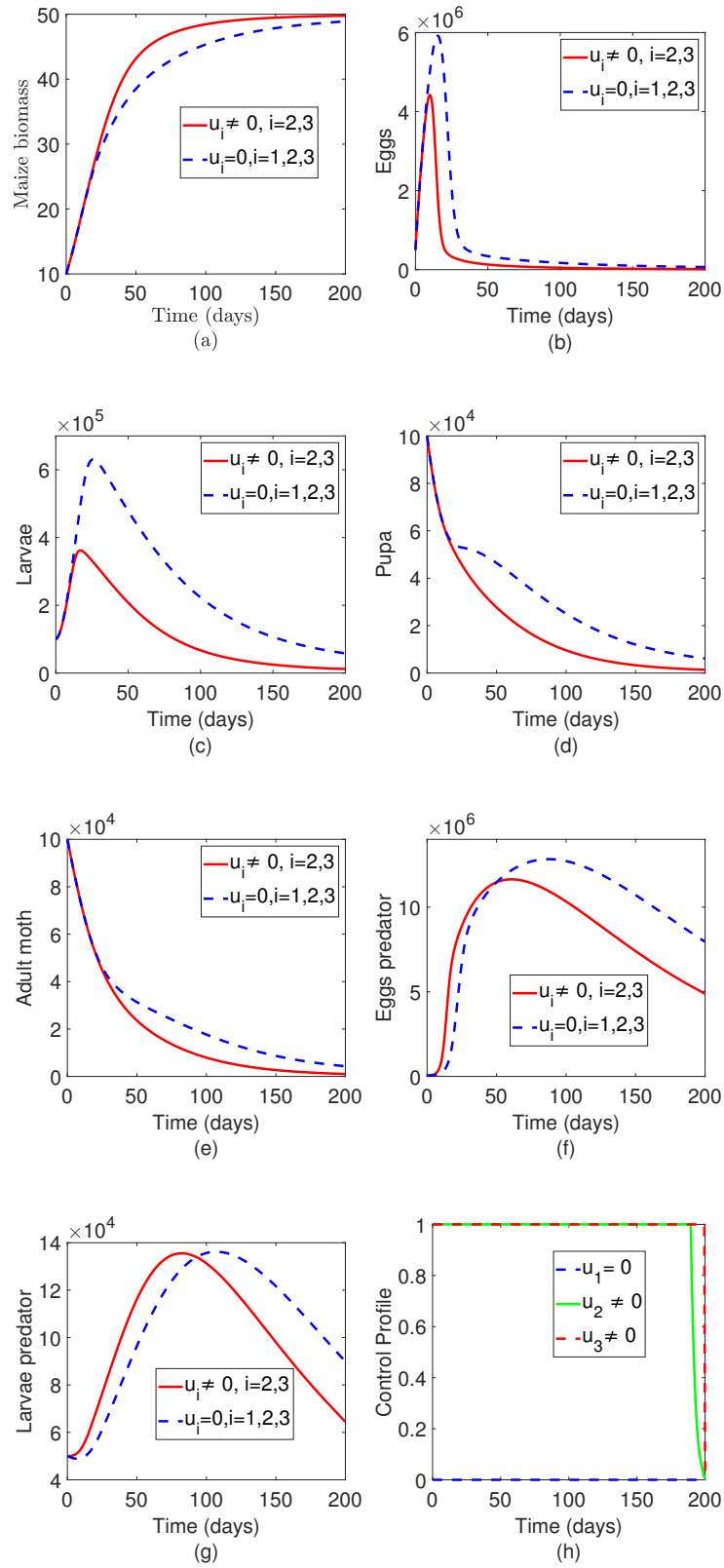


Figure 11: Impact of predator selection awareness on eggs and larval

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Invasive plants, insects, and diseases pose a formidable threat to the achievement of the SDGs on promoting zero hunger. *Spodoptera frugiperda*, an invasive insect, has rapidly spread and inflicted substantial damage on maize crops in African and Asian nations since 2016. As SF is a recent arrival in Africa, there is limited information available about its effects on final maize biomass under various conditions. The mathematical model, which was both proposed and rigorously analyzed, evaluates how factors such as farming education campaign and the predation awareness on reducing SF population, as well as the resulting maize biomass.

Since the first outbreak of SF in Africa in 2016, African governments have taken emergency measures, primarily involving the use of chemical insecticides. African governments face financial constraints and high costs of chemical insecticide spraying and genetically modified crops, making effective pest management a significant challenge. Understanding the importance of agricultural education for farmers as well as how natural enemies affect SF dynamics is vital. These educational campaigns, often disseminated through various media outlets, including radio, newspapers, and television, not only raise awareness among farmers about SF outbreaks but also foster trust in the IPM strategies advocated by each nation (Abraha *et al.*, 2021).

To investigate the influence of biological control agent a novel deterministic model was developed and thoroughly analyzed. This model incorporated elements such as the presence of egg and larval predators, as well as best farming practices. It was revealed to have six equilibrium points, all of which could achieve universal asymptotic stability under specific outlined conditions. Simulations of the model, considering time-dependent farming education campaign and the selection of predator agents, demonstrated that achieving the desired maize biomass is feasible when the farming education campaign and predator awareness are incorporated in early stages. The study found that the best farming practices and natural enemies are vital factors in controlling SF. Likewise, utilizing farming practices in the early stages of a season reduces SF by 152 days (76%) and biological controls by 81 days (40.5%). This increase in maize biomass by 48 (96%) occurs within 70 to 80 days.

5.2 Recommendations

Cereal crops hold a crucial place in the daily diets of African populations, providing up to 46% of their daily calorie intake. Among these crops, maize and sorghum are the primary staple foods cultivated across diverse agro-ecological zones and farming systems in SSA (Niassy *et al.*, 2021). In this research, mathematical model was developed and analyzed to investigate the effects of SF infestations in maize fields on the final maize biomass. Drawing from the results of this study, the following recommendations emerge:

- (i) Based on the outcomes of the sensitivity analysis and simulation results, it is clear that the populations of eggs and larvae pose a substantial threat to maize biomass.
- (ii) Collaboration with local communities and indigenous groups to apply farming practices and biological methods.
- (iii) Policy reinforcement on farming education campaigns and biological control methods including PPT technology.
- (iv) We recommend that more efforts be directed to reduce eggs and larval populations and improving farming methods through education campaign.
- (v) This study is not comprehensive; we suggest that once data is accessible, the model should undergo validation to improve its reliability and suitability for real-world situations. Additionally, incorporating cost-effectiveness and machine learning tools would be beneficial.

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APPENDICES

Appendix 1: MATLAB Code for Numerical Simulation

```
1      %place your matlab code here
2      %Sensitivity analysis
3      %% The results should be compared to the PRCC results
4      section in
5      %% runs in the script below
6      clear all;
7      close all;
8
9      %% Sample size N
10     runs=200;
11
12     %% LHS MATRIX %%
13     Parameter_settings_LHS;
14
15     alpha_E_LHS=LHS_Call(0.1,alpha_E,0.5, 0 ,runs,'unif');%
16     baseline 0.5;
17     alpha_L_LHS=LHS_Call(0.01,alpha_L,0.09, 0 ,runs,'unif');%
18     baseline 1/14;
19     alpha_P_LHS=LHS_Call(0.01,alpha_P,0.08, 0 ,runs,'unif');%
20     baseline 1/5;
21     h_LHS=LHS_Call(0.01,h,0.9, 0 ,runs,'unif');% baseline 0.8;
22     delta_LHS=LHS_Call(0.05,delta,0.000005, 0 ,runs,'unif');%
23     baseline 160 ;
24     sigma_LHS=LHS_Call(0.05,sigma,0.000005, 0 ,runs,'unif');%
25     baseline 160 ;
26     %% LHS MATRIX and PARAMETER LABELS
27     LHSmatrix=[alpha_E_LHS, alpha_L_LHS, alpha_P_LHS, h_LHS,
28     delta_LHS, sigma_LHS];% dummy_LHS];
29     for x=1:runs %Run solution x times choosing different values
30     f=@ODE_LHS;
31     x;
32     LHSmatrix(x,:)
33     [t,y]=ode15s(@(t,y) f(t,y,LHSmatrix,x,runs),tspan,y0,[]);
34     A=[t y]; % [time y]
35     %% Save only the outputs at the time points of interest [
36     time_points]:
37     %% MORE EFFICIENT
38     TI_lhs(:,x)=A(time_points+1,1);
39     MB_lhs(:,x)=A(time_points+1,2);
40     EE_lhs(:,x)=A(time_points+1,3);
41     LL_lhs(:,x)=A(time_points+1,4);
42     PP_lhs(:,x)=A(time_points+1,5);
43     AA_lhs(:,x)=A(time_points+1,6);
```

```

37 EP_lhs(:,x)=A(time_points+1,7);
38 LP_lhs(:,x)=A(time_points+1,8);
39 end
40 %% Save the workspace
41 save Model_LHS.mat;
42 %figure(1)
43 % CALCULATE PRCC
44 [prcc1 sign sign_label]=PRCC(LHSmatrix,MB_lhs,1:length(
    time_points),PRCC_var,0.01);
45 prcc1(100,:) '
46 figure(1) %subplot(1,2,1),
47 bar(prcc1(100,:));%,title(c1);
48 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
    %title('PRCCs');
49 ylabel('PRCC for Maize biomass');
50 print('Sensitivity_fig_1.eps','-depsc','-r600');
51 figure(2)
52 ax = axes;
53 ax.LineStyleOrder = {'-','--'};
54 hold on
55 x = [0 200 200 0];
56 y = [-0.3 -0.3 0.3 0.3];
57 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');
58 hold on
59 pt = plot(prcc1,'linewidth',2);
60 legend([pt], '$\alpha_{E}$', '$\alpha_{L}$', '$\alpha_{P}$', '$h$', '$\delta$', '$\sigma$', ...
    'Location','eastoutside','Interpreter','LaTeX','fontweight',
    'bold','fontsize',14);
61 ylabel('PRCC for Maize biomass','fontsize',12); xlabel({'
    Time (days)'},'fontsize',12);
62 xlim([0,200]);
63 box on;
64 print('Sensit_fig_1.eps','-depsc','-r600');
65 %% Save the workspace
66 save Model_LHS.mat;
67 % CALCULATE PRCC
68 [prcc2 sign sign_label]=PRCC(LHSmatrix,EE_lhs,1:length(
    time_points),PRCC_var,0.01);
69 prcc2(10,:) '
70 figure(3) %subplot(1,2,1),
71 bar(prcc2(100,:));%,title(c1);
72 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
    %title('PRCCs');
73 ylabel('PRCC for Egg population');
74 print('Sensitivity_fig_2.eps','-depsc','-r600');
75 figure(4)
76 ax = axes;
77 ax.LineStyleOrder = {'-','--'};
78

```

```

79 hold on
80 x = [0 200 200 0];
81 y = [-0.3 -0.3 0.3 0.3];
82 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');
83 hold on
84 pt = plot(prcc2,'linewidth',2);
85 hold off
86 legend([pt], '$\alpha_E$', '$\alpha_L$', '$\alpha_P$', '$h$', '$\delta$', '$\sigma$', ...
87 'Location','eastoutside','Interpreter','LaTeX','fontweight',
88 'bold','fontsize',14);
89 ylabel('PRCC for Egg population','fontsize',12); xlabel({'
90 Time (days)', '(a)'}, 'fontsize',12);
91 xlim([0,200]);
92 box on;
93 print('Sensit_fig_2.eps','-depsc','-r600');
94 %% Save the workspace
95 save Model_LHS.mat;
96 % CALCULATE PRCC
97 [prcc3 sign sign_label]=PRCC(LHSmatrix,LL_lhs,1:length(
98 time_points),PRCC_var,0.01);
99 prcc3(10,:)'
100 figure(5) %subplot(1,2,1),
101 bar(prcc3(100,:));%,title(c1);
102 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
103 %title('PRCCs');
104 ylabel('PRCC for Larvae population');
105 print('Sensitivity_fig_3.eps','-depsc','-r600');
106 figure(6)
107 ax = axes;
108 ax.LineStyleOrder = {'-','--'};
109 hold on
110 x = [0 200 200 0];
111 y = [-0.3 -0.3 0.3 0.3];
112 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');
113 hold on
114 pt = plot(prcc3,'linewidth',2);
115 hold off
116 legend([pt], '$\alpha_E$', '$\alpha_L$', '$\alpha_P$', '$h$', '$\delta$', '$\sigma$', ...
117 'Location','eastoutside','Interpreter','LaTeX','fontweight',
118 'bold','fontsize',14);
119 ylabel('PRCC for Larvae population','fontsize',12); xlabel({'
120 Time (days)', '(b)'}, 'fontsize',12);
121 xlim([0,200]);
122 box on;
123 print('Sensit_fig_3.eps','-depsc','-r600');
124 %% Save the workspace
125 save Model_LHS.mat;

