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Mathematical Modelling and Analysis of Corruption Dynamics with Control Measures in Tanzania

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Abstract. Corruption is a worldwide problem that affects many countries where by individuals loses their rights, lower community confidence in public authorities, absence of peace and security, misallocation of resources and termination of employment. Despite various measures which have been taken by various countries to control corruption, the problem still exists. In this paper, we formulate and analyze a mathematical model for the dynamics of corruption in the presence of control measures. Analysis of the model shows that both Corruption Free Equilibrium (CFE) and Corruption Endemic Equilibrium (CEE) exist. The next generation matrix method was used to compute the effective reproduction number (R_0) which is used to study the corruption dynamics. The results indicate that CFE is both locally and globally asymptotically stable when $R_0 < 1$ whereas CEE is globally asymptotically stable when $R_0 > 1$. The normalized forward sensitivity method was used to describe the most sensitive parameters for the spread of corruption. The most positive sensitive parameters are κ and ν while the most negative sensitive parameters are α and β. Therefore, the parameters of mass education α and religious teaching β are the best parameters for control of corruption. The model was simulated using Runge-Kutta fourth order method in MATLAB and the results indicate that the combination of mass education and religious teaching is effective to corruption control within short time compared to when each control strategy is used separately. Therefore, this study recommends that more efforts in providing both mass education and religious teaching should be applied at the same time to control corruption.

***Keywords**:* Corruption, Mathematical model, effective reproduction number, equilibria, stability, numerical simulation, bribe, eigenvalue.

AMS Mathematics Subject Classification (2010): 91B26

1. Introduction

Corruption can be defined as "abuse of public office for private gain". It involves misappropriation of public authority for personal advantage. Corruption involves bribes, illegal gratitude, conflict of interests, embezzlement of public funds and economic extortion [1, 2]. Corruption can affect small group of people (petty corruption) or affect part or the entire government (grand corruption), individuals may be exposed to corruption slowly and steadily, as a result they turn out to be most corrupt [3, 4]. Corruption is considered as one of the Intimidating factors for the Supportable economic growth, ethical values and justice as it destroys the society and disturb the rule of law [5, 6]. Corruption is a major globally problem nowadays, but the highly affected countries are from low economies mostly in the southern part of Sahara desert [2, 3, 4, 5, 6, 10, 11]. Corruption in the society spread like diseases from one infected (corrupt) person to a susceptible individual. Corruption has a detrimental effect on the development of a country because it degrades national economy as well as international peace and security [8, 9, 12].

Most of the studies on corruption such as [4–7] and [14, 15,16, 17] concluded that corruption mostly undermines the economic, political as well as social development. It is a major cause of poverty around the world, especially in Africa. Corruption in Tanzania has resulted to economic challenges among its people, as the money which would have used to reduce poverty in the country is embezzled by few individuals. The most affected areas are land administration, taxation, and public procurement [18]. Great efforts are being made by the current government to reduce the rate of corruption in the country so as to bring rapid development to the citizens. Therefore, this study intends to determine the dynamics of corruption in Tanzania. To achieve this, compartmental model, based on ordinary differential equations is presented, and solutions determined using fourth order Runge-Kutta method to find the best way to help the government in controlling corruption.

2. Materials and methods

2.1. Model formulation

The corruption dynamics model is developed by modifying the work done by Athithan et al. [19] by including compartment of immune individuals with intervention strategies through combination of mass education and religious teaching. Our corruption model divided the population into four classes/compartments namely: Susceptible individuals, Corrupt individuals, Recovered individuals and Immune individuals. Also the corruption dynamics model is formulated by letting the total population $N(t)$, Susceptible $S(t)$, Corrupt $C(t)$, Reformed $R(t)$ and Immune $I(t)$ as described below:

a) Susceptible Class: These are individuals who have never been involved in corruption but are vulnerable for being influenced by the corrupt practice in the community.
b) Corrupt Class: This class consists of individuals who frequently engage in corrupt practices and are able to influence the susceptible individuals to become corrupt.
c) Recovered Class: This class consists of people who previously engaged in corruption but later on adapted ethical ways. However, they are either susceptible to corruption or immune.
d) Immune Class: These are individuals who can never engage in corruption irrespective of the environments around them.

The susceptible compartment comprises individuals who are obtained through everyday recruitment and who are born with a good conduct and can likely be vulnerable to being infected by corrupt individuals at the rate of $(\kappa+\nu)$ whereas immune compartment consists of individuals who are ethically good from their homes at the rate of $\Lambda(1-\varepsilon)$ and can never engage in corruption practice.

Susceptible individual increases due to birth rate of ε while the immune individual increases due to birth rate of $\Lambda(1-\varepsilon)$. Susceptible individual gets into corruption practice after contact and being convinced by the corrupt individuals at the rate of $(\kappa+\nu)$. The susceptible, corrupt, recovered and immune individuals are all subject to a natural removal rate of μ. The corrupt individuals are recovered at the rate of $(\alpha+\beta+\rho)$. Individuals who are in the recovery compartment due to natural recovery do not get into corrupt compartment rather go to susceptible compartment first at the rate of $(1-\gamma)$ due to human behaviour. γ is the rate at which recovered individuals become immunized. The model for this study is formulated using ordinary differential equations.

2.2. Model assumptions

The assumptions used to formulate the model are listed as follows:

- **a.** Susceptible individuals are equally likely to be corrupt.
- **b.** The corrupt individual compels susceptible individuals into corruption practice as they interact.
- **c.** All model parameters are non-negative.
- **d.** Upon being recovered after some time, individuals can either become susceptible or immune.
- **e.** The corruption spread in the society is analogous to spread of infectious disease.
- **f.** The recruitment rate of a new individual into the susceptible and immune classes is through birth and immigration.

2.3. Corruption dynamics model flow diagram

Figure 1 is a diagrammatic representation of the model for this study showing the corruption flow between the compartments: S- Susceptible, C- Corrupt, R- Recovered and I- Immune.

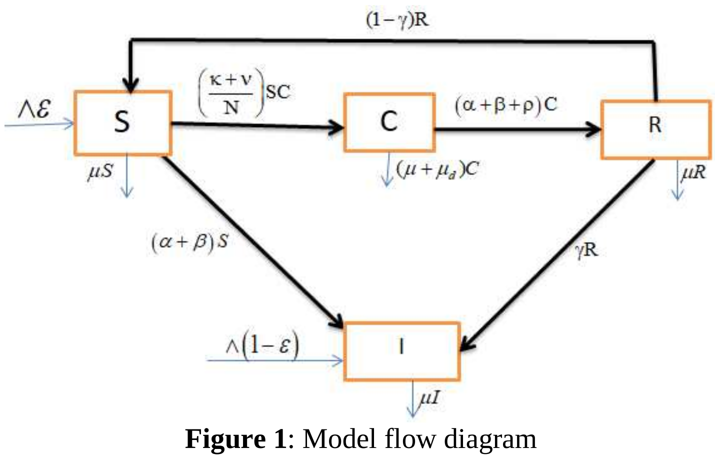

Figure 1: Model flow diagram

2.4. Model equations

From Figure 1 the model's system of differential equations is deduced as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S \\
\frac{dC}{dt} &= \left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C \\
\frac{dR}{dt} &= (\alpha+\beta+\rho)C + (\mu+1)R \\
\frac{dI}{dt} &= \Lambda(1-\varepsilon) + (\alpha+\beta)S + \gamma R - \mu I \\
& S(0) > 0\ ; C(0) > 0\ ;\ R(0) > 0\ ;\ I(0) > 0
\end{aligned} \tag{1}
$$

2.5. Definition of the model variables

Table 1 define the model variables.

