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Corruption dynamics in Tanzania: Modeling the effects of control strategies

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**CORRUPTION DYNAMICS IN TANZANIA: MODELING THE
EFFECTS OF CONTROL STRATEGIES**

Oscar Danford

**A Dissertation Submitted in Partial Fulfilment of the Requirements for the Degree of
Master's in Mathematical and Computer Science and Engineering of the Nelson
Mandela African Institution of Science and Technology**

Arusha, Tanzania

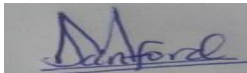
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ABSTRACT

Corruption can be defined generally as taking bribes, forcing out benefit, receiving money or gifts as favour in exchange for doing one's job. Worldwide corruption is considered a problem that affects many countries in the world currently. Individuals loses their rights, lower community confidence in public authorities, termination of employment, absence of peace and security and misallocation of resources. Despite various measures which have been taken by different countries to control corruption, the problem still persists. The negative impacts of corruption are well known; however, little has been done on mathematical modelling on corruption, particularly the inclusion of mass education and religious teaching as a strategy for controlling corruption spread. In this study, the mathematical model for the dynamics of corruption in the absence and presence of these control measures are presented and analysed to determine which parameter are very sensitive to the spread of corruption and how will these control strategies help to reduce corruption. Most parameters used in this work are from different literature and some were assumed. The next generation matrix method is used to compute the basic and effective reproduction number which used to study the corruption dynamics. Sensitivity analysis results shows that, parameters of mass education and religious teaching are the most sensitive to the control of corruption. Also, stability analysis for equilibrium states by linearization, Lyapunov function and use of the Lassalle's invariance principle was derived and the results shows that the corruption free equilibrium is locally and globally asymptotically stable when $R_0 < 1$ and endemic equilibrium is globally asymptotically stable when $R_0 > 1$. The model was simulated by using Runge-Kutta 4th order in MATLAB and the results depict that when mass education α and religious teaching β are combined together as a control measures, corruption within the country is successfully controlled in a short period of time. The study recommends to invest more in provision of mass education to the citizens through creating general awareness to all and including it in education curriculum from pre-primary to university as well as to use religious leaders to teach their followers seriously about the impact of corruption. The numerical simulation results agreed with the analytical results.


DECLARATION

I, Oscar Danford, do hereby declare to the senate of Nelson Mandela African Institution of Science and Technology that this dissertation is my own original and it has neither been submitted nor presented for similar award in any other institution.

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CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the Nelson Mandela African Institution of Science and Technology the dissertation entitled: Corruption dynamics in Tanzania: Modelling the Effects of Control Strategies, in fulfilment of the requirements for the degree of Masters in Mathematical and Computer Science and Engineering of the Nelson Mandela African Institution of Science and Technology.

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DEDICATION

I dedicate this work to Danford's family.

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LIST OF ABBREVIATIONS AND SYMBOLS

AfDB	African Development Bank
CFE	Corruption Free Equilibrium
CoCSE	Computational and Communication Science and Engineering
CPI	Corruption Perception Index
DRC	Democratic Republic of Congo
EE	Endemic Equilibrium
NM-AIST	The Nelson Mandela Institution of Science and Technology
ODE	Ordinary Differential Equation
OECD	Organization for Economic Co-operation and Development
PCB	Prevention of Corruption Bureau
PCCB	Prevention and Combating Corruption Bureau
R_0	Basic Reproduction Number
R_e	Effective Reproduction Number
RHS	Right Hand Side
SCRI	Susceptible, Corrupt, Recovered and Immune
TANESCO	Tanzania Electricity and Supply Company

CHAPTER ONE

INTRODUCTION

1.1 Background of the Problem

According to Hill (2006) corruption means, abuse of public office for private gain. Corruption involves bribes, illegal gratitude, conflict of interests, embezzlement of public funds and economic extortion (Nathan & Jakob, 2019). Corruption can affect small group of people (petty corruption) or affect part or the entire government (grand corruption), individuals may be exposed to corruption then slowly and steadily, they turn out to be most corrupt (Abdulrahman, 2014) Most of the studies on corruption such as (Cherrier, 2009; Rose-Ackerman, 1975; Gould & Amaro-Reyes, 1983; Sobel, 2008; Murphy, 2000; & Klitgaard, 1991) in their studies concerning corruption, politics and development they summarised that corruption mostly distort the social, political and economic development. It is believed that corruption is one of the major sources of poverty around the world, especially in Africa (Gebeye *et al.*, 2012).

Corruption seen to be one of the threatening factors for the sustainable economic development, ethical values and justice, it destroys the society and distract the rule of law (Muhammad *et al.*, 2012). Corruption is among of the very critical issues in the world today, (Bevir & Letki, 2012; Jucá *et al.*, 2016).

The problem of overcoming corruption issues involved social scientists, policy makers and philosophers since the age of Aristotle, historically some countries seemed to have been able to carry out substantial reductions of corruption but perceived levels of corruption remain too high throughout the world (Uslaner & Rothstein, 2013). Some recent studies have defined corruption as a behaviour that is opposite to ethical universalism in the exercise of public power, means that turning public goods into their own benefit. Corruption has a detrimental effect on the development of a country; it is pervasive and exists, with varying degrees in every country in the world. Below is the world map indicating the prevalence of corruption in the world by the year 2019.