```

```

120 % CALCULATE PRCC
121 [prcc4 sign sign_label]=PRCC(LHSmatrix,PP_lhs,1:length(
    time_points),PRCC_var,0.01);
122 prcc4(100,:) '
123 figure(7) %subplot(1,2,1),
124 bar(prcc4(100,:));%,title(c1);
125 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
    %title('PRCCs');
126 ylabel('PRCC for Pupa population');
127 print('Sensitivity_fig_4.eps','-depsc','-r600');
128 figure(8)
129 ax = axes;
130 ax.LineStyleOrder = {'-','--'};
131 hold on
132 x = [0 200 200 0];
133 y = [-0.3 -0.3 0.3 0.3];
134 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');
135 hold on
136 pt = plot(prcc4,'linewidth',2);
137 hold off
138 legend([pt],'$\alpha_E$','$\alpha_L$','$\alpha_P$','$h$','$\delta$','$\sigma$',...
    'Location','eastoutside','Interpreter','LaTeX','fontweight',
    'bold','fontsize',14);
139 ylabel('PRCC for Pupa population','fontweight','bold','
    fontsize',14); xlabel({'Time (days)','(c)'},...
    'fontweight','bold','fontsize',14);
140 xlim([0,200]);
141 box on;
142 print('Sensit_fig_4.eps','-depsc','-r600');
143 %% Save the workspace
144 save Model_LHS.mat;
145 % CALCULATE PRCC
146 [prcc5 sign sign_label]=PRCC(LHSmatrix,AA_lhs,1:length(
    time_points),PRCC_var,0.01);
147 prcc5(10,:) '
148 figure(9) %subplot(1,2,1),
149 bar(prcc5(100,:));%,title(c1);
150 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
    %title('PRCCs');
151 ylabel('PRCC for Adult moth');
152 print('Sensitivity_fig_5.eps','-depsc','-r600');
153 figure(10)
154 ax = axes;
155 ax.LineStyleOrder = {'-','--'};
156 hold on
157 x = [0 200 200 0];
158 y = [-0.3 -0.3 0.3 0.3];
159 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');

```

```

162 hold on
163 pt = plot(prcc5,'linewidth',2);
164 hold off
165 legend([pt], '$\alpha_E$', '$\alpha_L$', '$\alpha_P$', '$h$', '$\delta$', ...
166         '$\sigma$', 'Location', 'eastoutside', 'Interpreter', 'LaTeX', 'fontweight', 'bold', 'fontsize', 14);
167 ylabel('PRCC for Adult moth', 'fontweight', 'bold', 'fontsize', 14); xlabel({'Time (days)', '(d)'}, ...
168         'fontweight', 'bold', 'fontsize', 14);
169 xlim([0,200]);
170 box on;
171 print('Sensit_fig_5.eps', '-depsc', '-r600');
172 %% Save the workspace
173 save Model_LHS.mat;
174 % CALCULATE PRCC
175 [prcc6 sign sign_label]=PRCC(LHSmatrix, EP_lhs, 1:length(
176     time_points), PRCC_var, 0.01);
177 prcc6(100,:) '
178 figure(11) %subplot(1,2,1),
179 bar(prcc6(100,:)); %title(c1);
180 set(gca, 'XTickLabel', PRCC_var, 'XTick', [1:6], 'fontsize', 14);
181 %title('PRCCs');
182 ylabel('PRCC for Egg predator');
183 print('Sensitivity_fig_6.eps', '-depsc', '-r600');
184 figure(12)
185 ax = axes;
186 ax.LineStyleOrder = {'-', '--'};
187 hold on
188 x = [0 200 200 0];
189 y = [-0.3 -0.3 0.3 0.3];
190 patch(x,y, [0.5 0.5 0.5], facecolor='#cccccc');
191 hold on
192 pt = plot(prcc6,'linewidth',2);
193 hold off
194 legend([pt], '\alpha_E', '\alpha_L', '\alpha_P', 'h', '\delta', '\sigma', 'Location', ...
195         'eastoutside', 'fontweight', 'bold', 'fontsize', 14);
196 ylabel('PRCC for Egg predator', 'fontweight', 'bold', 'fontsize', 14); xlabel({'Time (days)', ...
197         '(e)'}, 'fontweight', 'bold', 'fontsize', 14);
198 xlim([0,200]);
199 box on;
200 print('Sensit_fig_6.eps', '-depsc', '-r600');
201 %% Save the workspace
202 save Model_LHS.mat;
203 % CALCULATE PRCC
204 [prcc7 sign sign_label]=PRCC(LHSmatrix, LP_lhs, 1:length(
205     time_points), PRCC_var, 0.01);

```

```

203 prcc7(10,:) '
204 figure(13) %subplot(1,2,1),
205 bar(prcc7(100,:));%,title(c1);
206 set(gca,'XTickLabel',PRCC_var,'XTick',[1:6],'fontsize',14);
    %title('PRCCs');
207 ylabel('PRCC for Larva predator');
208 print('Sensitivity_fig_7.eps','-depsc','-r600');
209 figure(14)
210 ax = axes;
211 ax.LineStyleOrder = {'-','--'};
212 hold on
213 x = [0 200 200 0];
214 y = [-0.3 -0.3 0.3 0.3];
215 patch(x,y,[0.5 0.5 0.5],facecolor='#cccccc');
216 hold on
217 pt = plot(prcc7,'linewidth',2);
218 hold off
219 legend([pt],'\alpha_E','\alpha_L','\alpha_P','h','\delta','\
    sigma','Location',...
220 'eastoutside','fontweight','bold','fontsize',14);
221 ylabel('PRCC for Larva predator','fontweight','bold','
    fontsize',14); xlabel({'Time (days)','(f)'},...
222 'fontweight','bold','fontsize',14);
223 xlim([0,200]);
224 box on;
225 print('Sensit_fig_7.eps','-depsc','-r600');
226
227
228 %OPTIMAL CONTROL
229 clc
230 close all
231 format long
232 t0 = 0; tf=200; N=1000;
233 time =linspace(t0,tf,N);
234 % initial condition for STATE SYSTEM
235 % y0=[1000 100 1000 100 2700 200 100];
236 y0 = [10,500,100,100,100,50,50];
237 %% parameter values
238 constant=[0.003 0.04 0.04 0.01 0.00005 0.00005 10^5 10^3 50
    0.000005...
239 125 0.01 0.2 0.01 0.07 0.09 0.5 0.01 0.1 1/100 1/100 0.1];
240 alpha_E=constant(1); %senstive curve
241 alpha_L = constant(2);
242 alpha_P = constant(3);
243 d = constant(4);
244 delt = constant(5);
245 sigma = constant(6);
246 K_E = constant(7);
247 K_L = constant(8);

```

```

248 K_M =constant(9);
249 beta= constant(10);
250 a= constant(11);
251 mu_E = constant(12);
252 e = constant(13);
253 mu_L = constant(14);
254 mu_A=constant(15);
255 r =constant(16);
256 v =constant(17);
257 mu_P =constant(18);
258 psi =constant(19);
259 gamma = constant(20);
260 eta = constant(21);
261 epsilon =constant(22);
262
263 lf = [0 0 0 0 0 0 0];
264 %%
265 init =y0;
266 init2 =lf;
267 h = (tf-t0)/N;
268 u = linspace(0,0,N+1);
269 u1=u'; u2=u'; u3=u';
270 U = [u1 u2 u3];
271
272 %% IMPLIMENTATION OF THE ALGORITHM
273 %Test 1 stoping condition 1
274 delta = 0.001;
275 X=init;
276 i=0; %Initialize iteration counter
277 mm=size(X);
278 NumXX =10e10;
279 Xnew = rand(N+1,mm(2)).*(repmat(X,N+1,1));
280 DenXnew=norm(Xnew);
281 while NumXX/DenXnew>delta
282 Xold = Xnew;
283 oldu = U;
284 %FORWARD RUNGE KUTTA FOR STATES
285 [Tx, X]=rk4foward(@kims,t0, tf,N, init,U,constant);
286
287 % BACKWARD RUNGEKUTA FOR COSTATES
288 [Tp, B]=rk4back(@kims_costate,t0,tf,N,init2,U,X,constant);
289 M = X(1,:);E = X(2,:);L = X(3,:);P = X(4,:); A= X(5,:);R = X
    (6,:);Q = X(7,:);
290 lambda1 = B(1,:); lambda2 = B(2,:); lambda3 = B(3,:);
    lambda4 = B(4,:);
291 lambda5 = B(5,:); lambda6 = B(6,:); lambda7 = B(7,:);
292
293 %% Graph plotting
294 % Case0:No control,

```

```

295 % u1 =zeros(1,N+1);
296 % u2 =zeros(1,N+1);
297 % u3 =zeros(1,N+1);
298
299
300 %% Case1: u1/=0, u2=0, u3=0,
301 % u1 =max(0,min(1, (d*(lambda2.*E + lambda3.*L + lambda4
    .*P + lambda5.*A))./W1));
302 % u2 =zeros(1,N+1);
303 % u3 =zeros(1,N+1);
304
305 %% Case2:u1=0, u2/=0, u3=0,
306 % u1 = zeros(1,N+1);
307 % u2 = max(0,min(1, (sigma.*epsilon.*Q.*L.*(lambda3 -
    lambda7))./W2));
308 % u3 = zeros(1,N+1);
309
310
311 %% Case3:u1=0, u2=0, u3/=0,
312 % u1 =zeros(1,N+1);
313 % u2 =zeros(1,N+1);
314 % u3 =max(0,min(1, (delt.*psi.*E.*R.*(lambda2 - lambda6))./
    W3));
315
316 %% Case4:u1/=0, u2/=0, u3=0,
317 % u1 =max(0,min(1, (d*(lambda2.*E + lambda3.*L + lambda4
    .*P + lambda5.*A))./W1));
318 % u2 = max(0,min(1, (sigma.*epsilon.*Q.*L.*(lambda3 -
    lambda7))./W2));
319 % u3 =zeros(1,N+1);
320
321 %% Case5:u1/=0, u2=0, u3/=0,
322 % u1 =max(0,min(1, ((lambda2.*E + lambda3.*L + lambda4.*
    P + lambda5.*A))./W1));
323 % u2 =zeros(1,N+1);
324 % u3 =max(0,min(1, (delt.*psi.*E.*R.*(lambda2 - lambda6))
    ./ W3));
325
326 %% Case6:u1=0, u2/=0, u3/=0,
327 % u1 =zeros(1,N+1);
328 % u2 = max(0,min(1, (sigma.*epsilon.*Q.*L.*(lambda7 - lambda3
    ))./W2));
329 % u3 =max(0,min(1, (delt.*psi.*E.*R.*(lambda6 - lambda2))./
    W3));
330
331 %% Case7:u1/=0, u2/=0, u3/=0,
332 u1 =min(1,max(0, ((h*(lambda2.*E + lambda3.*L + lambda4.*P +
    lambda5.*A))./W1)));