Table 1: Description of Model Variables

Symbols	Definition
N(t)	Total population at time t
S(t)	Number of Susceptible individuals at time t
C(t)	Number of Corrupt individuals at time t
R(t)	Number of Recovered individuals at time t
I(t)	Number of Immune individuals at time t

2.6. Definition of the model parameters

Table 2 describe the parameters used in the system of equation (1) as per month.

Table 2: Description of parameters

Parameters	Definition	Values	Source
α	Rate of change of corruption due to Mass education	0.0095	Assumed
β	Rate of change of corruption due to Religious teaching	0.0125	Lemacha &Feyissa (2018)
ρ	Natural recovery rate	0.01	Binuyo (2019)
κ	Rate of change of corruption transmission due to desire (greed-driven).	0.13	Assumed
ν	Rate of change of corruption transmission due to poverty (need-driven)	0.22	TICPI (2017)
ε	Percentage of humans not born immune	0.7	Eguda *et al.* (2017)
Λ	Recruitment number	50	Assumed
μ	Natural removal rate	0.008	Assumed

$1-\gamma$	Rate at which reformed individuals become Susceptible	0.94	Binuyo (2019)
γ	Rate at which reformed individuals become Immune	0.06	Athithan etal(2018)

2.7. Model properties

To test the positivity and boundedness of the model is very crucial. The model is said to be mathematically meaningful if the solutions are positive and they are bounded.

2.7.1. Positivity of the solution

Here we prove that our model system (1) is mathematically meaningful and well posed. From the system of equation (1) above, we need to show that $S(t)$, $C(t)$, $R(t)$ and $I(t)$ are always positive. First, we omit all terms from the equation which do not contain the variable S, C, R and I respectively because our intention is to test for positivity of each compartment.

$$\frac{ds}{dt} \geq -\left(\left(\frac{\kappa+\nu}{N}\right)C + \alpha + \beta + \mu\right)S$$

$$\int \frac{ds}{dt} \geq -\int \left(\left(\frac{\kappa+\nu}{N}\right)C + \alpha + \beta + \mu\right)S$$

$$\int \frac{ds}{S(t)} \geq -\int \left(\left(\frac{\kappa+\nu}{N}\right)C + \alpha + \beta + \mu\right)dt$$

$$lnS(t)|_0^t \geq -\int \left(\left(\frac{\kappa+\nu}{N}\right)C + \alpha + \beta + \mu\right)dt$$

$$ln\,S\,(t) - ln\,S\,(0) \geq -\left[\int_0^t (\mu+\alpha+\beta)dt + \int_0^t \left(\frac{\kappa+\nu}{N}\right)Cdt\right]$$

$$\frac{S(t)}{S(0)} \geq -e^{-(\mu+\alpha+\beta)t} - \int_0^t \left(\frac{\kappa+\nu}{N}\right)Cdt \qquad (2)$$

$$S(t) \geq S(0)e^{-(\mu+\alpha+\beta)t - \int_0^t \left(\frac{\kappa+\nu}{N}\right)Cdt} \geq 0 for \forall_t \geq 0 \qquad \square$$

Similarly, when the same procedure applied in Corrupt, Recovered and Immune compartment, after integrating and simplifying we obtain the following positive solution.

$$C(t) \geq C(0)e^{-(\alpha+\beta+\rho+\mu+\mu_d)t} \geq 0 \; for \; \forall_t \geq 0$$

$$R(t) \geq R(0)e^{-(\mu+1)t} \geq 0 \quad \text{for} \quad \forall_t \geq 0, \; \text{and} \; I(t) \geq I(0)e^{-\mu t} \geq 0 \quad \text{for} \quad \forall_t \geq 0 \quad \square \qquad (3)$$

2.7.2. Boundedness of solution

To check for boundedness of the solution of our model system (1) we need to add all derivatives of the model and simplify it as follows:

$$\frac{dS}{dt} + \frac{dC}{dt} + \frac{dR}{dt} + \frac{dI}{dt} = \varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S +$$

$$\left(\frac{\kappa+\nu}{N}\right)SC-(\mu+\mu_d)C-(\alpha+\beta+\rho)C+(\alpha+\beta+\rho)C+(\mu+1)R+$$
$$\Lambda(1-\varepsilon)+(\alpha+\beta)S+\gamma R-\mu I$$

Simplifying we have

$$\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{N(t)})=\Lambda-\mu\mathrm{N}-\mu_{\mathrm{d}}\mathrm{C}$$

Then we have to find the integrating factor (I.F)

$\frac{dN}{dt}\leq\Lambda-\mu N$

$$\frac{dN}{dt}+\mu N\leq\Lambda$$

Therefore, the integrating factor (I.F) is given by

$$IF=\ell^{\int\mu dt}=\ell^{\mu t}$$

From $\frac{dN}{dt}+\mu N\leq\Lambda$, multiplying both sides by I.F

$$\ell^{\mu t}\left(\frac{dN}{dt}+\mu N\right)\leq(\Lambda)\ell^{\mu t}$$
$$\frac{d}{dt}\left(N\ell^{\mu t}\right)\leq\Lambda\ell^{\mu t}$$
$$\int_0^t Ne^{\mu t}\,dt\leq\int_0^t\Lambda\ell^{\mu t}dt$$

$Ne^{\mu t}\leq\frac{\wedge}{\mu}e^{\mu t}+A$ dividing by $e^{\mu t}$ both sides we have $N(t)\leq\frac{\wedge}{\mu}+Ae^{-\mu t}$

But $N(t)=S(t)+C(t)+R(t)+I(t)$

Now when we solve for A with the initial condition $t=0$ we obtain $N(0)\leq\frac{\wedge}{\mu}+Ae^{0}$

this implies $A\geq N(0)-\frac{\wedge}{\mu}$, and therefore $N(t)\leq\frac{\wedge}{\mu}+\left(N(0)-\frac{\wedge}{\mu}\right)e^{-\mu t}$

$N(t)\leq\max\left\{\frac{\wedge}{\mu},N(0)\right\}$ if $N(0)\leq\frac{\wedge}{\mu}$ means that $N(t)\leq\frac{\wedge}{\mu}$, otherwise $N(0)$ is the maximum boundary of $N(t)$

Therefore $\omega=\left\{(S,C,R,I)\in R_+^4: N(t) if N(0)\leq\frac{\Lambda}{\mu}\right\}$ □ (4)

This shows that solution to model system (1) which start at the boundary of the region ω converge to the region and remain bounded. For this case our model is mathematically meaningful and hence the model can be considered for analysis.