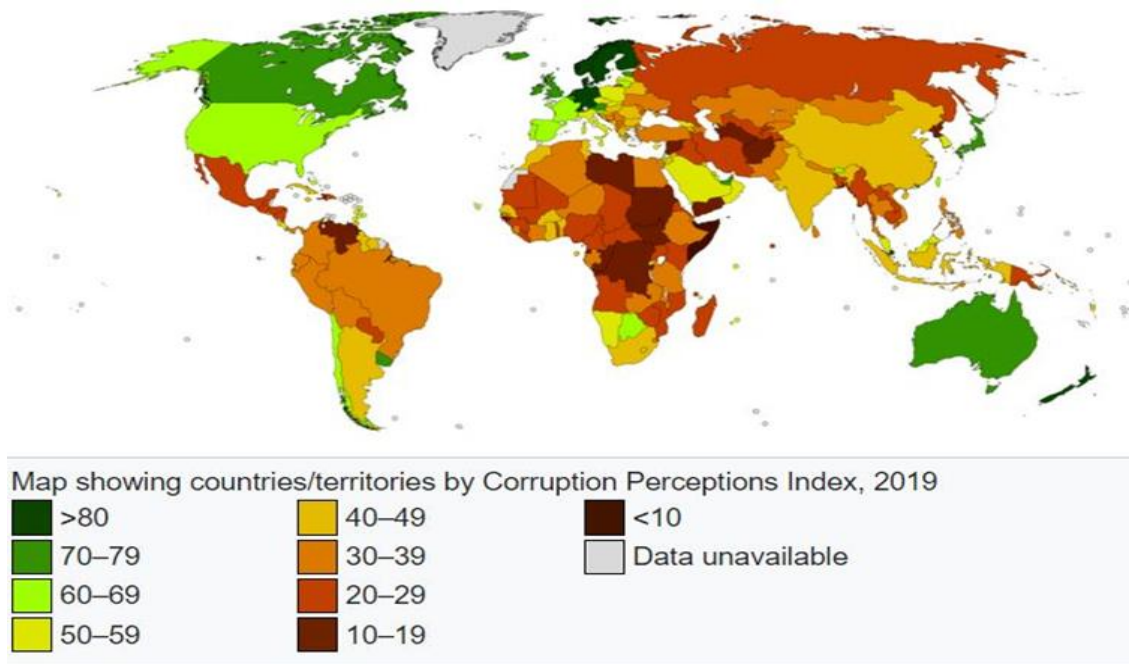


Figure 1: Rate of Corruption World Wide 2019

https://en.wikipedia.org/wiki/Corruption_Perceptions_Index

According to Index (2019) presented in Fig. 1, it shows that scores are on the scale of 0-100, where 0 means that a country is highly affected with corruption practices, thus, a country with index 9 is more corrupt than a country with index 60. Furthermore, the report shows that the world's most corrupt continent is Africa. Corruption is a major reason accelerated and contributed to the stagnation of development and prevailing poverty of many African countries. For example, countries considered most corrupt in Africa are notably: South Sudan, Libya, Somalia, DRC, Burundi and North Sudan just few to mention. Also, the three at least clean countries in Africa with the CPI score enclosed in brackets are Botswana (61), Rwanda (53) and Namibia (52) whereas in other continents the top seven clean countries are Denmark (87), New Zealand (87), Finland (86), Singapore (85), Switzerland (85), Sweden (85) and Norway (84).

According to Schoeman (2006) in the African Union study, it is known that the issue of corruption undermines the African continent and causes a loss of about 150 billion annually. Economists argue that African governments required to fight corruption rather than relying on foreign assistance. But the initiative and efforts from anti-corruption on the continent have shown some encouraging outcome in the recent years, though some analysis showed that there is fear that the international partners are unwilling to exert and invest over African countries.

Tanzania's population is still impoverished, observers refer to corruption as a major contributor to unequal development as well as the role of development assistance with lax fiduciary controls in providing low-risk rent-seeking possibilities to the ruling elite and sectors of the private sector (Cooksey, 2011).

Tanzania's attempts to combat corruption, according to reports, remain critical to the country's ambitions to eradicate poverty and achieve equitable growth (Xinhua, 2017a). Poverty, high living costs, poor wages, and severe workloads are all seen as important barriers to overcoming corruption by Tanzanians (Camargo, 2017).

When the fifth President Magufuli, came to power, citizens praised him for his tough stance against corruption, which he earned throughout his tenure as the former public works minister and his general anti-graft tactics (Allison, 2015). Since then, he has transferred cash from expensive state banquets to hospitals in need, fired 16 000 government ghost workers, and prohibited all government officials except the president, vice president, and prime minister from traveling abroad (Allison, 2015; Robi, 2016).

Corruption of the highest order especially from the government procurement, privatization processes, election finance, taxation, and customs clearance are all affected by grand corruption in Tanzania (Kilimwiko, 2019). The country had been faced with several corruption scandals before Magufuli entered power in 2015, particularly between 2005 and 2015.

Mr. Msahaba, the former Minister of Energy, and his successor, Mr. Nazir Karamagi, as well as former Prime Minister Edward Lowassa, are said to have been the main political supporters of Richmond (Gray, 2015). When allegations about problems in the tendering process arose, the PCCB found no evidence of corruption; but, due to Richmond's non-performance, a legislative commission was appointed to investigate the subject (Gray, 2015; Cooksey, 2017).

The Corruption Perception Index, 2020, evaluates 180 nations and territories according to their perceived levels of public sector corruption, based on 13 expert assessments and business executive polls. It employs a 0 to 100 scale, with zero representing the most corrupt. A score of less than 50 implies that the public sector is rife with corruption. Rwanda was classified as the least corrupt country in East Africa, with 54 points, followed by Tanzania with 38, Kenya with 31, Uganda with 27, and Burundi with 19, putting Tanzania at 92 out of 180 countries in Africa. Corruption has been seen as a significant impediment to economic growth, effective government, and fundamental freedoms.

According to Bevir and Letki (2012) corruption in Tanzania is prevalent and is among the challenging issue across most of the sectors of the economy despite the great effort made by the government and PCCB to control it. The sectors affected much are taxation, government procurement, land administration and customs. Also, corruption seemed to be an obstacle to the principles of democracy, good governance and human rights and hence accelerate danger to the peace and security in the society.

Andrzej (2019) formulated a mathematical model on corruption using game theory which imitated a mechanism of corruption and cooperation patterns. The model discussed the modelling approach of prisoner dilemma to understand the collapse of cooperation and increase of corruption in post-communistic countries. The building a cooperative society involves moral education, values, norms of the society, post-communism transformation led to protection of political elites with implementation democratic procedures with huge mistrust levels among citizens. The results of the study indicate that bureaucratic rules, licenses, lack of transparency and mistrust lead to the occurrence of defecting and corruption.

Despite of the great efforts made by the government of Tanzania to prevent and combat corruption in the form of anti-corruption policies and strategies spearheaded by PCCB, the problem still exists (Controller *et al.*, 2017). Currently this is the national issue that need to be addressed seriously mostly in sub-Sahara Africa in order to bring rapid development (Parrish *et al.*, 2002).

1.1.1 Mathematical Models

Mathematical modelling is the relationship between the rest of the world and mathematical world. Below is Fig. 2 showing the translation from a real-world problem to mathematical world (Waykar, 2020).