```

```

333 u2 = min(1,max(0,((sigma.*epsilon.*Q.*L.*(lambda3 - lambda7)
334 ) ./W2)));
335 u3 =min(1,max(0,((delt.*psi.*E.*R.*(lambda2 - lambda6))./ W3
336 )));
337 %%
338 Uu=[u1' u2' u3'];
339 U = 0.5*Uu + 0.5*oldu; % Convex combination of the controls
340
341 Xnew = X';
342 NumXX =abs(norm(Xnew-Xold));
343 DenXnew =norm(Xnew);
344 i=i+1; %Update iteration counter
345 end
346
347 %% PLOTTING
348 X=X';
349 Tx =Tx';
350 MM=X(:,1); EE=X(:,2); LL=X(:,3); PP=X(:,4); AA=X(:,5); RR=X
351 (:,6); QQ=X(:,7);
352
353 Up = [0 0 0];
354 [T,Y] = ode45(@kims,time,y0,[],Up,constant);
355 J=sum ((D1.*EE(1:end) + D2.*LL(1:end) +D3.*PP(1:end)+...
356 D4.*AA(1:end)+(W1/2)*Uu(:,1).*Uu(:,1)+(W2/2)*Uu(:,2).*Uu
357 (:,2)+...
358 (W3/2)*Uu(:,3).*Uu(:,3))));
359 %%
360 S=[Tx,X];
361 save case4;
362
363 %%FULL FIGURE PLOT
364
365 figure(2)
366 %subplot(3,3,1)
367 plot(Tx,X(:,1),'r',T, Y(:,1),'--b', 'LineWidth',3);
368 ylabel('Maize biomass','Interpreter','LaTeX','fontsize',12);
369 xlabel({'Time (days)','(a)'},'Interpreter','LaTeX','fontsize
370 ',12);
371 % legend('u_i\neq 0, i=1','u_i=0,i=1,2,3')
372 % legend('u_i\neq 0, i=2','u_i=0,i=1,2,3')
373 % legend('u_i\neq 0, i=3','u_i=0,i=1,2,3')
374 % legend('u_i\neq 0, i=1,2','u_i=0,i=1,2,3')
375 % legend('u_i\neq 0, i=1,3','u_i=0,i=1,2,3')
376 % legend('u_i\neq 0, i=2,3','u_i=0,i=1,2,3')
377 legend('u_i\neq 0, i=1,2,3','u_i=0,i=1,2,3')

```

```

377 print('Opt12.eps','-depsc','-r600');
378 %%
379 figure(3)
380 %subplot(3,3,2)
381 plot(Tx,X(:,2)*1000,'r', T, Y(:,2)*1000,'--b','LineWidth',3)
382 ;
383 ylabel('Eggs');
384 xlabel({'Time (days)','(b)'});
385 % legend('u_i\neq 0, i=1','u_i=0,i=1,2,3')
386 % legend('u_i\neq 0, i=2','u_i=0,i=1,2,3')
387 % legend('u_i\neq 0, i=3','u_i=0,i=1,2,3')
388 % legend('u_i\neq 0, i=1,2','u_i=0,i=1,2,3')
389 % legend('u_i\neq 0, i=1,3','u_i=0,i=1,2,3')
390 % legend('u_i\neq 0, i=2,3','u_i=0,i=1,2,3')
391 legend('u_i\neq 0, i=1,2,3','u_i=0,i=1,2,3')
392 print('Opt13.eps','-depsc','-r600');
393 %%
394 figure(4)
395 %subplot(3,3,3)
396 plot(Tx,X(:,3)*1000,'r', T, Y(:,3)*1000,'--b','LineWidth',3);
397 ylabel('Larvae');
398 xlabel({'Time (days)','(c)'});
399 % legend('u_i\neq 0, i=1','u_i=0,i=1,2,3')
400 % legend('u_i\neq 0, i=2','u_i=0,i=1,2,3')
401 % legend('u_i\neq 0, i=3','u_i=0,i=1,2,3')
402 % legend('u_i\neq 0, i=1,2','u_i=0,i=1,2,3')
403 % legend('u_i\neq 0, i=1,3','u_i=0,i=1,2,3')
404 % legend('u_i\neq 0, i=2,3','u_i=0,i=1,2,3')
405 legend('u_i\neq 0, i=1,2,3','u_i=0,i=1,2,3')
406 print('Opt14.eps','-depsc','-r600');
407 %%
408 figure(5)
409 %subplot(3,3,4)
410 plot(Tx,X(:,4)*1000,'r', T, Y(:,4)*1000,'--b','LineWidth',3);
411 ylabel('Pupa');
412 xlabel({'Time (days)','(d)'});
413 % legend('u_i\neq 0, i=1','u_i=0,i=1,2,3')
414 % legend('u_i\neq 0, i=2','u_i=0,i=1,2,3')
415 % legend('u_i\neq 0, i=3','u_i=0,i=1,2,3')
416 % legend('u_i\neq 0, i=1,2','u_i=0,i=1,2,3')
417 % legend('u_i\neq 0, i=1,3','u_i=0,i=1,2,3')
418 % legend('u_i\neq 0, i=2,3','u_i=0,i=1,2,3')
419 legend('u_i\neq 0, i=1,2,3','u_i=0,i=1,2,3')
420 print('Opt15.eps','-depsc','-r600');
421 %%
422 figure(6)
423 %subplot(3,3,5)

```

```

423 plot(Tx,X(:,5)*1000,'r', T, Y(:,5)*1000,'--b', 'LineWidth'
    ,3);
424 ylabel('Adult moth');
425 xlabel({'Time (days)', '(e)'});
426 % legend('u_i\neq 0, i=1', 'u_i=0,i=1,2,3')
427 % legend('u_i\neq 0, i=2', 'u_i=0,i=1,2,3')
428 % legend('u_i\neq 0, i=3', 'u_i=0,i=1,2,3')
429 % legend('u_i\neq 0, i=1,2', 'u_i=0,i=1,2,3')
430 % legend('u_i\neq 0, i=1,3', 'u_i=0,i=1,2,3')
431 % legend('u_i\neq 0, i=2,3', 'u_i=0,i=1,2,3')
432 legend('u_i\neq 0, i=1,2,3', 'u_i=0,i=1,2,3')
433 print('Opt16.eps', '-depsc', '-r600');
434 %%
435 figure(7)
436 %subplot(3,3,6)
437 plot(Tx,X(:,6)*1000,'r', T, Y(:,6)*1000,'--b', 'LineWidth'
    ,3);
438 ylabel('Eggs predator');
439 xlabel({'Time (days)', '(f)'});
440 % legend('u_i\neq 0, i=1', 'u_i=0,i=1,2,3')
441 % legend('u_i\neq 0, i=2', 'u_i=0,i=1,2,3')
442 % legend('u_i\neq 0, i=3', 'u_i=0,i=1,2,3')
443 % legend('u_i\neq 0, i=1,2', 'u_i=0,i=1,2,3')
444 % legend('u_i\neq 0, i=1,3', 'u_i=0,i=1,2,3')
445 % legend('u_i\neq 0, i=2,3', 'u_i=0,i=1,2,3')
446 legend('u_i\neq 0, i=1,2,3', 'u_i=0,i=1,2,3')
447 print('Opt17.eps', '-depsc', '-r600');
448
449 %%
450 figure(8)
451 %subplot(3,3,7)
452 plot(Tx,X(:,7)*1000,'r', T, Y(:,7)*1000,'--b', 'LineWidth'
    ,3);
453 ylabel('Larvae predator');
454 xlabel({'Time (days)', '(g)'});
455 % legend('u_i\neq 0, i=1', 'u_i=0,i=1,2,3')
456 % legend('u_i\neq 0, i=2', 'u_i=0,i=1,2,3')
457 % legend('u_i\neq 0, i=3', 'u_i=0,i=1,2,3')
458 % legend('u_i\neq 0, i=1,2', 'u_i=0,i=1,2,3')
459 % legend('u_i\neq 0, i=1,3', 'u_i=0,i=1,2,3')
460 % legend('u_i\neq 0, i=2,3', 'u_i=0,i=1,2,3')
461 legend('u_i\neq 0, i=1,2,3', 'u_i=0,i=1,2,3')
462 print('Opt18.eps', '-depsc', '-r600');
463 %% control profile
464 figure(9)
465 %subplot(3,3,8)
466 plot(Tx,Uu(:,1),'--b',Tx,Uu(:,2),'--g',Tx,Uu(:,3),'--r', '
    LineWidth', 3);
467 ylabel('Control Profile');

```

```

468 % xlabel('Time (Year)');
469 xlabel({'Time (days)', '(h)'});
470 % legend('u_1\neq 0', 'u_2= 0', 'u_3=0')
471 % legend('u_1= 0', 'u_2\neq 0', 'u_3=0')
472 % legend('u_1= 0', 'u_2= 0', 'u_3 \neq 0')
473 % legend('u_1\neq 0', 'u_2 \neq 0', 'u_3 = 0')
474 % legend('u_1\neq 0', 'u_2= 0', 'u_3\neq 0')
475 % legend('u_1= 0', 'u_2 \neq 0', 'u_3\neq 0')
476 legend('u_1 \neq 0', 'u_2 \neq 0', 'u_3\neq 0')
477 box
478 print('Profile1.eps', '-depsc', '-r600');
479 box
480 %%
481 % collect all the incidence terms in the ODE
482 X=X; % solution of the optimal control
483 U =[0 0 0]; % when no control
484 [Tx,Y] = ode45(@kims,time,y0,[],U,constant);
485 Y=(Y);
486
487 Inew=sum(Y(:,2))-sum(X(:,2))+sum(Y(:,3))-sum(X(:,3))+sum(Y
488 (:,4))-sum(X(:,4))...
489 +sum(Y(:,5))-sum(X(:,5)) % averted
490
491 function ydot = kims_costate(t,y,U,C,constant)
492 %this function solves a MU infection on human type three
493 equations
494 lambda1 = y(1);
495 lambda2 = y(2);
496 lambda3 = y(3);
497 lambda4 = y(4);
498 lambda5 = y(5);
499 lambda6 = y(6);
500 lambda7 = y(7);
501
502 alpha_E=constant(1);
503 alpha_L = constant(2);
504 alpha_P = constant(3);
505 d = constant(4); %Used as h in the model
506 delt = constant(5);
507 sigma = constant(6);
508 K_E = constant(7);
509 K_L = constant(8);
510 K_M =constant(9);
511 beta= constant(10);
512 a= constant(11);
513 mu_E = constant(12);
514 e = constant(13);
515 mu_L = constant(14);
516 mu_A=constant(15);

```

```

515 r =constant(16);
516 v =constant(17);
517 mu_P =constant(18);
518 psi =constant(19);
519 gamma = constant(20);
520 eta = constant(21);
521 epsilon =constant(22);
522 %%
523 D1 = 0.005; %weight
524 D2 = 0.005; %weight
525 D3 = 0.005;
526 D4 = 0.005;
527 W1 = 0.005;
528 W2 = 0.005;
529 W3 = 0.005;
530
531 u1 = U(1); u2=U(2); u3=U(3);
532 M = C(1,:);E = C(2,:);L = C(3,:);P = C(4,:);A= C(5,:);R = C
    (6,:);Q = C(7,:);
533 %%
534 ydot1=((-r+2*r*M/K_M)+ beta*L)*lambda1 - e*beta*L*lambda3; %
    first ODE
535 ydot2=( a*v*A/K_E + (alpha_E+mu_E+ u1*d)+ (1+ u3)*delt*psi*R
    )*lambda2 - (alpha_E - alpha_E*L/K_L)*lambda3 - (1+u3)*
    delt*psi*R*lambda6 -D1; % second ODE
536 ydot3= beta*M*lambda1 + (alpha_E*E/K_L - e*beta*M + (mu_L +
    alpha_L+ u1*d)+ (1+u2)*sigma*epsilon*Q)*lambda3 - alpha_L
    *lambda4 - (1+u2)*sigma*epsilon*Q*lambda7 - D2;
537 ydot4=(mu_P + alpha_P+ u1*d)*lambda4 - alpha_P*lambda5 - D3;
538 ydot5= -(1 - E/K_E)*a*v*lambda2 + (mu_A + u1*d)*lambda5 - D4
    ;
539 ydot6= (1+u3)*delt*psi*E*lambda2 - ((1+u3)*delt*psi*E -
    gamma)*lambda6;
540 ydot7= (1+u2)*sigma*epsilon*L*lambda3 - ((1+u2)*sigma*
    epsilon*L - eta)*lambda7;
541 ydot = [ydot1; ydot2;ydot3;ydot4;ydot5;ydot6;ydot7];
542 %
    %*****
543
544 function [t, w]=rk4back(f,t0, tf, N, init2,U,YY,constant)
545 h = -(tf-t0)/N; %the step size
546 t(N) = tf;
547 w(:,N) = init2; %initial conditions
548
549 for i = N:-1:1
550 uu = U(i,:);
551 yy = YY(:,i);
552 k1 = h*f(t(i),w(:,i),uu,yy,constant);