3. Model analysis

In this part, we derive the effective reproduction number, equilibrium states and determine their stability.

3.1. Corruption Free Equilibrium (CFE)

Corruption free equilibrium (CFE) is the state when there is no corruption. In other words, the corruption-free equilibrium is the state in which the population is free of corruption, in which we have only susceptible and immune individuals existing. For this case we get the corruption free equilibrium when the right-hand side (RHS) of the system is set to zero [20]. At this point susceptible and immune individuals are the only population existing, hence we set the system of equation (1) to zero and then we compute it.

$$\varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S = 0,$$

$$\left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C = 0,$$

$$(\alpha+\beta+\rho)C + (\mu+1)R = 0,$$

$$\Lambda(1-\varepsilon) + (\alpha+\beta)S + \gamma R - \mu I = 0$$

Therefore, after solving it we have corruption free equilibrium (CFE) given by

$$CFE = S, C, R\, I = \left(\frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}, 0, 0, \frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)}\right) \quad (5)$$

3.2. Effective reproduction number (R_0)

The reproduction number is the average number of hosts that become corrupted as a result of the entry of one corrupt host into a completely susceptible population in the absence of intervention or control. Effective reproduction number (R_0) is very important for investigating the effect of the control measures. It is used to understand the capability of the corruption to disseminate in the entire community when the control measures are used [21]. For corruption to establish and propagate in the population R_0 must be greater than 1. The foremost component of this ratio includes the transmission probability duration of infectious and the contact pattern between hosts. When there is corruption endemic, R_0 should be greater than 1, and when there is a control of corruption R_0 must be less than 1, while at $R_0 = 1$ shows that the system has reached an endemic equilibrium (steady state). It should be noted that sometimes corruption occurs even when R_0 is less than 1 as a result of random fluctuations in the number of new corrupt individuals generated at any given time instance [19].

For our model we consider the following equation of corrupt humans:

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C$$

Now we take the first term and equate it to f and the remaining terms equal to ν, then we multiply by negative one for the function ν.

Therefore $\boldsymbol{V = (\mu+\mu_d)C + (\alpha+\beta+\rho)C = (\alpha+\beta+\rho+\mu+\mu_d)C}$

$$f = \left(\frac{\kappa+\nu}{S+C+R+I}\right)SC \text{ and } V = (\alpha+\beta+\rho+\mu+\mu_d)C$$

since $N(t) = S(t) + C(t) + R(t) + I(t)$

At the corruption free equilibrium (CFE) $S \neq 0$ & $I \neq 0$, $C = R = 0$,

If $F = \frac{\partial f}{\partial C} = \frac{(\kappa+\upsilon)S}{(S+I)}$ and $V = \frac{\partial v}{\partial C} = (\alpha+\beta+\rho+\mu+\mu_d)$

But from equation (5) $S = \frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}$ and $I = \frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)}$

Therefore $F = \frac{(\kappa+\nu)S}{(S+I)} = \frac{\frac{(\kappa+\nu)\varepsilon\Lambda}{(\alpha+\beta+\mu)}}{\left(\frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}+\frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)}\right)}$

Simplifying we have

$F = \frac{(\kappa+\nu)\varepsilon\mu}{\varepsilon\mu+(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon}$ and $V^{-1} = \frac{1}{(\alpha+\beta+\rho+\mu+\mu_d)}$

Hence, the Next generation matrix given by FV^{-1}which is the same as R_0. Therefore, the effective reproduction number $R_0$is given by:

$$R_0 = FV^{-1} = \frac{(\kappa+\nu)\varepsilon\mu}{(\alpha+\beta+\rho+\mu+\mu_d)[\varepsilon\mu+(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon]}$$

$$R_0 = \frac{(\kappa+\nu)\varepsilon\mu}{(\alpha+\beta+\rho+\mu+\mu_d)(\alpha+\beta+\mu)} \quad (6)$$

From equation (6) it shows that the effective reproduction number (R_0) depends on the infection rate between the rate of change of corruption due to poverty (ν) and rate of change of corruption due to desire (κ). Again when the sum of $(\kappa+\nu)$ are greater than the sum of the rate of change of corruption due to mass education (α) , rate of change of corruption due to religious teaching (β), natural removal rate μ, corruption induced death rate μ_d, then the reproduction number is greater than 1 hence corruption endemic . Note that α and β are the parameters representing effectiveness on the fight of corruption in the country respectively. Therefore according to the model the increase of α and β while keeping other parameter constant is a successful strategy to control corruption since it will reduce R_0 and hence the reproduction number will be less than 1.

3.3. Local stability of corruption free equilibrium points

To check for local stability of equilibrium points we take into consideration all model equations and find the Jacobian matrix that will be used to evaluate whether the equilibrium point is stable or not depending on the sign of the eigenvalues. Therefore, we linearize the model system (1) by computing the Jacobian matrix in the system with respect to the state variable S, C, R, I. If all eigenvalues are negative, then the equilibrium points are stable, otherwise it is unstable. Here we use Jacobian matrix to determine local stability of corruption free equilibrium (CFE) which is obtained by first letting the given four equations as functions $f, g, h,$ and z as follows:

$$\begin{aligned}
f(S,C,R,I) &= \varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S \\
g(S,C,R,I) &= \left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C \\
h(S,C,R,I) &= (\alpha+\beta+\rho)C - (\mu+1)R \\
z(S,C,R,I) &= \Lambda(1-\varepsilon) + (\alpha+\beta)S + \gamma R - \mu I
\end{aligned} \quad (7)$$

where $N(t) = S(t) + C(t) + R(t) + I(t)$, then its Jacobian Matrix (J) at corruption free equilibrium (CFE) is given by

$$J_{CFE} = \begin{pmatrix} \frac{\partial f}{\partial S} & \frac{\partial f}{\partial C} & \frac{\partial f}{\partial R} & \frac{\partial f}{\partial R} \\ \frac{\partial g}{\partial S} & \frac{\partial g}{\partial C} & \frac{\partial g}{\partial R} & \frac{\partial g}{\partial I} \\ \frac{\partial \square}{\partial S} & \frac{\partial \square}{\partial C} & \frac{\partial \square}{\partial R} & \frac{\partial \square}{\partial I} \\ \frac{\partial z}{\partial S} & \frac{\partial z}{\partial C} & \frac{\partial z}{\partial R} & \frac{\partial z}{\partial I} \end{pmatrix} \tag{8}$$

and

$$J_{CFE} = \begin{pmatrix} -(\mu+\alpha+\beta) & -(k+v) & (1-\gamma) & 0 \\ 0 & (k+v)-(\alpha+\beta+\rho+\mu+\mu_d) & 0 & 0 \\ 0 & (\alpha+\beta+\rho) & -(\mu+1) & 0 \\ (\alpha+\beta) & 0 & \gamma & -\mu \end{pmatrix} \tag{9}$$