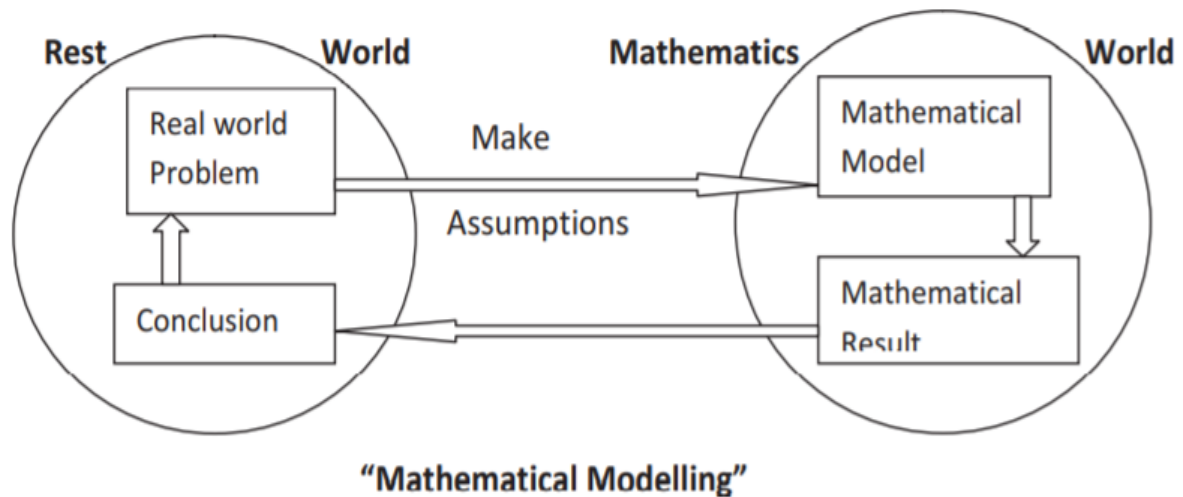


Figure 2: Mathematical Flow Diagram involving Imagination and Skills

Mathematical models can play an important role in understanding the dynamics of corruption which in turn can help in planning, implementing and evaluating for making the right decision on how to control it. Different mathematical studies have been done to understand the causes and control of corruption worldwide and proposed some of ways to control corruption (Athithan *et al.*, 2018a; Nathan & Jakob, 2019; Binuyo, 2019; Ullah *et al.*, 2012; Escalante & Odehna, 2020; Eguda *et al.*, 2017, Obsu & Science, 2019), investigated the causes and control of corruption dynamics by considering how corruption spread and how to control it.

Ullah *et al.* (2012) developed an initial system dynamics model to study the hidden dynamics of corruption and the analysis showed that the level of corruption can decline by increasing incentives towards honest behaviour, increasing public sector salaries, and effective check and balances among public officials. Furthermore, in their mathematical studies they found that corruption is the current threatening issue for both developing as well as developed countries and recommended that the best ways to control corruption is to give punishment to the victim, enforcing of law, the use of technology and transferring the victim from one working station to another. Most of these studies have not considered the combination of mass education and religious teaching as control against vice. This study intends to formulate a mathematical model for corruption transmission dynamics taking into consideration the combination of mass education and religious teaching to curb the corruption.

1.2 Statement of the Problem

In Tanzania, the issue of corruption is a very serious problem though many initiatives and efforts such as anti-corruption policies are being done to control corruption in the country. The

government of the United Republic of Tanzania has been fighting against corruption since the early independence, and the efforts have been increased in the last thirteen years by adaptation of new and effective anti-corruption techniques (United Republic of Tanzania [URT], 2018). In order to deal with the problem of corruption, the government of the United Republic of Tanzania has strengthened and improved the Prevention and Combating of Corruption Bureau (PCCB) department by educating the citizen on the negative impact of corruption to the growth of the economy of the country (Olesuya, 2018).

Despite great efforts that have been done by the first president up to the current president of the United Republic of Tanzania and all its control authorities to reduce corruption but the problem still exists. Since corruption is prevalent in our country, understanding the transmission dynamics of corruption and designing appropriate measures for its control is very important for the development of our country. The problem brought by corruption are well known; however, little has been done on mathematical modelling on corruption, especially the inclusion of mass education and religious teaching as the control measure of corruption. Due to existence of this problem in the country this research is conducted to study the corruption dynamics by taking into consideration the combination of mass education and religious teaching as the control measures.

1.3 Rationale of the Study

Corruption is a form of dishonesty conducted by a person or group of people to acquire illicit benefit for personal interest. Corruption blinds those who see and twist the words of the innocent. Corruption is one of the causes of under development among different countries in the world today. For this case there is a need to study issues of corruption as it affects the standard of living of the people. In order to control the problem of corruption there is a necessity of understanding its transmission dynamics. The mathematical models played an important role in understanding the transmission dynamics of corruption and drawing the correct decision on the corruption control approaches.

Mathematical models assist in the planning, implementing and evaluating the corruption detection, combat and prevention measures. The negative impacts of corruption to the community are well known, however, few studies have been conducted on mathematical modelling concerning corruption particularly the inclusion of mass education and religious teaching as the strategy in combating the spread of corruption. Therefore, this study intended

to develop the mathematical model on corruption taking into consideration the combination of mass education and religious teaching which will create the awareness to the stake holders on the corruption transmission dynamics and its control.

1.4 Research Objectives

1.4.1 General Objective

The main objective of this study is to develop and analyze the mathematical model for corruption dynamics with control strategies in Tanzania.

1.4.2 Specific Objectives

To accomplish this study, the following specific objectives are carried out:

- (i) To formulate a mathematical model for corruption dynamics in absence and presence of control strategies.
- (ii) To determine the existence and stability of equilibrium points.
- (iii) To compute reproduction number and determine the parameters that drive corruption.
- (iv) To assess the impact of corruption control strategies.

1.5 Research Questions

The following questions intended to be answered by this research:

- (i) How will the dynamics of corruption be mathematically formulated?
- (ii) What is the criteria for stability and existence of equilibrium points?
- (iii) How can reproduction number be computed? What are the mostly sensitive parameters of the model?
- (iv) Which control parameters have significant impact on corruption control?

1.6 The Significance of the Study

The study of corruption dynamics is very important as it will benefit the following stakeholders:

- (i) Policy makers to apply the correct intervention strategies to reduce or prevent corruption in the country.
- (ii) Provision of mathematical understanding framework for researchers in the applied mathematics to use the concept of mathematics through modelling in controlling corruption in the country.

1.7 Delineation of the Study

Modelling the corruption transmission dynamics is a wide and sensitive area of study. This study intended not to cover each and everything on the transmission dynamics of corruption problem. Rather, it concentrated on the modelling the transmission dynamics of corruption taking into consideration how corruption spread in the society and its control. Furthermore, this study does not include all the control measures used to control the problem of corruption in the country. Rather, it used only the combination of mass education and religious teaching as the corruption control measures.

CHAPTER TWO

LITERATURE REVIEW

2.1 Mathematical Model for Corruption Dynamics

Different mathematical studies have been done to understand the causes and effects of corruption worldwide and proposed some of ways by investigating the causes, spread, effects and control of corruption dynamics. Abdulrahman (2014) formulated and analysed mathematical model by considering constant recruitment rate and the transmission dynamics of corruption which considered as a disease, the results indicates that when $R_0 < 1$, corruption-free equilibrium is globally asymptotically stable noting that naturally corruption cannot allow for the total eradication, but it can be reduced slowly to a reasonable degree and concluded that there is necessary to apply the model to a country of interest incorporating high Anti-Corruption Feasible Reforms (HAPR), which involves public awareness on the danger and consequence of corruption, transparency and accountability.