```

```

553 k2 = h*f(t(i)+h/2, w(:,i)+0.5*k1,uu,yy,constant);
554 k3 = h*f(t(i)+h/2, w(:,i)+0.5*k2,uu,yy,constant );
555 k4 = h*f(t(i)+h, w(:,i)+k3,uu,yy,constant);
556 if i>1
557 w(:,i-1) = w(:,i) + (k1 + 2*k2 + 2*k3 + k4)/6;
558 t(i-1) = tf + i*h;
559 else
560 ww = w(:,i) + (k1 + 2*k2 + 2*k3 + k4)/6;
561 tt = tf + i*h;
562 end
563 end
564 w = [ww w];
565 t = [tt t];
566
567 function [t, w]=rk4foward(f,t0, tf, N, init,U,constant)
568 h = (tf-t0)/N; %the step size
569 t(1) = t0;
570 w(:,1) = init; %initial conditions
571 for i = 1:N
572 uu = U(i,:);
573 k1 = h*f(t(i),w(:,i),uu,constant);
574 k2 = h*f(t(i)+h./2, w(:,i)+0.5*k1,uu,constant);
575 k3 = h*f(t(i)+h./2, w(:,i)+0.5*k2,uu,constant);
576 k4 = h*f(t(i)+h, w(:,i)+k3,uu,constant);
577 w(:,i+1) = w(:,i) + (k1 + 2*k2 + 2*k3 + k4)/6;
578 t(i+1) = t0 + i*h;
579 end

```

Listing 1: MATLAB code for basic model

RESEARCH OUTPUTS

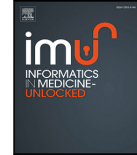
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Predatory effects on the dynamics of *Spodoptera Frugiperda* infestations in maize

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ABSTRACT

Maize remains in demand due to its nutritive value, capacity to provide food for a growing world population, contribution to food security, and increase in worldwide investments in ethanol as a biofuel. However, the invasion and widespread infestation of *Spodoptera frugiperda* result in significant maize yield losses, leading to a lower standard of living and a weakened economy for maize producers. This study builds differentiable equations to simulate the behavior of the *Spodoptera frugiperda*-maize biomass model, incorporating predators and best farming practices. The model exhibits six points of equilibrium, all of which are locally asymptotically stable if the necessary requirements are met. Latin Hypercube Sampling (LHS) and PRCC multivariate analysis were employed to identify the sensitive parameters affecting the pest. Numerical simulations suggest that, in the early stages, integrating natural enemies with best farming practices proves to be an effective intervention as it directly reduces the pest population and promotes sustainable pest control.

1. Introduction

Maize is still a significant staple food for millions of people in sub-Saharan Africa (SSA) and numerous Asian nations [1]. It has developed into a vital industrial commodity that guarantees the nutritional value of food, feed, and a source of income. Among all sources, maize accounts for 19.5% of global calorie intake. More than 116 million tonnes of maize are thought to be used annually, with 30% of that consumption taking place globally and 21% in SSA, respectively [1–3].

As stated by the Food and Agriculture Organization (FAO), food security is the state in which people consistently and adequately have access to enough nourishing meals that satisfy dietary needs and personal preferences, ensuring their well-being and an active lifestyle [4]. Likewise, by 2050, maize production must quadruple to ensure global food security [5]. However, the main biotic factors limiting maize output in these nations are pests and diseases [1].

Spodoptera frugiperda (SF), commonly recognized as the fall armyworm, is a highly polyphagous pest that poses a significant threat to global food security. This endemic agricultural pest has been identified in North and South America, where it is known for its rapid and destructive spread [6]. SF primarily infects members of the *Poaceae* family, which includes important food crops such as corn, sorghum, rice, wheat, maize, and a variety of pastures [7]. According to earlier

studies, SF exhibits a preference for maize over other crops and pastures [8]. It targets every stage of maize growth, from germination to ear development, causing defoliation, whorl injury, and reductions in grain yields and quality [9,10].

In the Americas, SF has caused substantial maize damage, resulting in yield reductions of 34% in Brazil, 72% in Argentina, and 40% in Honduras [11,12]. In Africa, the impact has been even more severe, with up to 90% of corn harvests lost due to epidemics that originated in Nigeria and spread to sub-Saharan African countries. The first SF outbreak in West Africa occurred in 2016, and it reached East African nations in 2017, affecting 44 countries across the continent [8,13–15]. Farm-level estimates reveal significant yield losses due to SF infestations, such as 22%–67% in Ghana, 47% in Kenya, and 9.4% in Zimbabwe.

Integrated pest management (IPM) techniques use multiple strategies, including biological agents like predators and cultural control approaches [1,6,16]. Moreover, sprays of pesticides, such as Bt maize, and genetically engineered foods have been common management methods in South America, where the insect has been a concern for some time [6,13,17]. The cost of extensive chemical insecticide spraying programs and the use of genetically engineered crops, combined

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with the financial difficulties faced by the majority of African countries, make it difficult to effectively manage this pest on the continent [6,9]. Additionally, the overuse of chemical insecticides is linked to adverse environmental impacts and may result in the emergence of pest resistance [18]. In this context, IPM is receiving increased attention from researchers as it reduces SF infestation and dependency on pesticides [13,16,19].

In addition, best farming practices include methods such as hand picking, weeding, soil management, sanitation, the use of biological extracts, intercropping, host plant resistance, monitoring, and cropping systems. Nevertheless, it is advised to consider a regional strategy as one of the most effective options for controlling this invasive pest [20–23].

The primary target of this project is to create a mathematical framework that would evaluate how natural enemies might affect SF outbreaks. A variety of mathematical models have recently been formulated to investigate the relationship between plants and pests. Daudi et al. [13] developed a fractional-order model analyzing the contribution of natural enemies and farming awareness that is important in controlling SF. The study recommends the use of farming awareness campaigns to reduce SF impacts. In a similar vein, Daudi et al. [24] considered a non-autonomous model to learn the impact of seasonal fluctuations on SF. The study includes traditional methods as well as the use of chemical pesticides as a framework for developing efficient SF control measures.

Ogunfunmilayo et al. [25] employed a statistical model to determine whether using natural enemies, like predators or parasitoids, is more environmentally safe and economically practical for controlling SF than the currently recommended synthetic pesticides. The report recommends a public awareness campaign promoting the use of integrated pest management techniques. Moreover, Ntare et al. [16] investigated the effect of current smallholder crop production practices on SF infestation in Uganda. These results provide a foundation for understanding how intercropping systems significantly interact to reduce SF infestations. In a different study, Faithpraise et al. [11] formulated a mathematical model to analyze how naturally beneficial insects may be used to manage African armyworm (AAW) pest infestations in cereal crop areas. The results of the study demonstrated how helpful the suggested model was in supplying prospective solutions for containing AAW outbreaks using natural enemies.

The studies described above and a number of others that were cited therein undoubtedly provided numerous beneficial findings and enhanced our understanding of plant–insect interactions. Despite all of these efforts, there are very few mathematical models representing SF management options during an outbreak, and those that do exist have some limitations: The bulk of these few models were generic, indicating that their findings were also generic. The models presented used integer-order and fractional differential equations and few biological factors were captured. Likewise, several studies did not include a numerical model for describing the patterns of SF infestations. In this study, we utilize a deterministic model to evaluate the impacts of egg and larval predation with best farming practices on pest survival and elimination in a maize field.

This paper is organized as follows: Section 2 provides the model formulation and various analytical techniques. Section 3 focuses on an in-depth global analysis of the model parameters. Section 4 presents numerical results to validate the analytical findings. Finally, Section 5 summarizes the key findings of the study and provides a conclusion based on the results obtained. Following this structured approach, the paper ensures a logical flow of information, enabling readers to understand the model formulation, analyze its behavior, examine numerical results, and draw meaningful conclusions, along with recommendations for further studies.

2. Material and methods

2.1. Model formulation

We proposed a nonlinear model that utilizing biological control to reduce the population of SF [11,26]. Our model incorporates three populations: SF, predators and maize biomass. The SF population consists of four classes: egg (E), larva (L), pupa (P), and adult (A). The predators for the egg and larval stages are denoted as (R) and (Q), respectively. Furthermore, maize biomass is represented as (M).

Concerning maize biomass, we assume logistic growth with a carrying ability of K_M and a gross growth rate of r . Biomass conversion efficiency is represented by e , and the rate of SF larvae consumption is denoted by β .

The dynamics of the SF egg and larva populations are expected to follow a logistic growth model in the presence of maize. The carrying capacities for the SF egg and larva populations are represented by K_E and K_L respectively. The net growth rates for the SF egg and larva populations are denoted by a and α_E respectively. It is anticipated that the egg will eventually develop into a larva after $\frac{1}{\alpha_E}$ days. The natural mortality rates for the SF egg and larva are μ_E and μ_L , respectively. Furthermore, the population decrease resulting from deaths or removals due to farmers adopting best farming practices is represented by h .

The eggs and larvae are easily attacked by a range of parasitoids and predators, including birds, rodents, beetles, and earwigs [13,27,28]. Let ϵ be the conversion efficiency and σ be attack frequency of the predation on larvae. Predators are estimated to have an average lifespan of $\frac{1}{\eta}$ days. Similarly, for the impact of egg predation, define ψ as the conversion efficiency and δ as the rate at which predators attack the egg. Egg predators are estimated to have an average lifespan of $\frac{1}{\gamma}$ days.

Maize seeds are planted in one day and grow at a constant rate. Weather, climate, and how maize seeds were placed are assumed to promote seed germination and maize biomass growth with no natural mortality rate prior to harvest. SF moths remain in the field and do not depart before the harvesting process. Additionally, it is assumed that maize is the main source of sustenance for SF. Moreover, in the absence of maize, SF larvae perish at an exponential rate. The dynamics of SF, maize biomass, and predators are organized by a flow diagram as shown in Fig. 1, and are governed by a set of differential equations, Eq. (1).

$$\begin{cases} \frac{dM}{dt} = rM \left(1 - \frac{M}{K_M}\right) - \beta LM \\ \frac{dE}{dt} = a \left(1 - \frac{E}{K_E}\right) vA - (\alpha_E + \mu_E + h)E - \delta \psi RE \\ \frac{dL}{dt} = \alpha_E \left(1 - \frac{L}{K_L}\right) E + e\beta ML - (\alpha_L + \mu_L + h)L - \sigma \epsilon QL \\ \frac{dP}{dt} = \alpha_L L - (\mu_P + \alpha_P + h)P \\ \frac{dA}{dt} = \alpha_P P - (\mu_A + h)A \\ \frac{dR}{dt} = \delta \psi ER - \gamma R \\ \frac{dQ}{dt} = \sigma \epsilon LQ - \eta Q \end{cases} \quad (1)$$

where $M(0) \geq 0$, $E(0) \geq 0$, $L(0) \geq 0$, $P(0) \geq 0$, $A(0) \geq 0$, $R(0) \geq 0$, $Q(0) \geq 0$.

2.2. Positivity of the solution

Theorem 1. Let $M(0) \geq 0$, $E(0) \geq 0$, $L(0) \geq 0$, $P(0) \geq 0$, $A(0) \geq 0$, $R(0) \geq 0$ and $Q(0) \geq 0 \in I$. Then, all solutions of the model system Eq. (1) above are non-negative for all $t \geq 0$.

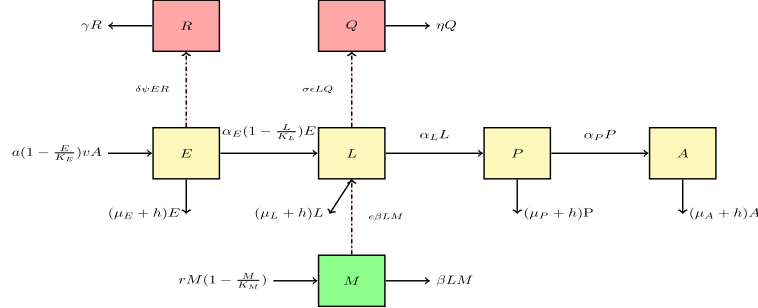


Fig. 1. Model flow diagram.