By observation, the first eigenvalue is $\lambda_1 = -\mu < 0$. Therefore, the reduced Jacobian matrix J^1 at CFE is given by:

$$J^1{}_{CFE} = \begin{pmatrix} -(\mu+\alpha+\beta) & -(\kappa+\nu) & (1-\gamma) \\ 0 & (k+v)-(\alpha+\beta+\rho+\mu+\mu_d) & 0 \\ 0 & (\alpha+\beta+\rho) & -(\mu+1) \end{pmatrix} \tag{10}$$

Also, by observation the second eigenvalue is, $\lambda_2 = -(\mu+\alpha+\beta) < 0$, and hence the reduced Jacobian matrix, J^2 at CFE is given below.

$$J^2{}_{CFE} = \begin{pmatrix} (\kappa+\nu)-(\alpha+\beta+\rho+\mu+\mu_d) & 0 \\ (\alpha+\beta+\rho) & -(\mu+1) \end{pmatrix} \tag{11}$$

Again, through observation the third eigenvalue is $\lambda_3 = -(\mu+1) < 0$ and hence the fourth eigenvalue is $\lambda_4 = (\kappa+\nu)-(\alpha+\beta+\rho+\mu+\mu_d)$.

By considering λ_4 we find that the corruption free equilibrium (CFE) will be asymptotically stable only if $\lambda_4 < 0$, which means $(\kappa+\nu) < (\alpha+\beta+\rho+\mu+\mu_d)$ must hold true for stability of CFE point.

3.4. Global stability of corruption free equilibrium

3.4.1. Lyapunov stability theorem

Let $(x^*, y^*) = (0,0)$ be the equilibrium point of $x' = f(x,y)$ and $V(x,y)$ be continuously differentiable positive definite function in the neighbourhood of the origin [22]. The function $V(x,y)$ is Lyapunov function if the following conditions hold:

a) $V(0,0) = (0,0)$
b) $V(x,y) > 0, \forall\, x,y \in \mu - \{0\}$
c) $V(x,y) \leq 0, \forall\, x,y \in \mu - \{0\}$ (12)
d) $V(x,y) < 0$ then $V(x,y)$ is strictly Lyapunov

There are different ways to solve global stability by Lyapunov function but for the purpose of this study we shall use the LaSalle's Invariance Principle [23].

Let $V=\frac{1}{2}C^2$ and $\frac{dV}{dt}=\frac{\partial V}{\partial C}*\frac{\partial C}{\partial t}$ chain rule

From $V=\frac{1}{2}C^2$ then $\frac{dV}{dC}=\frac{1}{2}*2C=C$ and from the system of equation (1) above $\frac{dC}{dt}=(\frac{\kappa+\nu}{N})SC-(\mu+\mu_d)C-(\alpha+\beta+\rho)C$

$\therefore \frac{dV}{dt}=\frac{\partial V}{\partial C}*\frac{\partial C}{\partial t}$

$$\frac{dV}{dt}=C^2\left[(\kappa+\nu)\frac{S}{N}-(\alpha+\beta+\rho+\mu+\mu_d)\right]$$

From $R_0=\frac{(\kappa+\nu)\varepsilon\mu}{(\alpha+\beta+\rho+\mu+\mu_d)(\alpha+\beta+\mu)}$ and making $(\alpha+\beta+\rho+\mu+\mu_d)$ the subject we have $(\alpha+\beta+\rho+\mu+\mu_d)=\frac{(\kappa+\nu)\varepsilon\mu}{R_0(\alpha+\beta+\mu)}$

Therefore $\frac{dV}{dt}=C^2\left[(\kappa+\nu)\frac{S}{N}-\frac{(\kappa+\nu)\varepsilon\mu}{R_0(\alpha+\beta+\mu)}\right]$

$$\frac{dV}{dt}\leq C^2(\kappa+\nu)\left[1-\frac{\varepsilon\mu}{R_0(\alpha+\beta+\mu)}\right]$$

From equation (5) at corruption free equilibrium $S=\frac{\varepsilon\Lambda}{\alpha+\beta+\mu}$ which implies $\frac{\varepsilon}{(\alpha+\beta+\mu)}=\frac{S}{\Lambda}$ then

$$\frac{dV}{dt}\leq C^2(\kappa+\nu)\left[1-\frac{S\mu}{\Lambda R_0}\right]$$

If $\frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}<\Lambda$ then $\frac{dV}{dt}\leq C^2(\kappa+\nu)\left[1-\frac{1}{R_0}\right]$, hence $\frac{dV}{dt}\leq 0$ when $R_0\leq 1$

Therefore, corruption free equilibrium is globally asymptotically stable if $R_0<1$

3.5. Existence of Corruption Endemic Equilibrium

The corruption endemic equilibrium is the stable state solution of the corruption transmission model where corruption persists in the population and all compartments are positive. Therefore here we need to set the given equations equal to zero so that we solve for S, C, R , and I , this means:

$$\varepsilon\Lambda+(1-\gamma)R-\left(\frac{\kappa+\nu}{N}\right)SC-(\alpha+\beta)S-\mu S=0 \quad \text{(i)}$$

$$\left(\frac{\kappa+\nu}{N}\right)SC-(\mu+\mu_d)C-(\alpha+\beta+\rho)C=0 \quad \text{(ii)}$$

$$(\alpha+\beta+\rho)C+(\mu+1)R=0 \quad \text{(iii)}$$

$$\Lambda(1-\varepsilon)+(\alpha+\beta)S+\gamma R-\mu I=0 \quad \text{(iv)}$$

From equation (iii) we get the value of C such that

$$(\alpha+\beta+\rho)C-\mu R-R+\gamma R-\gamma R=0$$

$$(\alpha+\beta+\rho)C-(\mu+1)R=0$$

$$\therefore C^{*}=\frac{(\mu+1)R}{(\alpha+\beta+\rho)}$$

Now we need to check for equation (i) and substituting $C^{*}=\frac{(\mu+1)R}{(\alpha+\beta+\rho)}$

We have

$$\left(\frac{\kappa+\nu}{N}\right)S\frac{(\mu+1)R}{(\alpha+\beta+\rho)}-(\alpha+\beta+\rho)\frac{(\mu+1)R}{(\alpha+\beta+\rho)}-(\mu+\mu_{d})\frac{(\mu+1)R}{(\alpha+\beta+\rho)}=0$$

In order to simplify computational, we first let $B=\left(\frac{\kappa+\nu}{N}\right)$ now

$$BS\frac{(\mu+1)R}{(\alpha+\beta+\rho)}-(\alpha+\beta+\rho)\frac{(\mu+1)R}{(\alpha+\beta+\rho)}-(\mu+\mu_{d})\frac{(\mu+1)R}{(\alpha+\beta+\rho)}=0$$

$$\therefore \quad S^{*}=\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B},$$

where $\boldsymbol{B}=\left(\frac{\boldsymbol{\kappa}+\boldsymbol{\nu}}{\boldsymbol{N}}\right)$