Maslii *et al.* (2018) developed a model to study the intellectual innovative information technology for preventing the bribes and corruption. The intellectual innovative information technology uses statistical data, logical and probabilistic risk models and knowledge base. The study shows that, without technology scientists will fail to overcome the problem of controlling bribes and corruption in the society. The study concluded that corruption can be detected and controlled by the use of technology, system approach and application of anticorruption platforms.

Shah and Yeolekar (2017) formulated and analysed the mathematical model for dynamics of corruption by using a non-linear differential equation with four different compartments on Susceptible, Exposed, Infected and Punished. The model considered punishment in terms of transfer of the victim from one working station to another as among of the best way to control corruption. The threshold in terms of reproduction number was computed to make society corruption free and the local and global stability was analysed. The result showed that the use of punishment should be a must to help the decline of corruption in the society.

Musa (2020) formulated mathematical model for controlling the spread of corruption through social media, in their study they treated the spread of corruption as an infection that spreads through social interaction. The model analyzed with constant recruitment rate, incorporating

standard incidence rate, effort rate against corruption through social media. Findings indicate that if at least 40% of the corrupt individuals are embarrassed on social media, corruption can be significantly controlled. It further concluded that this will not be a short time effects but will take years to be achieved.

Eguda *et al.* (2017) developed and analysed mathematical model to control the dynamics of corruption and suggested techniques or methods to prevent the wide spread of corruption in the community. The result showed that the equilibrium free state is locally asymptotically stable since the reproduction number was less than 1 whereas corruption endemic equilibrium exist and is locally asymptotic stable for $R_0 > 1$.

Lemacha (2018) developed and analysed mathematical model to investigate the dynamical nature of corruption governing the mathematical model with constant recruitment rate from the total population. The developed corruption mathematical model taken into consideration the awareness created by anti-corruption and counselling in jail. Corruption free equilibrium was proved to be locally asymptotically stable whenever reproduction number is less than one. The analytical results were verified using numerical simulation and they recommended that the future work should base on the designing optimal strategy.

Afrobarometer (2017) conducted a round 7 survey in Tanzania for the selected number of the citizens who were asked questions whether the corruption rate in the country is: Increasing? Decreasing? Stayed the same? or don't know anything? The results of the survey were recorded and shown well in Fig.3 below by including the results taken in the year 2014.

Figure 3 indicates that the citizen are aware about the presence of corruption in the country and that they are aware on the measures taken by the government as we can see that about 13% of respondents in 2014 agreed that corruption is decreasing while in 2017 about 72% of the respondents agreed that the corruption rate in the country is decreasing as the measures taken by the government and its authorities (PCCB) to control it are seen to them. For the results in Fig. 3 it shows that the issue of corruption in the country is existing and this encourage researchers to find the solution for this problem.

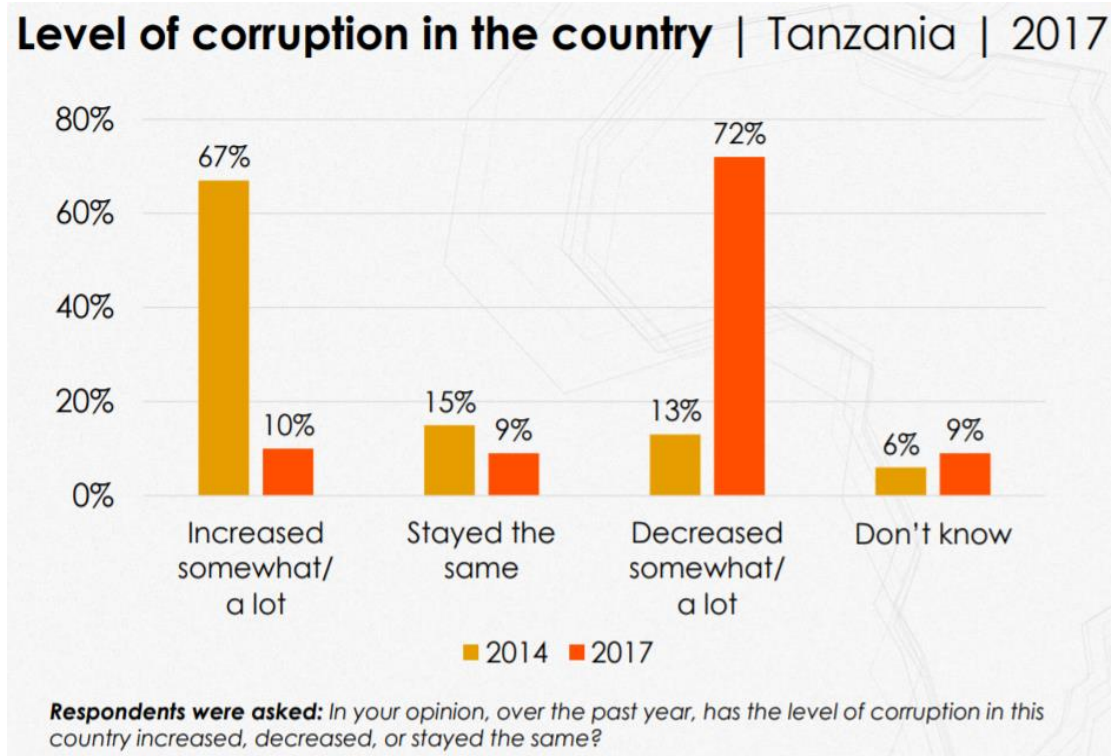


Figure 3: Perceived level of corruption in Tanzania in 2017

According to Bevir and Letki (2012) and Pring and Vrushi (2019) who conducted a research to measure the level of corruption on 180 countries with its territories, drawing a survey of business people and specialist on assessments. The score index is considered from zero scale (highly corrupt) to one hundred (very clean), results indicates that more than two-third of countries score below 50, while the average score was 43 as corruption continue to go largely unchecked, democracy is under threat around world due to corruption.

Trend of Corruption and transformation of PCB-PCCB in Tanganyika /Tanzania from 1958 to 2018 is attached with this paper to show the trend and transformation of PCCB and Corruption itself and measures taken by the government. Refer Appendix1 as part of literature review extracted from the review of the Performance of Tanzania’s Prevention and Combating of Corruption Bureau, 2007-16. Also refer Appendix 2 for action taken for corrupt individuals.

According to the literatures review seen, there is no study has used the combination of mass education and religious teaching as a control measure for corruption dynamics. These variables were considered in this study to analyse their impact on corruption control.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Materials and Methods

The model of this research consists of Susceptible, Corrupt, Reformed and Immune classes (SCRI). Susceptible class are recruited from birth and migration. The model is formulated using ordinary differential equations. The MATLAB Software has been used for numerical simulation and analysis, whereas as MATHEMATICA and MAPLE 18 software used for computational of equilibrium points. Table1 below shows the summary of objectives and the respective methods for accomplishing each objective.