Proof. Consider the equation for the population of maize,

$$\frac{dM}{dt} = rM \left(1 - \frac{M}{K_M} \right) - \beta LM$$

Whose solution is,

$$M \leq Ce^{-rt},$$

where C is the constant. At $t = 0$, $M = M(0)$,

$$M \leq M(0)e^{-rt}.$$

This implies that, the maize population lies in the interval of $0 \leq M(t) \leq M(0)$, $\forall t \geq 0$. Using the same approach, it might be shown that $E(t) \geq 0$, $L(t) \geq 0$, $P(t) \geq 0$, $A(t) \geq 0$, $R(t) \geq 0$ and $Q(t) \geq 0$, $\forall t \geq 0$. Hence, it can be concluded that the model system Eq. (1) has non-negative solutions. $\square$

2.3. System boundedness

Theorem 2. All the solutions of the model system Eq. (1), which initiate in $\mathbb{R}_{+}^6$, are uniformly bounded. That is there exists a constant $G > 0$ such that $M(t) \leq G$, $E(t) \leq G$, $L(t) \leq G$, $P(t) \leq G$, $A(t) \leq G$, $R(t) \leq G$ and $Q(t) \leq G$ for each solution of the model system with all t large enough.

Proof. We check boundedness in the pest population equations. Consider $(E(t), L(t), P(t), A(t))$ with favorable initial conditions and,

$$N = E + L + P + A.$$

Thus,

$$\frac{dN}{dt} = \frac{dE}{dt} + \frac{dL}{dt} + \frac{dP}{dt} + \frac{dA}{dt}.$$

Therefore,

$$\frac{dN}{dt} \leq (av - \mu_A - h)A - (\mu_E + h)E - (\mu_L + h)L - (\mu_P + h)P,$$

implies,

$$\frac{dN}{dt} \leq -\theta^*(E + L + P + A) + 2(av - \mu_A - h)A.$$

where $\theta^* = \min \{(\mu_E + h), (\mu_L + h), (\mu_P + h), (av - (\mu_A + h))\} > 0$, provided $av > (\mu_A + h)$. Therefore,

$$\frac{dN}{dt} + \theta^* N \leq G_0,$$

where $G_0 = (av - \mu_A - h)^2 > 0$.

Applying the theory of differential inequality, we obtain,

$$N(t) \leq \frac{G_0}{\theta^*} (1 - e^{-\theta^* t}) + N(0)e^{-\theta^* t}.$$

For $t \rightarrow \infty$, we get $N(t) \leq \frac{G_0}{\theta^*}$. So $N(t)$ is ultimately bounded. Therefore, there exist a positive constant $G = \frac{G_0}{\theta^*}$ such that each solution of the model system Eq. (1), $E(t) \leq G$, $L(t) \leq G$, $P(t) \leq G$ and $A(t) \leq G$ when t is sufficient large. Again, considering predator population,

$$\frac{dN_1}{dt} \leq (\gamma - \delta\psi G)R - (\eta - \sigma\epsilon G)Q,$$

$$\frac{dN_1}{dt} \leq -\pi(R + Q).$$

where $\pi = \min \{(\gamma - \delta\psi G), (\eta - \sigma\epsilon G)\} > 0$, $N_1 = R + Q$, provided $\gamma > \delta\psi G$ and $\eta > \sigma\epsilon G$. Therefore,

$$N_1(t) \leq \frac{G_0^*}{\pi} (1 - e^{-\pi t}).$$

Using the same approach, solution of maize and predators are bounded as $0 \leq M_m \leq K_M$ and $N_1(t) \leq \frac{G_0^*}{\pi} = B$ as $t \rightarrow \infty$ respectively. $\square$

2.4. System of equilibrium points

The model system Eq. (1) has six equilibrium points as follows.

- The point of trivial equilibrium $\xi_1 : (0, 0, 0, 0, 0, 0)$, always exists. Biologically, the species defined in Eq. (1) are completely absent at this equilibrium point ξ_1 , either as a result of loss or through the use of conservation measures.
- The pest-extinction equilibrium point $\xi_2 : (k_M, 0, 0, 0, 0, 0)$. As a result, in the absence of SF larvae, high-yield maize biomass is anticipated at the conclusion of the season.
- The maize-extinction equilibrium point $\xi_3 : (0, E_3, L_3, P_3, A_3, R_3, Q_3)$, where:

$$E_3 = \frac{\gamma}{\delta\psi}; L_3 = \frac{\eta}{\sigma\epsilon}; P_3 = \frac{\eta\alpha_L}{\sigma\epsilon m_3}; A_3 = \frac{\eta\alpha_L\alpha_P}{\sigma\epsilon m_3 m_4},$$

$$R_3 = \frac{(\sigma\epsilon m_1 m_3 m_4 K_E + a\eta v \alpha_L \alpha_P) (\mathcal{R}_1 - 1)}{\delta\epsilon \psi \sigma K_E m_3 m_4},$$

$$Q_3 = \frac{(\delta\psi m_2 K_L + \gamma \alpha_E) (\mathcal{R}_2 - 1)}{\delta\psi \epsilon \sigma K_L},$$

with $\mathcal{R}_1 = \frac{a\alpha_L \alpha_P v \psi \eta \delta K_E}{\gamma(\sigma\epsilon m_1 m_3 m_4 K_E + a\eta v \alpha_L \alpha_P)}$, $\mathcal{R}_2 = \frac{\sigma\gamma \epsilon \alpha_E K_L}{\eta(\delta\psi m_2 K_L + \gamma \alpha_E)}$, $m_1 = (\alpha_E + \mu_E + h)$, $m_2 = (\alpha_L + \mu_L + h)$, $m_3 = (\alpha_P + \mu_P + h)$ and $m_4 = (\mu_A + h)$. This implies that, existence of the equilibrium point ξ_3 depends on the satisfactions of the following conditions

$$\mathcal{R}_i > 1, \forall i \in \{1, 2\} \quad (2)$$

- (d) The equilibrium between maize and predator extinction: $\xi_4 : (0, E_4, L_4, P_4, A_4, 0, 0)$, where:

$$E_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{\alpha_E (a v K_L \alpha_L \alpha_P + K_E m_1 m_3 m_4)}, \quad L_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{a v \alpha_L \alpha_P (K_L m_2 + K_E \alpha_E)},$$

$$P_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{a v \alpha_P m_3 (K_E \alpha_E + K_L m_2)}, \quad A_4 = \frac{K_E K_L m_1 m_2 m_3 m_4 (\mathcal{R} - 1)}{a v m_3 m_4 (K_E \alpha_E + K_L m_2)} \quad (3)$$

and $\mathcal{R} = \frac{a v \alpha_E \alpha_L \alpha_P}{m_1 m_2 m_3 m_4}$. Thus the equilibrium point ξ_4 occurs and exist if and only if $\mathcal{R} > 1$. This implies that, the SF population exist if the threshold quantity $\mathcal{R}$ is greater than unity.

- (e) The predator-extinction equilibrium point is: $\xi_5 : (M_5, E_5, L_5, P_5, A_5, 0, 0)$, where:

$$M_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2 r a \beta e v K_L \alpha_L \alpha_P}, \quad E_5 = \frac{K_E (\mathcal{R}_3 - 1) \sqrt{h_1}}{2 a \beta^2 e v K_L^2 \alpha_L \alpha_P K_M},$$

$$L_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2 a \beta^2 e v K_L \alpha_L \alpha_P K_M}, \quad P_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2 a \beta^2 e v K_L \alpha_P K_M m_3},$$

$$A_5 = \frac{(\mathcal{R}_3 - 1) \sqrt{h_1}}{2 a \beta^2 e v K_L K_M m_3 m_4}, \quad (4)$$

$$\text{and } \mathcal{R}_3 = \frac{h_2}{\sqrt{h_1}},$$

$$h_1 = ((-\beta e K_M + m_2)^2 r + 4 \alpha_E e \beta^2 K_E K_M) K_L^2 + 2 a_E r K_E (-\beta e K_M + m_2) K_L + \alpha_E^2 r K_E^2 a^2 v^2 r \alpha_P^2 - 2 m_1 m_3 K_L \beta^2 a e v K_E K_M m_4 r \alpha_P$$

$$((-\beta e K_M + m_2) K_L - \alpha_E K_E) \alpha_L + e^2 \beta^4 K_E^2 K_M^2 m_1^2 m_3^2 m_4^2,$$

$$h_2 = ((\beta e K_M + m_2) v r a K_L + a \alpha_E r v K_E) \alpha_P \alpha_L + \beta^2 e K_E K_L K_M m_1 m_3 m_4.$$

As a result, the equilibrium point ξ_5 is real and has biological significance if $h_1 > 0$ and $\mathcal{R}_3 > 1$.

- (f) The endemic equilibrium point: $\xi^* : (M^*, E^*, L^*, P^*, A^*, R^*, Q^*)$, where:

$$M^* = \frac{\beta \eta K_M (\mathcal{R}_4 - 1)}{r \sigma \epsilon}, \quad E^* = \frac{\gamma}{\delta \psi}; \quad L^* = \frac{\eta}{\epsilon \sigma}; \quad P^* = \frac{\eta \alpha_L}{\sigma \epsilon m_3}; \quad A^* = \frac{\alpha_L \alpha_P \eta}{\sigma \epsilon m_3 m_4},$$

$$R^* = \frac{(\sigma \epsilon m_1 m_3 m_4 K_E + a \alpha_L \alpha_P v \eta) (\mathcal{R}_5 - 1)}{\delta e \psi \sigma K_E m_3 m_4},$$

$$Q^* = \frac{\beta^2 \delta e \eta \psi K_L K_M + \delta e \psi r \sigma K_L m_2 + \epsilon \gamma r \sigma \alpha_E}{\psi \delta \epsilon^2 \sigma^2 r K_L} (\mathcal{R}_6 - 1), \quad (5)$$

with $\mathcal{R}_4 = \frac{\sigma r \epsilon}{\eta \beta}$, $\mathcal{R}_5 = \frac{a \alpha_L \alpha_P v \eta \psi \delta K_E}{\sigma \epsilon m_1 m_3 m_4 K_E \gamma + a \alpha_L \alpha_P v \eta \gamma}$ and $\mathcal{R}_6 = \frac{\epsilon r \sigma K_L (\epsilon \gamma \sigma \alpha_E + \beta \delta e \eta \psi K_M)}{\beta^2 \delta e \eta^2 \psi K_L K_M + \delta e \eta \psi r \sigma K_L m_2 + \epsilon \eta \gamma r \sigma \alpha_E}$. Therefore, the equilibrium point ξ^* exists and has ecological significance if $\mathcal{R}_i > 1 \forall i \in \{4, 5, 6\}$.

2.5. Local stability analysis

The Jacobian matrix used to compute the local stability analysis of the model system Eq. (1) is as follows:

$$J = \begin{pmatrix} -\theta_1 & 0 & -\beta M & 0 & 0 & 0 & 0 \\ 0 & -\theta_2 & 0 & 0 & \theta_4 & -\delta \psi E & 0 \\ 0 & \theta_5 & -\theta_3 & 0 & 0 & 0 & -\sigma \epsilon L \\ e \beta L & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & R \delta \psi & 0 & 0 & 0 & \theta_6 & 0 \\ 0 & 0 & Q \sigma \epsilon & 0 & 0 & 0 & \theta_7 \end{pmatrix}, \quad (6)$$

$$\text{with } \theta_1 = -r \left(1 - \frac{M}{K_M} \right) + \frac{r M}{K_M} + \beta L, \quad \theta_2 = \frac{a v A}{K_E} + m_1 + R \delta \psi,$$

$$\theta_3 = \frac{\alpha_E E}{K_L} - e \beta M + m_2 + Q \sigma \epsilon, \quad \theta_4 = a v \left(1 - \frac{E}{K_E} \right), \quad \theta_5 = \alpha_E \left(1 - \frac{L}{K_L} \right),$$

$\theta_6 = E \delta \psi - \gamma$ and $\theta_7 = L \epsilon \sigma - \eta$. Criteria conditions of Lemma [Lemmas 1](#) and [6](#) are used to ascertain the local stability of the system.