Now, since we have the values of S^{*} and C^{*}we use equation (i) to get R^{*}, therefore

$\varepsilon\Lambda+(1-\gamma)R-\left(\frac{\kappa+\nu}{N}\right)SC-(\alpha+\beta)S-\mu S=0$ but $C^{*}=\left(\frac{(\mu+1)}{(\alpha+\beta+\rho)}\right)$

and $S^{*}=\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B}$ hence

$$\varepsilon\Lambda+(1-\gamma)R-B\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B}\frac{(\mu+1)R}{(\alpha+\beta+\rho)}-(\alpha+\beta)\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B}-\mu\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B}=0$$

Let $M=\alpha+\beta+\rho$ and $N=\mu+\mu_{d}$

Simplifying we have $R^{*}=\frac{M\left((\alpha+\beta)(M+N)+\mu(M+N)\right)}{B((M-M\gamma)-(M+N)(u+1))}-\varepsilon\Lambda$

Now, to get I we substitute S and R into equation (iv)

$\Lambda(1-\varepsilon)+(\alpha+\beta)S+\gamma R-\mu I=0$ we have

$$\Lambda(1-\varepsilon)+(\alpha+\beta)\frac{(\alpha+\beta+\rho)+(\mu+\mu_{d})}{B}+\gamma\frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))}-(\varepsilon\Lambda+\mu I)=0$$

$$I^* = \frac{\Lambda}{\mu}(1-\varepsilon) + (\alpha+\beta)\frac{(\alpha+\beta+\rho+\mu+\mu_d)}{B} - \gamma\frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(\mu+1))} - \varepsilon\Lambda$$

Generally, when $R_0 > 1$ then our model system (1) above will experience the corruption endemic equilibrium point at which $E^* = \left(S^*, C^*, R^*, I^*\right)$

$$S^* = \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B}$$

$$C^* = \frac{(\mu+1)R}{(\alpha+\beta+\rho)} = \frac{(\mu+1)}{(\alpha+\beta+\rho)} * \left(\frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda\right)$$

$$R^* = \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda$$

$$I^* = \frac{1}{\mu}\left(\Lambda(1-\varepsilon) + (\alpha+\beta)\frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} + \gamma\frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda\right)$$

But $M = (\alpha+\beta+\rho)$, $N = (\mu+\mu_d)$ and $B = \left(\frac{\kappa+\nu}{N}\right)$

3.6. Global stability of corruption endemic equilibrium point

If $R_0 > 1$ therefore the corruption-endemic equilibrium, E^* of the system of equation (1) is globally asymptotically stable in ω when $\frac{S}{S^*} = \frac{C}{C^*}$ [3].

Proof, by considering the Lyapunov function:

$$U = a_1\left(S - S^* - S^* \ln\frac{S}{S^*}\right) + a_2\left(C - C^* - C^* \ln\frac{C}{C^*}\right) + a_3\left(R - R^* - R^* \ln\frac{R}{R^*}\right) + a_4\left(I - I^* - I^* \ln\frac{I}{I^*}\right)$$

Now we take the derivatives along the solutions of the model equations as follows:

$$U' = a_1\left(1-\frac{S^*}{S}\right)S' + a_2\left(1-\frac{C^*}{C}\right)C' + a_3\left(1-\frac{R^*}{R}\right)R' + a_4\left(1-\frac{I^*}{I}\right)I' \tag{13}$$

But

$$S' = \varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S,$$

$$C' = \left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C$$

$$R' = (\alpha+\beta+\rho)C + (\mu+1)R,$$

$$I' = \Lambda(1-\varepsilon) + (\alpha+\beta)S + \gamma R - \mu I$$

Then we substitute these derivatives into U' of equation (13)

$$U' = a_1(1-\frac{S^*}{S})(\varepsilon\Lambda + (1-\gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\beta)S - \mu S)$$
$$+ a_2(1-\frac{C^*}{C})(\left(\frac{\kappa+\nu}{N}\right)SC - (\mu+\mu_d)C - (\alpha+\beta+\rho)C)$$
$$+a_3(1-\frac{R^*}{R})((\alpha+\beta+\rho)C + (\mu+1)R)$$
$$+a_4(1-\frac{I^*}{I})(\grave{U}(1-\varepsilon) + (\alpha+\beta)S + \gamma R - \mu I)$$

At endemic equilibrium, taking into consideration of E^* and $a_i > 0$ for $i = 1, 2, 3, 4$ in ω and its constants a_i are continuous and differentiable in ω and $U(\omega^*) = 0$ for $\omega = (S^*, C^*, R^*, I^*)$. Global stability for endemic equilibrium holds if $U' \leq 0$, therefore at endemic equilibrium we have the following:

$$U' = a_1(1-\frac{S^*}{S})\left[(\frac{k+v}{N})S^*C^* + \mu S^* + (\alpha+\beta)S^* - (\frac{k+v}{N})SC - \mu S - (\alpha+\beta)S\right]$$
$$+ a_2(1-\frac{C^*}{C})\left[(\alpha+\beta+\rho)C^* + (\mu+\mu_d)C^* - (\alpha+\beta+\rho)C - (\mu+\mu_d)C\right] \quad (14)$$
$$+ a_3(1-\frac{R^*}{R})\left[(\mu+1)R^* - (\mu+1)R\right] + a_4(1-\frac{I^*}{I})\left[(\mu I^* - \mu I)\right]$$

By rearranging the terms in equation (14) leads to the following:

$$U' = a_1(1-\frac{S^*}{S})\left[(\frac{\kappa+\nu}{N})(SC - S^*C^*) + (\alpha+\beta)(S-S^*) + \mu(S-S^*)\right]$$
$$+ a_2(1-\frac{C^*}{C})\left[(\alpha+\beta+\rho)(C-C^*) + (\mu+\mu_d)(C-C^*)\right]$$
$$+ a_3(1-\frac{R^*}{R})\left[(\mu+1)(R-R^*)\right] + a_4(1-\frac{I^*}{I})\left[\mu(I-I^*)\right]$$

By simplifying the system above leads into the following equations:

$$U' = a_1(1-\frac{S^*}{S})\left[(\frac{\kappa+\nu}{N})SC(1-\frac{S^*}{S}) + (\alpha+\beta)S(1-\frac{S^*}{S}) + \mu S(1-\frac{S^*}{S})\right] +$$

$$a_2(1-\frac{C^*}{C})((\alpha+\beta+\rho)C(1-\frac{C^*}{C}) + (\mu+\mu_d)C(1-\frac{C^*}{C})) +$$

$$a_3(1-\frac{R^*}{R})\left[(\mu+1)R(1-\frac{R^*}{R})\right] + a_4(1-\frac{I^*}{I})\left((\mu I(1-\frac{I^*}{I})\right)$$

$$= a_1(\frac{S-S^*}{S})\left[(\frac{\kappa+\nu}{N})(S^*C^* - SC) + (\alpha+\beta)(S^* - S) + \mu(S^* - S)\right] +$$

$$a_2(\frac{C-C^*}{C})\left[(\mu+\mu_d)(C-C^*) + (\alpha+\beta+\rho)(C-C^*)\right] +$$

$$a_3(\frac{R-R^*}{R})\left[(\mu+1)(R^* - R)\right] + a_4(\frac{I-I^*}{I})\left[\mu(I^* - I)\right]$$