Table 1: Summary of Specific Objectives and Methods

S/No	Specific Objectives	Methods
i	To formulate a mathematical model for corruption dynamics in absence and presence of control strategies.	The ordinary differential equations are used to formulate the corruption dynamics model
ii	To determine the existence and stability of equilibrium points.	Linearization method by Jacobian Matrix for local stability of corruption free equilibrium and Lyapunov function for global stability of equilibrium point
iii	To compute reproduction number and determine the parameters that drive corruption.	Next Generation Matrix and Normalized forward sensitivity index
iv	To assess the impact of corruption control strategies.	Numerical simulation using MATLAB Software

3.2 Basic Model Development

The corruption dynamics model is developed by extending the model which was formulated by Athithan *et al.* (2018) to include the combination of mass education and religious teaching. The Corruption basic model divided the population into three compartments which are Susceptible, Corrupt and Reformed individuals. Corruption dynamics model is formulated by

letting the total population be $N(t)$, Susceptible class $S(t)$, Corrupt Class $C(t)$ and Reformed Class $R(t)$.

Susceptible individuals increase due to birth rate of Λ and later decreases due to high interaction rate caused by force of infection at the rate of $(\kappa + \nu)$ and the removal rate of μS . Susceptible individuals get into corruption practices due to convincing power from corrupt humans. Corrupt humans decrease due to natural recovery rate and induced death rate of ρ and $(\mu + \mu_d)$ respectively.

Corruption induced death is the death of the corrupt individuals caused by either of the following: (a) Pressure caused by heart beat for fearing their future decision from the government (b) Shameful, that means they feel shameful to the society to be known as corrupt (c) Commit suicide, others they take a decision to lost their life for fearing the consequence of corruption cases; (d) discreditable in front of the society, as human being need to be respectable, thus to maintain their status they decide to lost their life.

At the beginning recovered individuals increases at the rate of ρ but later decreases since at the recovery compartment there is a temporally stay meaning that after short interval of time individuals leave the compartment at the rate of ϕ to susceptible humans and others undergo removal rate of μR as illustrated in Fig. 4.

3.2.1 Model Assumptions

The assumptions used to formulate the model are listed as follows:

- (i) Susceptible individuals are equally likely to be corrupt.
- (ii) The corrupt individuals compel susceptible individuals into corruption practice as they interact.
- (iii) All model parameters are non-negative.
- (iv) Upon being recovered, individuals can either become susceptible or immune after some time.
- (v) The corruption spread in the society is analogous to spread of infectious disease.
- (vi) The recruitment rate of a new individual into the susceptible and immune classes is through birth and immigration.

3.2.2 Basic Model Variable Description

Below is Table 2 explaining the variables used in this research:

Table 2: Description of Model Variable

Variable	Definition
$N(t)$	Total population at time t
$S(t)$	Susceptible individuals at time t
$C(t)$	Corrupt individuals at time t
$R(t)$	Recovered individuals at time t

3.2.3 Description of Basic Model Parameters

The parameters for model (1)- (3) are described in Table 3

Table 3: Parameters Descriptions

Parameters	Description
ρ	Natural recovery rate
κ	Rate of change of corruption transmission due to desire (greed-driven).
ν	Rate of change of corruption transmission due to poverty (need-driven)
Λ	Recruitment rate due to birth
μ	Natural removal rate
ϕ	Rate at which recovered individuals become Susceptible
μ_d	Corruption induced death rate

3.2.4 Model Flow Diagram

The model flow diagram is illustrated in Fig. 4

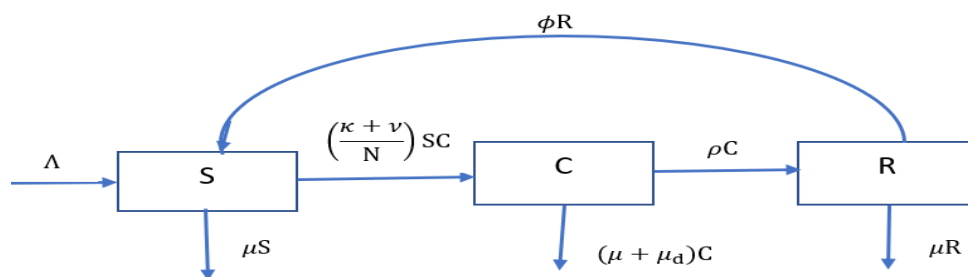


Figure 4: Flow Diagram of the Basic Compartmental Model for Corruption Dynamics

From the basic compartment in Fig. 4, the solid lines show the movement of humans from one compartment to another.

3.2.5 Basic Model Equations

From the compartmental diagram in Fig. 4 the formulated basic model's Equations system of differential equations shown as follows:

$$\frac{dS}{dt} = \Lambda + \phi R - \left(\frac{\kappa+\nu}{N}\right) SC - \mu S \quad (1)$$

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu}{N}\right) SC - (\rho + \mu + \mu_d)C \quad (2)$$

$$\frac{dR}{dt} = \rho C - (\phi + \mu)R \quad (3)$$

$$S > 0, C \geq 0, R \geq 0$$

3.2.6 Basic Model Properties

In this part we determine whether the model is mathematically meaningful by first checking if it has positive solutions which are bounded in a given region. The next part determines the invariant region (boundedness) and positivity of the solution.

(i) Boundedness of the Solutions

To check for boundedness of the solution of the model needed to add all of the model Equations as below:

$$\begin{aligned} \frac{dS}{dt} + \frac{dC}{dt} + \frac{dR}{dt} &= \Lambda + \phi R - \left(\frac{\kappa+\nu}{N}\right) SC - \mu S + \left(\frac{\kappa+\nu}{N}\right) SC - (\rho + \mu + \mu_d)C + \rho C - (\phi + \mu)R \\ &= \Lambda - \mu S - \mu C - \mu R - \mu_d C \\ &= \Lambda - (S + C + R)\mu - \mu_d C \\ \frac{d}{dt}(S + C + R) &= \Lambda - \mu N(t) - \mu_d C \\ \frac{dN}{dt} &\leq \Lambda - \mu N(t) \end{aligned} \quad (4)$$