Lemma 1 (Routh–Hurwitz Criteria). Given the polynomial $K(\lambda) = \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_{n-1} \lambda + a_n$, where the coefficients a_i are real constants ($i = 1, \dots, n$), the Hurwitz matrices $H_1, H_2, \dots, H_n$ can be defined as follows:

$$H_1 = [a_1], \quad H_2 = \begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix}, \quad H_3 = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 \end{bmatrix},$$

and

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & \dots & 0 \\ a_3 & a_2 & a_1 & 1 & \dots & 0 \\ a_5 & a_4 & a_3 & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & a_n \end{bmatrix}.$$

where $a_j = 0$ if $j > n$. All roots of the polynomial $K(\lambda)$ are negative or have a negative real portion if and only if all determinants of Hurwitz matrices are positive:

$$\det(H_j) > 0, \quad j = 1, 2, \dots, n.$$

The Routh–Hurwitz criteria for $n = 2, 3$, and 4 are as follows:

(C1) $n = 2$: $a_1 > 0$ and $a_2 > 0$,

(C2) $n = 3$: $a_1 > 0$, $a_3 > 0$, and $a_1 a_2 > a_3$,

(C3) $n = 4$: $a_1 > 0$, $a_3 > 0$, $a_4 > 0$, and $a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$.

(C4) $n = 5$: $\tilde{H}_1 : a_1 > 0$, $a_1 a_2 > a_3$, $\tilde{H}_2 : a_1 a_2 a_3 + a_1 a_5 - a_1^2 a_4 - a_3^2 > 0$, $\tilde{H}_3 : (a_3 a_4 - a_2 a_5)(a_1 a_2 - a_3) - (a_1 a_4 - a_4^2)^2 > 0$ and $a_4 \tilde{H}_3 > 0$.

These criteria establish the parameters under which stability can be evaluated based on the coefficients of a system's distinguishing polynomial.

The Jacobian matrix of system Eq. (1) evaluated at ξ_1 is,

$$J(\xi_1) = \begin{pmatrix} r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_1 & 0 & 0 & a v & 0 & 0 \\ 0 & \alpha_E & -m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta \end{pmatrix}.$$

Clearly, one of the eigenvalues of matrix $J(\xi_1)$ is $\lambda_1 = r > 0$, suggesting that the trivial point ξ_1 is locally asymptotically unstable.

Similarly, system of Eq. (1) evaluated at ξ_2 is,

$$J(\xi_2) = \begin{pmatrix} -r & 0 & -\beta K_M & 0 & 0 & 0 & 0 \\ 0 & -m_1 & 0 & 0 & a v & 0 & 0 \\ 0 & \alpha_E & \hat{p} & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta \end{pmatrix},$$

with $\hat{p} = -\beta e K_M + m_2$. Eigenvalues for the matrix $J(\xi_2)$ are $\lambda_1 = -r$, $\lambda_2 = -\gamma$ and $\lambda_3 = -\eta$ and the reduced matrix is used, with the following characteristic equation:

$$\lambda^4 + s_1^* \lambda^3 + s_2^* \lambda^2 + s_3^* \lambda + s_4^* \text{ with,}$$

$$s_1^* = m_4 + m_3 + \hat{p} + m_1, \quad s_2^* = \hat{p}(m_1 + m_3 + m_4) + m_1 m_3 + m_1 m_4 + m_3 m_4,$$

$$s_3^* = \hat{p}(m_1 m_3 + m_1 m_4 + m_3 m_4) + m_1 m_3 m_4,$$

$$s_4^* = \hat{p} m_1 m_3 m_4 - \alpha_P \alpha_E \alpha_L a v.$$

Following condition (C₃) of [Lemma 1](#), it can be observed that ξ_2 is locally asymptotically stable.

The Jacobian matrix of system Eq. (1) evaluated at ξ_3 is,

$$J(\xi_3) = \begin{pmatrix} r - \beta L_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta_1 & 0 & 0 & \theta_4 & -\theta_5 & 0 \\ e\beta L_3 & \theta_3 & -\theta_2 & 0 & 0 & 0 & -\theta_6 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & \delta\psi R_3 & 0 & 0 & 0 & -\theta_7 & 0 \\ 0 & 0 & \sigma\epsilon Q_3 & 0 & 0 & 0 & -\theta_8 \end{pmatrix},$$

with, $\theta_1 = \frac{avA_3}{K_E} + m_1 + R_3\delta\psi$, $\theta_2 = \frac{\alpha_E E_3}{K_L} + m_2 + Q_3\sigma\epsilon$, $\theta_3 = \alpha_E \left(1 - \frac{L_3}{K_L}\right)$, $\theta_4 = av \left(1 - \frac{E_3}{K_E}\right)$, $\theta_5 = \delta\psi E_3$, $\theta_6 = \sigma\epsilon L_3$, $\theta_7 = -E_3\delta\psi + \gamma$ and $\theta_8 = -L_3\epsilon\sigma + \eta$. Eigenvalues of matrix $J(\xi_3)$ are $\lambda_1 = r - \frac{\beta\eta}{\sigma\epsilon}$, $\lambda_2 = -L_3\epsilon\sigma + \eta$, and the reduced matrix is used to find the remaining eigenvalues, with the following characteristic equation: $\lambda^5 + \hat{s}_1\lambda^4 + \hat{s}_2\lambda^3 + \hat{s}_3\lambda^2 + \hat{s}_4\lambda + \hat{s}_5$ with,

$$\begin{aligned} \hat{s}_1 &= \theta_6 + m_4 + m_3 + \theta_1 + \theta_2, \quad \hat{s}_2 = (m_4 + \theta_1 + \theta_2 + \theta_6)m_3 \\ &+ (\theta_8 + \theta_1 + \theta_2 + \theta_6)m_4 + z_1, \quad \hat{s}_3 = ((\theta_8 + \theta_1 + \theta_2 + \theta_6)m_4 + z_1)m_3 \\ &+ z_1m_4 + z_2, \quad \hat{s}_4 = (h_1m_4 + z_2)m_3 + z_2m_4 - \alpha_P\alpha_L\theta_3\theta_4, \\ \hat{s}_5 &= h_2m_4m_3 - \theta_3\theta_4\theta_6\alpha_L\alpha_P. \end{aligned}$$

$$\begin{aligned} z_1 &= (\theta_1 + \theta_2)\theta_8 + Q\theta_6\sigma\epsilon + \theta_1\theta_2, \quad z_2 = (\theta_8 + \theta_1 + \theta_2)\theta_7 + R\theta_5\delta\psi, \\ z_3 &= (\theta_5 R\delta\psi + \theta_1\theta_2\theta_8 + Q\sigma\epsilon\theta_1\theta_6 + R\theta_5\theta_1\delta\psi, \\ z_4 &= (\theta_5 R\delta\psi + \theta_1\theta_7)(\theta_6 Q\sigma\epsilon + \theta_2\theta_8). \end{aligned}$$

Thus, if $\min\left(\frac{r}{\beta}, L_3\right) < \frac{\eta}{\epsilon\sigma}$ and condition (C₄) of Lemma 1 holds, then ξ_3 is locally asymptotically stable, otherwise it is unstable.

Following the same procedures, we have

$$J(\xi_4) = \begin{pmatrix} r - \beta L_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varpi_1 & 0 & 0 & \varpi_4 & -\delta\psi E_4 & 0 \\ e\beta L_4 & \varpi_3 & -\varpi_2 & 0 & 0 & 0 & -\sigma\epsilon L_4 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_4\delta\psi - \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_4\epsilon\sigma - \eta \end{pmatrix},$$

with, $\varpi_1 = \frac{avA_4}{K_E} + m_1$, $\varpi_2 = \frac{\alpha_E E_4}{K_L} + m_2$, $\varpi_3 = \alpha_E \left(1 - \frac{L_4}{K_L}\right)$ and $\varpi_4 = av \left(1 - \frac{E_4}{K_E}\right)$. Eigenvalues of matrix $J(\xi_4)$ are $\lambda_1 = r - \beta L_4$, $\lambda_2 = E_4\delta\psi - \gamma$, $\lambda_3 = L_4\epsilon\sigma - \eta$ and the reduced matrix is used to find the remaining eigenvalues, with the characteristic equation:

$$\lambda^4 + \bar{s}_1\lambda^3 + \bar{s}_2\lambda^2 + \bar{s}_3\lambda + \bar{s}_4 \text{ with,}$$

$$\begin{aligned} \bar{s}_1 &= m_4 + m_3 + \varpi_2 + \varpi_1, \quad \bar{s}_2 = \varpi_1\varpi_2 + \varpi_1m_3 + \varpi_1m_4 + \varpi_2m_3 \\ &+ \varpi_2m_4 + m_3m_4, \quad \bar{s}_3 = \varpi_1\varpi_2m_3 + \varpi_1\varpi_2m_4 + \varpi_1m_3m_4 + \varpi_2m_3m_4, \\ \bar{s}_4 &= \varpi_1\varpi_2m_3m_4 - \alpha_P\alpha_L\varpi_3\varpi_4. \end{aligned}$$

In this regards, if $\frac{r}{\beta} < L_4 < \frac{\eta}{\epsilon\sigma}$, $E_4 < \frac{\gamma}{\delta\psi}$ and condition (C₃) of Lemma 1 holds, then ξ_4 is locally asymptotically stable, otherwise it is unstable.

On another hand,

$$J(\xi_5) = \begin{pmatrix} -\bar{w}_5 & 0 & -\beta M_5 & 0 & 0 & 0 & 0 \\ 0 & -\bar{w}_1 & 0 & 0 & \bar{w}_4 & -\delta\psi E_5 & 0 \\ e\beta L_5 & \bar{w}_3 & -\bar{w}_2 & 0 & 0 & 0 & -\sigma\epsilon L_5 \\ 0 & 0 & \alpha_L & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_5\delta\psi - \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_5\epsilon\sigma - \eta \end{pmatrix},$$

with, $\bar{w}_1 = \frac{avA_5}{K_E} + m_1$, $\bar{w}_2 = \frac{\alpha_E E_5}{K_L} - e\beta M_5 + m_2$, $\bar{w}_3 = \alpha_E \left(1 - \frac{L_5}{K_L}\right)$, $\bar{w}_4 = av \left(1 - \frac{E_5}{K_E}\right)$ and $\bar{w}_5 = -r \left(1 - \frac{M_5}{K_E}\right) + \frac{rM_5}{K_M} + \beta L_5$. Eigenvalues of matrix $J(\xi_5)$ are $\lambda_1 = E_5\delta\psi - \gamma$ and $\lambda_2 = L_5\epsilon\sigma - \eta$ the reduced matrix is used, and the characteristic equation is as follows:

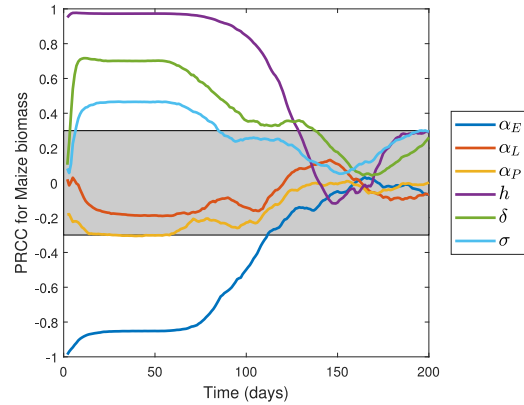


Fig. 2. PRCC values when measured against the increasing maize biomass population.

$$\lambda^5 + \hat{c}_1\lambda^4 + \hat{c}_2\lambda^3 + \hat{c}_3\lambda^2 + \hat{c}_4\lambda + \hat{c}_5 \text{ with,}$$

$$\begin{aligned} \hat{c}_1 &= m_3 + m_4 + \bar{w}_1 + \bar{w}_2 + \bar{w}_5, \quad \hat{c}_2 = (m_4 + \bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_3 \\ &+ (\bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_4 + \bar{n}_1\hat{c}_3 = ((\bar{w}_1 + \bar{w}_2 + \bar{w}_5)m_4 + \bar{n}_1)m_3 + \bar{n}_1m_4 + \bar{n}_2, \\ \hat{c}_4 &= ((\bar{n}_1m_4 + \bar{n}_2)m_3 + \bar{n}_2m_4 - \alpha_P\alpha_L\bar{w}_3\bar{w}_4), \quad \hat{c}_5 = \bar{n}_2\bar{w}_3\bar{w}_4 \\ &- \alpha_P\alpha_L\bar{w}_3\bar{w}_4\bar{w}_5, \quad \bar{n}_1 = (\bar{w}_2 + \bar{w}_5)\bar{w}_1 + e\beta^2 LM + \bar{w}_2\bar{w}_5, \\ \bar{n}_2 &= \bar{w}_1(LM\beta^2e + \bar{w}_2\bar{w}_5). \end{aligned}$$

Thus, if $E_5 < \frac{\gamma}{\delta\psi}$, $L_5 < \frac{\eta}{\epsilon\sigma}$ and condition (C₄) of Lemma 1, holds, then ξ_5 is locally asymptotically stable, otherwise it is unstable.