Now

$$U' = a_1(\frac{S-S^*}{S})\left[-(\frac{\kappa+\nu}{N})SC(1-\frac{S^*C*}{SC}) - (\alpha+\beta)S(1-\frac{S^*}{S}) - \mu S(1-\frac{S^*}{S})\right] +$$
$$a_2(\frac{C-C^*}{C})\left[-(\mu+\mu_d)C(1-\frac{C^*}{C}) + (\alpha+\beta+\rho)C(1-\frac{C^*}{C})\right] +$$
$$a_3(\frac{R-R^*}{R})\left[-(\mu+1)R(1-\frac{R^*}{R})\right] + a_4(\frac{I-I^*}{I})\left[-\mu I(1-\frac{I^*}{I})\right]$$

Therefore

$$U' = a_1(\frac{S-S^*}{S})\left[-(\frac{\kappa+\nu}{N})SC(\frac{SC-S^*C^*}{SC}) - (\alpha+\beta)S(\frac{S-S^*}{S}) - \mu S(\frac{S-S^*}{S})\right]$$
$$+$$
$$a_2(\frac{C-C^*}{C})\left[-(\mu+\mu_d)C(\frac{C-C^*}{C}) - (\alpha+\beta+\rho)C(\frac{C-C^*}{C})\right] +$$
$$a_3(\frac{R-R^*}{R})\left[-(\mu+1)R(\frac{R-R^*}{R})\right] + a_4(\frac{I-I^*}{I})\left[-\mu I(\frac{I-I^*}{I})\right] \quad (15)$$

By opening brackets, we have the following:

$$U' = -a_1(\alpha+\beta)S\left(\frac{S-S^*}{S}\right)^2 - a_1\mu S\left(\frac{S-S^*}{S}\right)^2 - a_2(\mu+\mu_d)C\left(\frac{C-C^*}{C}\right)^2$$

$$-a_2(\alpha+\beta+\rho)C\left(\frac{C-C^*}{C}\right)^2 - a_2(\alpha+\beta+\rho)C\left(\frac{C-C^*}{C}\right)^2$$

$$-a_3(\mu+1)(\frac{R-R^*}{R}) - a_4\mu I\left(\frac{I-I^*}{I}\right)^2 + F(\omega)$$

$$U' = -a_1(\frac{\kappa+\nu}{N})\frac{(S-S^*)(SC-S^*C^*)}{S} - a_1(\alpha+\beta)S\left(\frac{S-S^*}{S}\right)^2 - a_1\mu S\left(\frac{S-S^*}{S}\right)^2$$

$$-a_2(\mu+\mu_d)C\left(\frac{C-C^*}{C}\right)^2 - a_2(\alpha+\beta+\rho)C\left(\frac{C-C}{C}\right)^2$$

$$-a_3(\mu+1)R\left(\frac{R-R^*}{R}\right)^2 - a_4\mu I\left(\frac{I-I^*}{I}\right)^2$$

Note that all parameters are greater or equal to zero and all state variables $S, C, R, I, N \geq 0$

Also, $\left(\frac{S-S^*}{S}\right)^2 \geq 0$, $\left(\frac{C-C^*}{C}\right)^2 \geq 0$, $\left(\frac{R-R^*}{R}\right)^2 \geq 0$, $\left(\frac{I-I^*}{I}\right)^2 \geq 0$

Therefore

$$U' = -a_1(\alpha+\beta)S\left(\frac{S-S}{S}\right)^2 - a_1\mu S\left(\frac{S-S^*}{S}\right)^2 - a_2(\mu+\mu_d)C\left(\frac{C-C^*}{C}\right)^2$$

$$a_2(\alpha+\beta+\rho)C\left(\frac{C-C^*}{C}\right)^2 - a_2(\alpha+\beta+\rho)C\left(\frac{C-C^*}{C}\right)^2 \tag{16}$$

$$-a_3(\mu+1)R\left(\frac{R-R^*}{R}\right)^2 - a_4\mu I\left(\frac{I-I^*}{I}\right)^2 + F(\omega)$$

where $F(\omega) = -a_1(\frac{\kappa+\nu}{N})\frac{(S-S^*)(SC-S^*C^*)}{S}$ □

From La Salle Invariance Principle $F(\omega) \leq 0$ and hence $U' \leq 0$ as required

Therefore, corruption endemic equilibrium is stable if $R_0 > 1$.

3.7. The effective reproduction number sensitivity analysis and their analytical interpretation

Sensitivity analysis is the process performed when we want to determine the influence of each model parameter in the effective reproduction number R_0 [25, 26]. Thus, this process helps in the selection of corruption control measures in which the most sensitive parameters are seriously considered. This study used normalized forward sensitivity index to show the sensitivity of the model parameter in (1).

If R_0 is differentiable with respect to its parameter z, therefore its sensitivity index is given by

$$T_z^{R_0} = \frac{\partial R_0}{\partial z} x \frac{z}{R_0} \tag{20}$$

Since R_0 in equation (6) is differentiable to all its parameter, hence we apply equation (20) to compute the sensitivity indices of our model parameters by using the values in Table (2). By conducting partial derivatives of the effective reproduction number with respect to the parameters of this model and its interpretation is provided. Our basic reproduction number depends on eight parameters which are used to derive the analytical expression for every parameter. Therefore the partial derivatives of the effective reproduction number R_0 with respect to the parameters α, β, ρ, κ, ν, μ, μ_d and ε of the model conducted by the formula: $T_z^{R_0} = \frac{\partial R_0}{\partial z} x \frac{z}{R_0}$

From the derivative of R_0 with respect to model parameter we see that there are some values of the derivative of R_0 which are less than 0 and other values are greater than 0. Analytically it indicates that all values of R_0 which are less than 0 are the important factors for controlling corruption in the country. Therefore, increasing these parameters becomes the most control strategy of corruption in the country. Considering the most negative sensitivity indices which are α, β and μ_d. From Table 3, corruption induced death rate μ_d is the most negative sensitivity parameter but we will not recommend as one of the controls of corruption because it is against human right and unethical to increase corruption induced death rate [26]. Therefore, we consider the rest two parameters of mass education α and religious teaching β. Table 3 and Figure 2 below show the sensitive parameter values and its graph respectively.