Hence the integrating factor (I.F) is given by:

$$\text{I.F} = e^{\int \mu t} = e^{\mu t} \quad (5)$$

Then, when $e^{\mu t}$ is multiplied to both side of $\frac{dN}{dt} + \mu N \leq \Lambda$, we have the followings:

$$e^{\mu t} \left(\frac{dN}{dt} + \mu N \right) \leq \Lambda e^{\mu t}$$

$$\int_0^t \frac{d}{dt} (N e^{\mu t}) \leq \int_0^t \Lambda e^{\mu t}$$

$$(N e^{\mu t}) \leq \int_0^t \Lambda e^{\mu t} dt$$

$(N e^{\mu t}) \leq \frac{\Lambda}{\mu} e^{\mu t} + B$, by dividing by $e^{\mu t}$ both sides we have

$$N(t) \leq \frac{\Lambda}{\mu} + B e^{-\mu t} \text{ but } N(t) = S(t) + C(t) + R(t)$$

Solving for B with initial condition $t = 0$ we obtain:

$$N(0) \leq \frac{\Lambda}{\mu} + B \text{ this means } B \geq N(0) - \frac{\Lambda}{\mu}, \text{ therefore } N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu} \right) e^{-\mu t}$$

Now $N(t) \leq \max \left\{ \frac{\Lambda}{\mu}, N(0) \right\}$ if $N(0) \leq \frac{\Lambda}{\mu}$, otherwise $N(0)$ is the maximum boundary of $N(t)$.

$$\text{Therefore } \Omega = \left\{ (S, C, R) \in R_+^3 : N(t) \leq \frac{\Lambda}{\mu} \right\} \quad (6)$$

(ii) Positivity of Solutions

Here the model proved if it is mathematically meaningful and well posed. This conducted by looking for the positivity of the model solution so that there is no negative solution to the model variables.

From Equation (1), what required is to show that S (t) is always positive, by first omitting all terms from the Equation which do not contain the variable S because our intention is to test for positivity of only S (t) as follows:

$$\frac{ds}{dt} \geq - \left(\frac{\kappa + \nu}{N} \right) SC - \mu S$$

$$\int \frac{ds}{dt} \geq - \int \left(\left(\frac{\kappa + \nu}{N} \right) C - \mu \right) S$$

$$\int \frac{ds}{S(t)} \geq - \int \left(\left(\frac{\kappa + \nu}{N} \right) C + \mu \right) dt$$

$$\ln S(t) \Big|_0^t \geq - \int \left(\left(\frac{\kappa + \nu}{N} \right) C + \mu \right) dt$$

$$\ln S(t) - \ln S(0) \geq - \left[\int_0^t \mu dt + \int_0^t \left(\frac{\kappa + \nu}{N} \right) C dt \right]$$

$$\begin{aligned} \ln \frac{S(t)}{S(0)} &\geq -\mu t - \int_0^t \left(\frac{\kappa + \nu}{N} \right) C dt \\ S(t) &\geq S(0) e^{-\mu t - \int_0^t \left(\frac{\kappa + \nu}{N} \right) C dt} \geq 0 \quad \text{for } \forall t \geq 0 \end{aligned} \quad (7)$$

Consider Equation (2) corrupt humans, therefore:

$$\begin{aligned} \frac{dC}{dt} &\geq -(\rho + \mu + \mu_d)C \\ \int_0^t \frac{dC}{C(t)} &\geq -\int_0^t (\rho + \mu + \mu_d) dt \\ \ln C(t) \Big|_0^t &\geq -[\rho + \mu + \mu_d]t \\ \ln C(t) - \ln C(0) &\geq -[\rho + \mu + \mu_d]t \\ \ln \left(\frac{C(t)}{C(0)} \right) &\geq -[\rho + \mu + \mu_d]t \\ C(t) &\geq C(0) e^{-(\rho + \mu + \mu_d)t} \geq 0 \quad \text{for } \forall t \geq 0 \end{aligned} \quad (8)$$

Again, considering Equation (3) of recovered humans above we have to integrate by separating the variables, therefore:

$$\begin{aligned} \frac{dR}{dt} &\geq -(\phi + \mu)R \\ \int_0^t \frac{dR}{R(t)} &\geq -\int_0^t (\phi + \mu) dt \\ \ln R(t) \Big|_0^t &\geq -(\phi + \mu)t \\ \ln R(t) - \ln R(0) &\geq -(\phi + \mu)t \\ \ln \left(\frac{R(t)}{R(0)} \right) &\geq -(\phi + \mu)t \\ R(t) &\geq R(0) e^{-(\phi + \mu)t} \geq 0 \quad \text{for } \forall t \geq 0 \end{aligned} \quad (9)$$

This indicates that solution to our model system (1) - (3) which begins at the boundary of the region Ω converge to the region and remain bounded. For this results our model is mathematically meaningful and hence we can consider the model for analysis.

3.2.7 Basic Model Analysis

In this part we compute the equilibrium states and determine their stability. First, we compute for corruption free equilibrium and ended by computing endemic equilibrium.

(i) Basic Reproduction Number R_0

The basic reproduction number R_0 is the predictable number of secondary corrupt individuals that may stand up due to imposed one corrupt individual into a wholly susceptible population when no interventions are implemented. It plays a great role in determining whether corruption clears $R_0 < 1$ or persists $R_0 > 1$ within the given population under the conditions (Van de Driessche & Watmough, 2002). The basic reproduction number for this study is usually computed by using the next generation method which is done as follows:

First, by considering the differential Equation in (2)

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu}{N}\right)SC - (\rho + \mu + \mu_d)C, \text{ now we let the functions as follows:}$$

$$f = \left(\frac{\kappa+\nu}{N}\right)SC \text{ and } v = (\rho + \mu + \mu_d)C \text{ where } N = S + C + R$$

Thus, find the partial derivative of f and v

$$\text{Therefore, } F = \frac{\partial f}{\partial C} = \frac{(\kappa+\nu)}{N} \text{ and } V = \frac{\partial v}{\partial C} = (\rho + \mu + \mu_d)$$

At corruption Free Equilibrium (CFE) $S = \frac{\Lambda}{\mu}$, $R = C = 0$, substituting into these derivative and simplifying we have: $F = (\kappa + \nu)$ and $V = (\rho + \mu + \mu_d)$ and hence $V^{-1} = \frac{1}{(\rho+\mu+\mu_d)}$

Therefore, the next generation matrix is given by FV^{-1} likewise the basic reproduction number given by $R_0 = FV^{-1} = \frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)}$ (10)

Considering (10) it shows that R_0 depend much on the infection between rate of change of corruption due to desire κ , rate of change of corruption due to poverty ν , rate of change of corruption due to natural recovery (self-change) ρ , rate of change of corruption due to removal rate μ and rate of change of corruption due to induced death rate μ_d .

According to equation of R_0 in (3.10) the increase of κ and ν will increase the rate of corruption practice in the country while increasing natural recovery ρ when other factors remain constant will decrease the reproduction number and hence control of corruption in the country.