Since the endemic equilibrium point has non-zero values for every variable, it follows that matrix Eq. (6) is the Jacobian matrix. We observe, that $\lambda_1 = E^*\delta\psi - \gamma$, $\lambda_2 = L^*\epsilon\sigma - \eta$ and the reduced matrix is used to find the remaining eigenvalues,

$$\begin{pmatrix} -\Theta_1 & 0 & -\beta M^* & 0 & 0 \\ 0 & -\Theta_2 & 0 & 0 & \Theta_4 \\ e\beta L^* & \Theta_5 & -\Theta_3 & 0 & 0 \\ 0 & 0 & \alpha_L & -m_3 & 0 \\ 0 & 0 & 0 & \alpha_P & -m_4 \end{pmatrix},$$

The characteristic equation is as follows: $\lambda^5 + s_1\lambda^4 + s_2\lambda^3 + s_3\lambda^2 + s_4\lambda + s_5$ with,

$$\begin{aligned} s_1 &= m_4 + m_3 + \Theta_2 + \Theta_1 + \Theta_3, \quad s_2 = (m_4 + \Theta_1 + \Theta_2 + \Theta_3)m_3 \\ &+ (\Theta_3 + \Theta_2 + \Theta_1)m_4 + z_1^*, \quad s_3 = ((\Theta_3 + \Theta_2 + \Theta_1)m_4 + z_1^*)m_3 + z_1^*m_4 \\ &+ z_2^*, \quad s_4 = (z_1^*m_4 + z_2^*)m_3 + z_2^*m_4 - \alpha_P\alpha_L\Theta_4\Theta_5, \quad s_5 = z_2^*m_3m_4 \\ &- \alpha_P\alpha_L\Theta_5\Theta_4\Theta_1, \quad z_1^* = (\Theta_1 + \Theta_3)\Theta_2 + e\beta^2 L^*M^* + \Theta_1\Theta_3, \\ z_2^* &= \Theta_2(e\beta^2 L^*M^* + \Theta_1\Theta_3). \end{aligned}$$

Since $E^* < \frac{\gamma}{\delta\psi}$, $L^* < \frac{\eta}{\epsilon\sigma}$, it follows that condition (C₄) of Lemma 1 holds, then ξ^* is locally asymptotically stable, otherwise it is unstable.

2.6. Global stability analysis

The endemic equilibrium point is described in this section.

Theorem 3. The endemic equilibrium point ξ^* is globally asymptotically stable if the following conditions hold:

- (i) $(k_1 av)^2 \left(\frac{G}{K_E} - 1\right)^2 < B_1 m_4$,
- (ii) $(k_2 \alpha_E)^2 \left(\frac{L^*}{K_L} - 1\right)^2 < B_1 B_2$,
- (iii) $a_L^2 < m_3 B_2$,
- (iv) $\alpha_P^2 < m_3 m_4$,

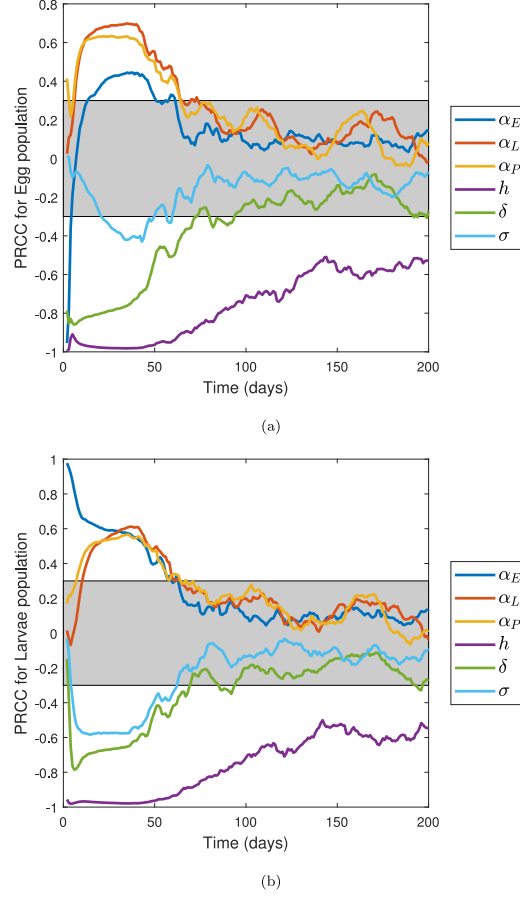


Fig. 3. PRCC values when measured against the increasing egg and larval SF population: 2(a) egg population, 2(b) larval population.

otherwise, they are unstable.

Proof. We consider the following positive definite function to prove the theorem

$$V = \left(M - M^* - M^* \ln \frac{M}{M^*} \right) + \frac{k_1}{2} (E - E^*)^2 + \frac{k_2}{2} (L - L^*)^2 + \frac{k_3}{2} (P - P^*)^2 + \frac{k_4}{2} (A - A^*)^2 + \frac{k_5}{2} (R - R^*)^2 + \frac{k_6}{2} (Q - Q^*)^2. \quad (7)$$

where $k_i > 0 \forall i \in \{1, 2, \dots, 6\}$ need to be appropriately selected. Changing the aforementioned equation with respect to t along the solution system of Eq. (1), we get

$$\frac{dV}{dt} = \left(1 - \frac{M^*}{M} \right) M' + k_1 (E - E^*) E' + k_2 (L - L^*) L' + k_3 (P - P^*) P' + k_4 (A - A^*) A' + k_5 (R - R^*) R' + k_6 (Q - Q^*) Q'.$$

By direct calculation, along the solution system Eq. (1), we arrive at the following result:

$$\frac{dV}{dt} = - \left[\frac{B_1}{2} (E - E^*)^2 + k_1 a v \left(\frac{E}{K_E} - 1 \right) (E - E^*) (A - A^*) + \frac{m_4}{2} (A - A^*)^2 \right] - \left[\frac{B_2}{2} (L - L^*)^2 + k_2 \alpha_E \left(\frac{L^*}{K_L} - 1 \right) (L - L^*) (E - E^*) + \frac{B_1}{2} (E - E^*)^2 \right]$$

$$- \left[\frac{m_3}{2} (P - P^*)^2 - \alpha_L (P - P^*) (L - L^*) + \frac{B_2}{2} (L - L^*)^2 \right] - \left[\frac{m_4}{2} (A - A^*)^2 - \alpha_P (A - A^*) (P - P^*) + \frac{m_3}{2} (P - P^*)^2 \right] - \left[\frac{r}{K_M} (M - M^*)^2 \right] - [(-\delta \psi E + \gamma)(R - R^*)^2] - [K_6 (-\sigma \epsilon E + \eta)(Q - Q^*)^2].$$

We now choose $k_1 = \frac{R^*}{E}$, $k_2 = \frac{1}{eL}$, $k_i = 1 \forall i \in \{3, 4, 5\}$, $k_6 = \frac{1}{eQ^*}$. Now $\frac{dV}{dt} \leq 0$, and is globally stable if the following criteria are satisfied:

- (i) $(k_1 a v)^2 \left(\frac{G}{K_E} - 1 \right)^2 < B_1 m_4$,
- (ii) $(k_2 \alpha_E)^2 \left(\frac{L^*}{K_L} - 1 \right)^2 < B_1 B_2$,
- (iii) $\alpha_L^2 < m_3 B_2$,
- (iv) $\alpha_P^2 < m_3 m_4$,

otherwise it is unstable, provided that $G < \frac{\gamma}{\delta \psi}$, $G < \frac{\eta}{\sigma \epsilon}$, $B_1 = k_1 \left(\frac{A^* a v G}{K_E} + m_1 + \delta \psi R^* \right)$,

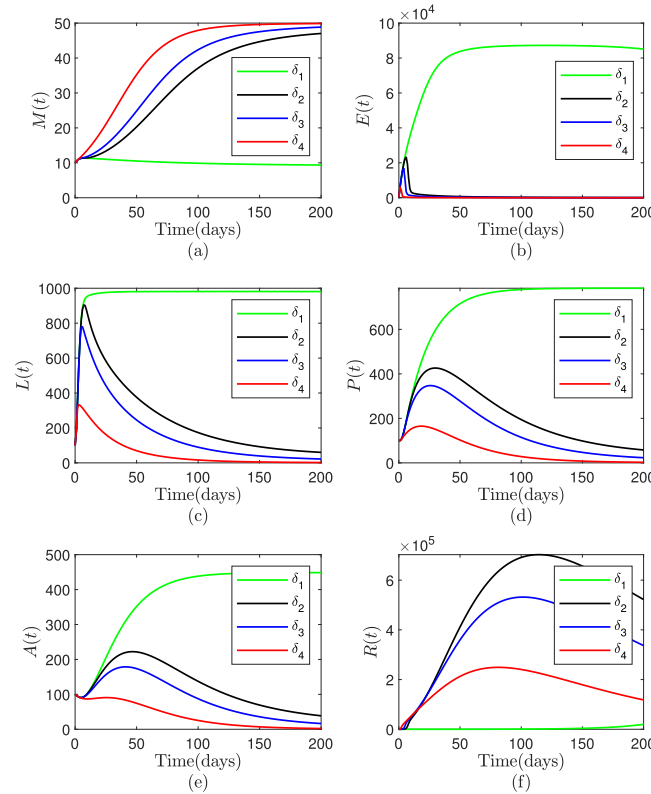


Fig. 4. Impact of varying predator attack rates on eggs, where $\delta_1 = 5 \times 10^{-6}$, $\delta_2 = 5 \times 10^{-5}$, $\delta_3 = 5 \times 10^{-4}$, $\delta_4 = 5 \times 10^{-3}$.

$$B_2 = k_2 \left(\frac{G\alpha_E}{K_L} + m_2 + \sigma\epsilon Q^* - M^* \right). \quad \square$$

3. Global sensitivity analysis

A global sensitivity analysis has been conducted to gain insights into the parameters influencing the dynamics of SF infestation in the presence of predators and the implementation of best farming practices. To identify the key parameters, we combined Latin Hypercube Sampling (LHS) and PRCC multivariate analysis. The results show that parameters h , δ , and σ have the most significant positive associations with maize biomass, indicating that improving these parameter values may potentially increase maize biomass. On the other hand, α_E has the most negative impact on maize biomass when increased. However, the gray zone ranges from -0.3 to 0.3 , indicating that the related input parameter has little or no influence on the output variable (see Fig. 2).

Similarly, it was found that the parameters h , δ , and σ significantly influence the reduction of SF egg and larval populations. This suggests that increasing predator attack rates and best farming practices can effectively reduce SF infestations: see Fig. 3(a)–(b).

4. Results and discussions

In this section, we begin by examining the dynamics of SF in the presence of natural enemies using numerical simulations. The model Eq. (1) is simulated with initial values $M(0) = 10$, $E(0) = 500$, $L(0) = 100$, $P(0) = 100$, $A(0) = 100$, $R(0) = 50$, and $Q(0) = 50$, employing literature based parameter values. Some parameter values fell within

Table 1
Parameter values used in numerical simulation.

Par	Value	Source	Par	Value	Source
a	125 eggs day ⁻¹	[29]	v	0.5	[29,30]
α_E^{-1}	3(2–5) days	[29,30]	α_L^{-1}	14(14–30) days	[29,30]
α_P^{-1}	9(8–30) days	[29,30]	r	0.05 day ⁻¹	[29]
K_M	50 Kg plant ⁻¹	[29]	K_E	10×10^8	[29]
K_L	10×10^6	[29]	μ_E^{-1}	0.09 day ⁻¹	[11]
μ_L^{-1}	0.014 day ⁻¹	[11,13]	μ_P^{-1}	0.01 day ⁻¹	[11,13]
μ_A^{-1}	18(15–21) day ⁻¹	[11]	β	5×10^{-8} day ⁻¹	[11,29]
e	0.2 day ⁻¹	[11,29]	h	0.01 day ⁻¹	[29,31]
η	100 days	[11,29]	γ	100 days	[11,29]
δ	5×10^{-5} day ⁻¹	[11,13]	ϵ	0.1 day ⁻¹	[11,29]
ψ	0.1 day ⁻¹	[11,29]	σ	5×10^{-5} day ⁻¹	[11,13]

credible and acceptable ranges, ensuring the creation of reasonably convincing scenarios. Refer to Table 1 for details.