Table 3: Sensitivity indices

Parameter	Sensitivity index
α	-0.4222
β	-0.5556
ρ	- 0.1111
μ	0. 6444
μ_d	-0.5556
ε	1.0000
κ	0.3714
ν	0.6286

From sensitivity indices in Table 3 we note that all positive indices indicates the proportional relationship with effective reproduction number R_0, every percentage increase in the parameters with positive indices will make also the same percentage increase in the effective reproduction number R_0 and therefore corruption prevalence. The parameters with most positive indices are rate of change of corruption due to birth rate and rate of change of corruption due to poverty ν. This shows that as the poverty level increases in the country the more people involve in corruption so as to speed up their personal development and meet their basic needs, therefore the government should make sure that strategies for reduction of poverty in the country are highly emphasized.

On other side, the negative indices show inverse relationship with effective reproduction number R_0. The percentage increase in parameters with the negative indices will make the same reduction of percentage of effective reproduction number R_0 [24]. From Table 3 the parameters with the most negative sensitivity indices are rate of change of corruption due to mass education α and rate of change of corruption due to religious teaching β and corruption induced death rate μ_d. For this case we also consider only α and β as to increase corruption induced death rate μ_d is unethical.

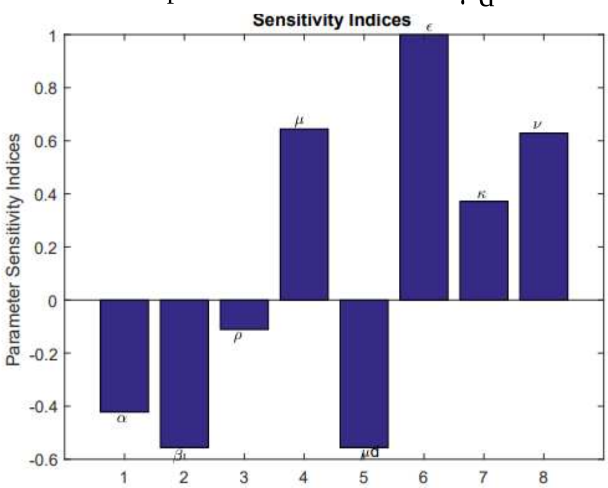

Figure 2: Sensitivity Indices for the model parameters

From Figure 2 it shows that parameters α, β and μ_d are most negative sensitive to the fight of corruption in the country this indicates that the more you increase these parameters the more you control the corruption in the country. Again when we consider the parameter κ and ν we find that these are the most positive influential of corruption in the country, therefore in the war to fight corruption we need to pay attention with these parameters as the more we increase the more the reproduction number increases which lead to high prevalence of corruption in the country. Therefore κ and ν must be fully controlled.

4. Numerical simulation and results

4.1. Numerical simulation of the model without control measures.

The simulation started by considering sensitivity to the parameters and concluded by including control strategies. Therefore, in this section it has been shown the dynamics of corruption with control strategies using the numerical simulation of the proposed model (1). Our model is simulated using the parameters value shown in Table 1. First the model is simulated when the control strategies are set to zero that is $\alpha = 0$ and $\beta = 0$ while values of other parameters are as stated in Table 1. The results are shown in Figure 1.4.

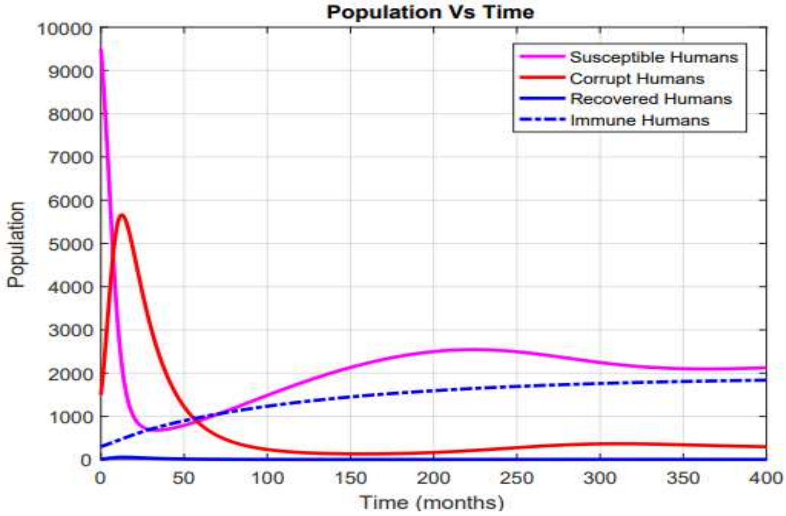

Figure 3: Corruption dynamics without control measures

From Figure 3 indicates that susceptible humans decreases asymptotically at the beginning due to high corruption contact rate of $(\kappa+\nu)$ and the remove rate of μ, then later increases due to birth rate of ε and the recovery rate of $(1-\gamma)$. Also, From the graph the number of corrupt humans increases at the very beginning up to when $t = 10$ due to increase rate of desire and poverty in the society which lead into most of humans to engage in corruption practices. After $t = 10$ the number of corrupt individuals decreases due to the fact that some individuals undergo corruption induced death rate and others are recovered due to natural recovery (self-change) of individuals.

Also figure 1.4 shows that immune individuals at the beginning increases due to birth rate of $(1-\varepsilon)$ and after $t = 350$the number of immune individuals remain constant without change. Recovered individuals from $t = 0$ up to $t = 10$ increases due to natural recovery and thereafter become zero throughout due to fact that some individuals goes to immune class at the rate of γ and others goes to susceptible class at the rate of $(1-\gamma)$ as a result the recovered class remain without individuals as from assumption that at the recovered class there is a temporally immunity. Corruption-free equilibrium state is given by:

$$CFE = (S, C, R, I) = \left(\frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}, 0, 0, \frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu) + (\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)}\right)$$

and the effective reproduction number $R_0 = \frac{(\kappa+\nu)\varepsilon\mu}{(\alpha+\beta+\rho+\mu+\mu_d)(\alpha+\beta+\mu)}$

4.2. Effects of mass education against corruption

In this part the model simulated to check the effect of mass education with the parameter value shown in Table 1.

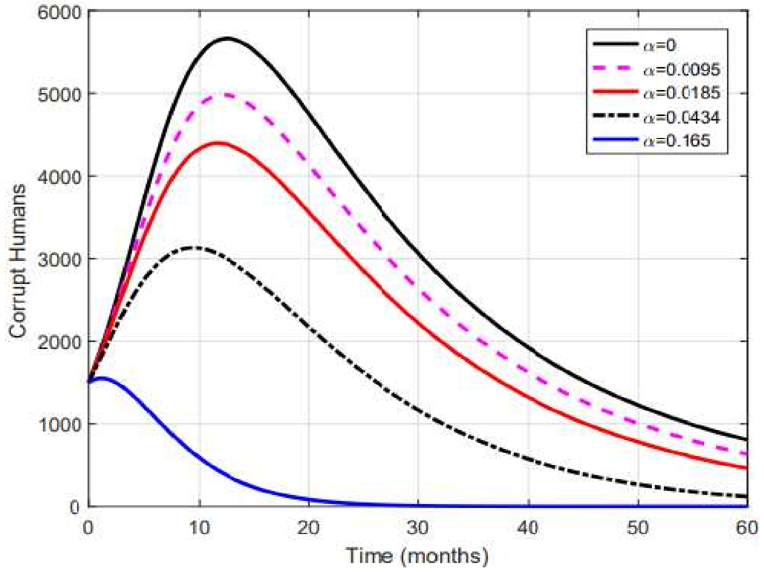

Figure 4: Number of corrupt humans with variation rate of mass education

From figure 4 it shows that the higher the rate of provision of education to the community concerning the effect of corruption to the development, the lower the number of corrupts humans in the community, this indicates that the government could invest more in providing education to the people about the negative impacts of corruption. The curriculum developers can be required to include corruption studies in the curriculum of all level of education in the country so that everyone will be aware of it. As we see at $t = 22$corruption decline which is almost after 2 years since its implementation.