3.2.8 Corruption Free Equilibrium (CFE)

Corruption free equilibrium (CFE) is the state when there is no corruption. In other words, the corruption-free equilibrium is the state in which the population is free from corruption, so that

we have only susceptible individuals where as corrupt and recovered individuals will be zero respectively. In this case we get the corruption free equilibrium when we set the right-hand side (RHS) of the system is set to zero. At this point only susceptible individuals exists in the population when equating system of the equations (1) -(3) set to zero we have the following:

$$\Lambda + \phi R - \left(\frac{\kappa+\nu}{N}\right) SC - \mu S = 0$$

$$\left(\frac{\kappa+\nu}{N}\right) SC - (\rho + \mu + \mu_d)C = 0$$

$$\rho C - (\phi + \mu)R = 0$$

To simplify computational, we let $W = \left(\frac{\kappa+\nu}{N}\right)$, hence we have the following system:

$$\Lambda + \phi R - WSC - \mu S = 0 \tag{11}$$

$$WSC - (\rho + \mu + \mu_d)C = 0 \tag{12}$$

$$\rho C - (\phi + \mu)R = 0 \tag{13}$$

From Equation (11) at CFE, $S = \frac{\Lambda}{\mu}$, also from Equation (3.13) at CFE, $R = 0$, as well as from equation (12) at CFE, $C = 0$

Therefore, corruption free equilibrium (CFE) is given by:

$$CFE = (S, C, R) = \left(\frac{\Lambda}{\mu}, 0, 0\right) \tag{14}$$

3.2.9 Local Stability of Corruption Free Equilibrium

The Corruption Free Equilibrium (CFE) is locally asymptotically stable when $R_0 < 1$ and unstable if $R_0 > 1$. The local stability of the CFE can be obtained by looking for the nature of the real parts of the eigenvalue of the matrix formulated from the system of differential Equation (1)- (3) with respect to state variables S, C, R. Here it is known that when it happens that all eigenvalues are negative, therefore the equilibrium point is stable otherwise it is unstable. To perform this, we need to linearize the model system by Jacobian matrix by letting these differential equations as functions of k, l, m as shown below:

$$k(S, C, R) = \Lambda + \phi R - \left(\frac{\kappa+\nu}{N}\right) SC - \mu S \tag{15}$$

$$l(S, C, R) = \left(\frac{\kappa+\nu}{N}\right)SC - (\rho + \mu + \mu_d)C \quad (16)$$

$$m(S, C, R) = \rho C - (\phi + \mu)R \quad (17)$$

Now, the Jacobian matrix at corruption free equilibrium is given by:

$$J_{CFE} = \begin{bmatrix} \frac{\partial k}{\partial S} & \frac{\partial k}{\partial C} & \frac{\partial k}{\partial R} \\ \frac{\partial l}{\partial S} & \frac{\partial l}{\partial C} & \frac{\partial l}{\partial R} \\ \frac{\partial m}{\partial S} & \frac{\partial m}{\partial C} & \frac{\partial m}{\partial R} \end{bmatrix} \quad (18)$$

Substituting its respective derivative value, we have:

$$J_{CFE}^0 = \begin{bmatrix} -\mu & -(\kappa + \nu) & \phi \\ 0 & (\kappa + \nu) - (\rho + \mu + \mu_d) & 0 \\ 0 & \rho & -(\phi + \mu) \end{bmatrix} \quad (19)$$

Through observation method, the first eigenvalue is $\lambda_1 = -\mu$, the reduced Jacobian matrix J^1 at CFE is given by:

$$J_{CFE}^1 = \begin{bmatrix} (\kappa + \nu) - (\rho + \mu + \mu_d) & 0 \\ \rho & -(\phi + \mu) \end{bmatrix} \quad (20)$$

Again, through observation the second Eigen value is $\lambda_2 = -(\phi + \mu)$ and hence the resulting reduced Jacobian matrix will give the third eigenvalue $\lambda_3 = (\kappa + \nu) - (\rho + \mu + \mu_d)$ taking Equation (10), $R_0 = \frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)}$ and rewriting as $(\kappa + \nu) = R_0(\rho + \mu + \mu_d)$ and we substitute into λ_3 we have

$$\lambda_3 = R_0(\rho + \mu + \mu_d) - (\rho + \mu + \mu_d) = (\rho + \mu + \mu_d)[R_0 - 1] \quad (21)$$

From Equation (21) if $R_0 < 1$ implies that $\lambda_3 < 0$

This indicate that CFE is locally asymptotically stable when $R_0 < 1$, otherwise it is unstable (for $R_0 \geq 1$).

3.2.10 Global Stability of Corruption Free Equilibrium (CFE)

There are different approaches to find the global stability of CFE by Lyapunov function, but for this study the LaSalle's invariance principle opted as shown below:

$$\text{First, let } V = \frac{1}{2}C^2, \frac{dV}{dC} = C$$

From Equation (2)

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu}{N}\right)SC - (\rho + \mu + \mu_d)C$$

By chain rule $\frac{dV}{dt} = \frac{dV}{dC} \cdot \frac{dC}{dt}$

Therefore, $\frac{dV}{dt} = C \cdot \left[\left(\frac{\kappa+\nu}{N}\right)SC - (\rho + \mu + \mu_d)C \right] = C^2 \left[\left(\frac{\kappa+\nu}{N}\right)S - (\rho + \mu + \mu_d) \right]$

From (10) reproduction number is given by $R_0 = \frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)}$, this implies $(\kappa + \nu) = R_0(\rho + \mu + \mu_d)$

Therefore, $\frac{dV}{dt} = C^2 \left[R_0(\rho + \mu + \mu_d) \frac{S}{N} - (\rho + \mu + \mu_d) \right] = C^2(\rho + \mu + \mu_d) \left[\frac{S}{N} R_0 - 1 \right]$,

but $\frac{S}{N} < 1$

This means that $\frac{dV}{dt} \leq C^2(\rho + \mu + \mu_d)[R_0 - 1]$

Hence $\frac{dV}{dt} \leq C^2(\rho + \mu + \mu_d)[R_0 - 1] \leq 0$ when $R_0 \leq 1$ (22)

For this case, the corruption free equilibrium is globally asymptotically stable if $R_0 < 1$

3.2.11 Corruption Endemic Equilibrium (CEE)

The corruption endemic equilibrium is the state solution of the corruption transmission model where corruption persists in the population and all compartments are positive. Therefore, here is required to set the given Equations in (1), (2) and (3) equal to zero so that the values of S, C and R obtained as follows:

$$\Lambda + \phi R - \left(\frac{\kappa+\nu}{N}\right)SC = 0, \quad \left(\frac{\kappa+\nu}{N}\right)SC - (\rho + \mu + \mu_d)C = 0 \text{ and } \rho C - (\phi + \mu)R = 0$$