4.1. Effects of varying the predator attack rate parameters

We present a numerical simulation of the effect of predators on reducing the SF impact on maize biomass. If the predator attack rate on eggs is higher, maize biomass increases with time from its initial quantity to near the expected maize biomass after 100 days (see Fig. 4(a)). Similarly, the impact of the SF population is minimal if the predator attack rate on larvae is higher, due to several factors like habitat management, temperature, attack behavior, and conservation

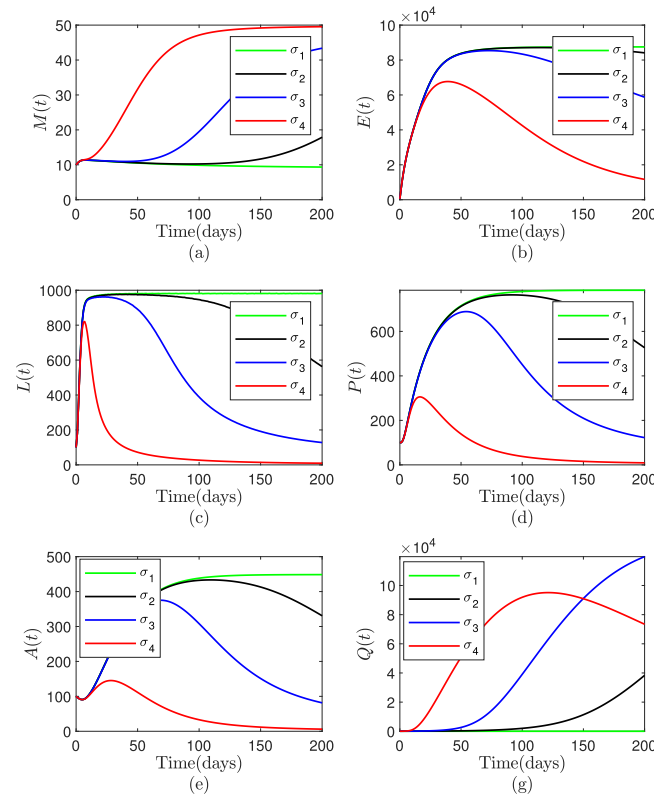


Fig. 5. Impact of varying predator attack rates on larval, where $\sigma_1 = 5 \times 10^{-6}$, $\sigma_2 = 5 \times 10^{-5}$, $\sigma_3 = 5 \times 10^{-4}$, $\sigma_4 = 5 \times 10^{-3}$.

biological control. However, the egg population will not decrease proportionally with time, as illustrated in Figs. 5(a) and (b). This suggests that managing the SF impact is possible if the efficacy of egg and larval predator attack rates are both higher.

4.2. Effects of constant predators and best farming practices

In this section, the model Eq. (1) is simulated as a results of using constant best farming practices effort h and predator interventions. From the simulation in Fig. 6(a)–(g), the effect of multiple predations at constant level (when $h = 0$) are observed. The biomass from maize would be higher from the beginning and converge to 45 units of biomass, which is lower than the expected 50 units of plant biomass. However, the biomass level of maize converges to the anticipated level when best farming practices, including handpicking, intercropping, crop rotations, sanitation, monitoring, and resistant varieties, are applied in the field by 50% (see Fig. 6(a)). Thus, as the level of adoption of best farming practices rises reaching levels nearly 100%, the SF population declines dramatically, and the ultimate maize biomass amounts to the anticipated levels.

5. Conclusion

In this study, a mathematical model incorporating natural enemies and best farming practices was developed. The model is examined to determine how well each strategy performs in lowering SF infestation. To find the most effective influencing parameters, including predator

attack rate and best farming practices that are safe for the environment and human health, a global sensitivity analysis is conducted. Parameter variations illustrated how multiple predator interventions are significant in reducing SF impacts when deployed in the early stages. The simulation outcomes due to the model using constant best farming practices h , showed the efficiency of incorporating best farming practices by 50% or more with multiple predation, which led towards the goal of the expected maize biomass and significantly reduced SF populations. Despite its limitations, this study has shown the value of natural enemies, best farming practices, and efficient management during SF outbreaks. In subsequent studies, we will investigate how migrations, seasonal fluctuations, and optimal control affect the dynamics of SF and how this affects maize biomass.

Declaration of competing interest

The authors declare that they have no conflicts of interest.

Data availability

Data used in this study were found from different similar studies and some were assumed.

Acknowledgments

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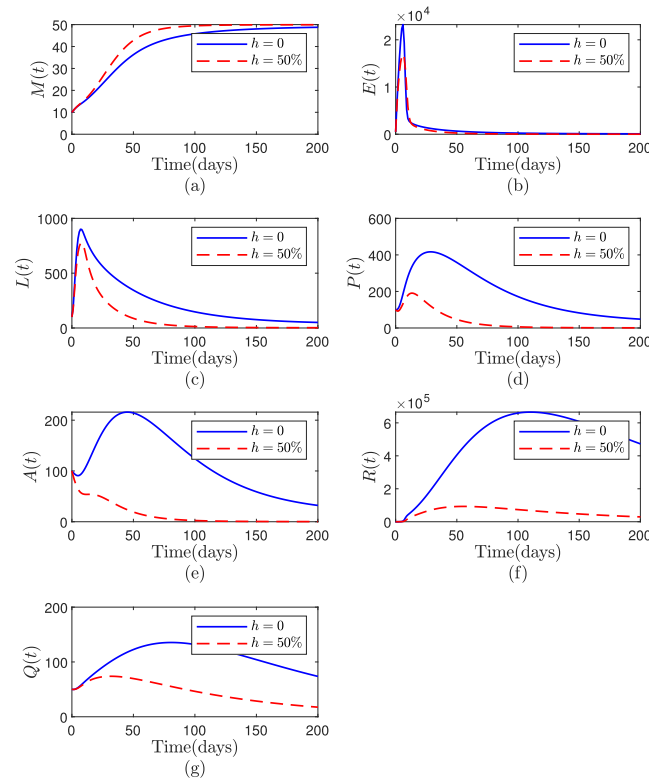


Fig. 6. Simulation results of model Eq. (1) with multiple predators and best farming practices.

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Modeling the dynamics of *Spodoptera frugiperda* infestation in maize production with control strategies

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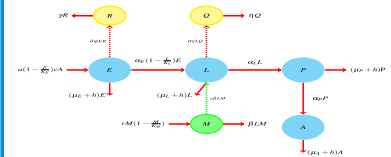
INTRODUCTION

Spodoptera frugiperda (SF), commonly referred to as the fall armyworm, is a highly polyphagous pest, posing a substantial risk to global food security.

This pest exhibits a distinct preference for maize over other crops and pastures, affecting every stage of maize growth with adverse effects on grain yields and quality. In the absence of control measures, these losses could potentially range from US\$2.5 billion to US\$6.2 billion annually. Such significant economic impact could otherwise provide food for 40 to 100 million people.

MODEL FORMULATION

Our model encompasses three distinct populations: Within the SF population, there are four classes: egg (E), larva (L), pupa (P), and adult moth (A). The predators targeting the egg and larval stages are denoted as (R) and (Q), respectively. Furthermore, we represent maize biomass as (M).



The depicted figure illustrates the flow diagram of SF, maize biomass, and predators. Based on this representation, we formulated the following nonlinear Ordinary Differential Equation (ODE) system:

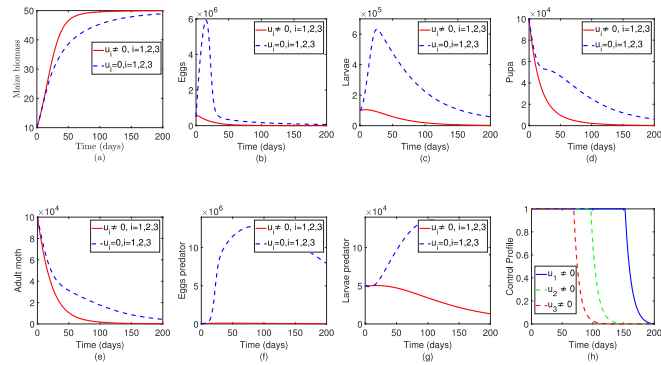
$$\begin{cases} \frac{dM}{dt} = rM \left(1 - \frac{M}{K_M}\right) - \beta LM, \\ \frac{dE}{dt} = a \left(1 - \frac{E}{K_E}\right) vA - (\alpha_E + \mu_E + u_1(t)h)E \\ \quad - (1 + u_3(t))\delta\psi RE, \\ \frac{dL}{dt} = \alpha_E \left(1 - \frac{L}{K_L}\right) E + e\beta ML \\ \quad - (\alpha_L + \mu_L + u_1(t)h)L \\ \quad - (1 + u_2(t))\sigma cQL, \\ \frac{dP}{dt} = \alpha_L L - (\mu_P + \alpha_P + u_1(t)h)P, \\ \frac{dA}{dt} = \alpha_P P - (\mu_A + u_1(t)h)A, \\ \frac{dR}{dt} = (1 + u_3(t))\delta\psi ER - \gamma R, \\ \frac{dQ}{dt} = (1 + u_2(t))\sigma cLQ - \eta Q. \end{cases} \quad (1)$$

with, $M(0) \geq 0$, $E(0) \geq 0$, $L(0) \geq 0$, $P(0) \geq 0$, $A(0) \geq 0$, $R(0) \geq 0$, $Q(0) \geq 0$.

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GRAPHICAL RESULTS



NUMERICAL PART

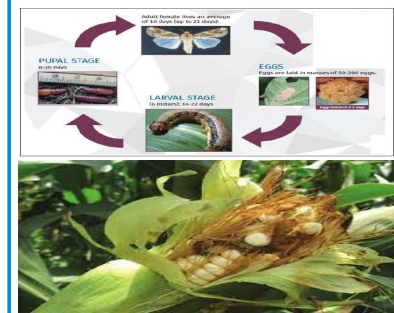
1. The model (1) is simulated with initial conditions $M(0) = 10$, $E(0) = 500$, $L(0) = 100$, $P(0) = 100$, $A(0) = 100$, $R(0) = 50$, and $Q(0) = 50$.
2. Implementing a farming education campaign with selective awareness of predators for eggs and larvae, as shown in Figure(top right), results in a significant decrease in both pest and predator populations when all controls applied early.
3. The study finds that the best farming practices and natural enemies are vital factors in controlling SF. Likewise, utilizing farming practices in the early stages of a season reduces SF by 152 days (76%) and biological controls by 81 days (40.5%). This increase in maize biomass by 48 (96%) occurs within 70 to 80 days.

SF CONTROLS

The benefits of employing farming practices (such as weeding, crop rotation, hand picking, sanitation, soil management, intercropping, and host plant resistance) along with biological controls include:

- (i) SF oviposition can be minimized by intercropping two or more crops or by introducing plants from non-host crops in the field by 80%.
- (ii) Predator utilization proves cost-effective in the long run, despite initial expenses associated with introducing a new species and its environment.
- (iii) Farming methods encourage the growth of natural enemies to effectively control SF.

S. frugiperda LIFE CYCLE



Maize infestation process illustrated: Immature larvae cause damage by feeding on plant material, creating semi-transparent windows. This affects kernel development in older plants, particularly those shielded from the sun.

REMARKS

- (ii) Based on the outcomes of the sensitivity analysis and simulation results, it is clear that the populations of eggs and larvae pose a substantial threat to maize biomass.
- (ii) Collaboration with local communities and indigenous groups to apply farming practices and biological methods.
- (iii) Policy reinforcement on farming education campaigns and biological control methods including PPT technology.
- (iv) We recommend that more efforts be directed to reduce eggs and larval populations and improving farming methods through education campaign.