4.3. Effects of religious teaching against corruption

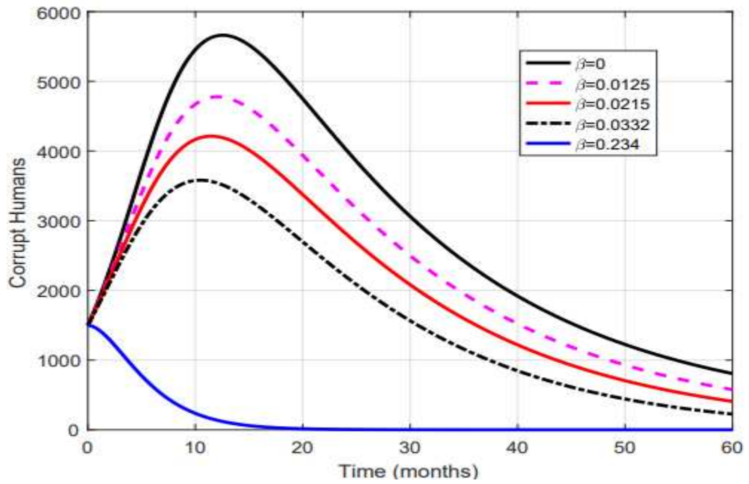

Figure 5: Number of corrupt humans with varying rate of religious teaching, in absence of mass education.

Considering figure 5 it shows that the higher the rate of provision of teaching through religious leaders to their followers about corruption as a vice, the number of corrupt humans in the community decline. The model assumed that most followers trust their religious leaders for this case we need to involve much religious leaders in the war of fighting corruption in the country. From the graph it indicates that corruption decline from $t = 16$.

4.4. Effects of combining mass education and religious teaching against corruption

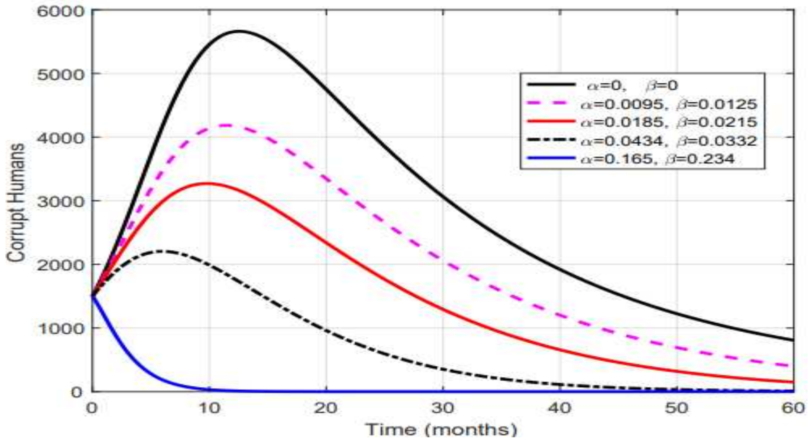

Figure 6: Number of corrupt humans by varying both Mass education and Religious teaching

From Figure 6 it shows that both mass education and religious teaching have significant impact on the community since when you increase their rates, the corruption tend to decrease rapidly compared to when mass education and religious teaching has been used separately as can be seen from figure 4 and figure 5 respectively. It should be noted that when mass education used as a single control measure the corruption decline at $t = 22$ whereas when religious used as a single control measure the corruption decline at $t = 16$ this indicates that religious teaching is more effective compared to mass education. It should be noted from figure 6 that when mass education and religious teaching are combined together as control measure, they tend to reduce corruption immediately at $t = 9$ which bring more success compared to a single control. From these results it shows that the government should invest more in giving education to the citizen as well as to use the religious leaders to teach their followers not to engage in corruption as it is a vice to engage in corruption. For this case the education curriculum developers should incorporate corruption studies in the education system from pre-school to Universities as serious topic to every Tanzanian in order to create awareness to everyone in the country.

5. Conclusion

This paper of corruption dynamics will positively add efforts to the current initiative established by the government of the United Republic of Tanzania to address the issue of corruption and way forward to control the prevalence of it in the country. From the analytical analysis of effective reproduction number (R_0) it shows that the effective reproduction number R_0 is directly affected by the parameters α ,β ,κ ,ρ μ, μ_d, κ and ν. The parameters κ and ν measures the force of infection of transmission rate from susceptible individuals to corrupt individuals, this means that the higher the corruption contact rate the higher the prevalence of corruption in the country since the effective reproduction number will be greater than 1. Whereas if parameters α and β which represent mass education and religious teaching respectively such that when these parameters increases while other parameters remain constant the basic reproduction number will be less than 1 as a result there will be a control of corruption in the community. For these results it suggests that there is a need to invest more efforts in mass education to the citizens and increase more emphasize to religious leaders to teach their followers effectively about corruption as it is against their faith and doctrines.

The effective reproduction number R_0 was obtained and the analysis indicates that for $R_0 < 1$ the corruption-free equilibrium is globally asymptotically stable. Due to the nature of corruption it is a bit difficult to eradicate it completely but can be reduced at a reasonable level which does not have harmful impact to the economic development of the country. Also, it should be noted that for whatever circumstances if $R_0 > 1$ the corruption free equilibrium point is unstable and endemic equilibrium persist. Again, it is well known that it is possible to reduce corruption in the country by reducing the level of poverty; realistic increase of the public sector wages to make sure the public servants do not get into temptation of involving in corruption during provision of services to the citizens. Great efforts have been made by the Prevention and Combating of Corruption Bureau (PCCB) to fight corruption in Tanzania, however the government should increase more effort in provision of education to all citizens and incorporate more religious leaders in the fight of corruption since most of Tanzanians are religious and respect very well their religious leaders. This paper recommends the inclusion of corruption studies in education curriculum as independent topic from pre-school to Universities so that citizens will be well informed about the impact of corruption to social, political and economic development of the country; all government and non-government offices should have surveillance cameras to deter any corrupt activities within offices during provision of services to the citizens; equal distribution of resources to the citizens in all regions in the country; improving working environment and Religious leaders should teach well their followers the impact of corruption. For the future work other researchers should check the following:

- To perform a cost benefit analysis of the suggested corruption control measures and;
- To determine the proper time to implement the control strategies.

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