From here in order to get the values of S^*, C^*, R^* we use Mathematica software by first letting:

$$H = \left(\frac{\kappa+\nu}{N}\right) \tag{23}$$

Now, from Equation (1) $S = \frac{\phi R + \Lambda}{CH + \mu}$, where by $H = \left(\frac{\kappa+\nu}{N}\right)$

Also, from Equation (3) $R = \frac{C\rho}{\phi + \mu}$, then we substitute the value of R into S we have:

$$S^* = \frac{\rho + \mu + \mu_d}{H} \tag{24}$$

$$C^* = \frac{(\phi + \mu)(H\Lambda - \mu(\mu + \rho + \mu_d))}{H((\phi + \mu)(\mu + \mu_d) + \rho\mu)} \tag{25}$$

$$R^* = \frac{\rho(H\Lambda - \mu(\mu + \rho + \mu_d))}{H(\rho\mu + (\phi + \mu)(\mu + \mu_d))} \tag{26}$$

Generally, when $R_0 > 1$ then our model (1) system above will experience the corruption endemic equilibrium point at which:

$$E^* = (S^*, C^*, R^*) = \left(\frac{(\rho + \mu + \mu_d)}{H}, \frac{(\phi + \mu)(H\Lambda - \mu(\mu + \rho + \mu_d))}{H((\phi + \mu)(\mu + \mu_d) + \rho\mu)}, \frac{\rho(H\Lambda - \mu(\mu + \rho + \mu_d))}{H(\rho\mu + (\phi + \mu)(\mu + \mu_d))} \right) \quad (27)$$

3.2.12 Global Stability of Endemic Equilibrium point

When $R_0 > 1$, the corruption endemic equilibrium point E^* of the system (1) -(3) is globally asymptotically in ω when $\frac{S}{S^*} = \frac{C}{C^*}$

We prove this using Lyapunov function as follows:

$$T = a_1 \left(S - S^* - S^* \ln \frac{S}{S^*} \right) + a_2 \left(C - C^* - C^* \ln \frac{C}{C^*} \right) + a_3 \left(R - R^* - R^* \ln \frac{R}{R^*} \right) \quad (28)$$

Taking the derivatives along the solutions of the model Equations (1) -(3) as follows:

$$T' = a_1 \left(1 - \frac{S^*}{S} \right) S' + a_2 \left(1 - \frac{C^*}{C} \right) C' + a_3 \left(1 - \frac{R^*}{R} \right) R' \quad (29)$$

But S', C', R' and I' are given in (1) -(3)

$$S' = \Lambda + \phi R - \left(\frac{\kappa + \nu}{N} \right) SC - \mu S$$

$$C' = \left(\frac{\kappa + \nu}{N} \right) SC - (\rho + \mu + \mu_d)C$$

$$R' = \rho C - (\phi + \mu)R$$

Then we substitute these derivatives into T' as given here underneath:

$$\begin{aligned} T' = a_1 \left(1 - \frac{S^*}{S} \right) & \left[\Lambda + \phi R - \left(\frac{\kappa + \nu}{N} \right) SC - \mu S \right] \\ & + a_2 \left(1 - \frac{C^*}{C} \right) \left[\left(\frac{\kappa + \nu}{N} \right) SC - (\rho + \mu + \mu_d)C \right] \\ & + a_3 \left(1 - \frac{R^*}{R} \right) [\rho C - (\phi + \mu)R] \end{aligned} \quad (30)$$

At equilibrium point

If we consider at E^* and $a_i > 0$ for $i = 1,2,3$ in ω and its constants a_i are continuous and differentiable in ω and $T(\omega^*) = 0$ for $\omega = (S^*, C^*, R)$. Global stability for endemic equilibrium holds if $T' \leq 0$, therefore at endemic equilibrium we have the following:

$$\begin{aligned} T' &= a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{k+v}{N}\right) S^* C^* + \mu S^* - \left(\frac{k+v}{N}\right) SC - \mu S \right] \\ &+ a_2 \left(1 - \frac{C^*}{C}\right) [(\rho + \mu + \mu_d) C^* - (\rho + \mu + \mu_d) C] \\ &+ a_3 \left(1 - \frac{R^*}{R}\right) [(\phi + \mu) R^* - (\phi + \mu) R] \end{aligned} \quad (31)$$

Rearranging the terms from Equation (31) we have:

$$\begin{aligned} T' &= a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{k+v}{N}\right) (S^* C^* - SC) + \mu (S^* - S) \right] \\ &+ a_2 \left(1 - \frac{C^*}{C}\right) [(\rho + \mu + \mu_d) (C^* - C)] \\ &+ a_3 \left(1 - \frac{R^*}{R}\right) [(\phi + \mu) R \left(\frac{R^*}{R} - 1\right)] \end{aligned} \quad (32)$$

By simplifying the system above leads into the following Equations:

$$\begin{aligned} T' &= a_1 \left(1 - \frac{S^*}{S}\right) \left[-\left(\frac{k+v}{N}\right) (SC - S^* C^*) - \mu (S - S^*) \right] + a_2 \left(1 - \frac{C^*}{C}\right) [-(\rho + \mu + \mu_d) (C - C^*)] \\ &+ a_3 \left(1 - \frac{R^*}{R}\right) [-(\phi + \mu) R \left(1 - \frac{R^*}{R}\right)] \end{aligned} \quad (33)$$

Again

$$\begin{aligned} T' &= -a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{k+v}{N}\right) (SC - S^* C^*) + \mu (S - S^*) \right] - a_2 \left(1 - \frac{C^*}{C}\right) [(\rho + \mu + \mu_d) (C - C^*)] \\ &- a_3 \left(1 - \frac{R^*}{R}\right) [(\phi + \mu) R \left(1 - \frac{R^*}{R}\right)] \end{aligned} \quad (34)$$

Expanding

$$\begin{aligned} T' &= -a_1 \left(\frac{S-S^*}{S}\right) \left[\left(\frac{k+v}{N}\right) SC \left(\frac{SC-S^*C^*}{SC}\right) - a_1 \left(\frac{S-S^*}{S}\right) \mu \left(\frac{S-S^*}{S}\right) \right] \\ &- a_2 \left(\frac{C-C^*}{C}\right)^2 [(\rho + \mu + \mu_d) C] \\ &- a_3 \left(\frac{R-R^*}{R}\right)^2 [(\phi + \mu) R] \end{aligned} \quad (35)$$

Further simplification,

$$T' = -a_1 \left(\frac{\kappa+\nu}{N} \right) \frac{(S-S^*)(SC-S^*C^*)}{S} - \mu a_1 \frac{(S-S^*)^2}{S} - a_2 \frac{(C-C^*)^2(\rho+\mu+\mu_d)}{C} - a_3 \frac{(R-R^*)^2(\phi+\mu)}{R} \quad (36)$$

Note that all the parameters and all state variables S, C, R, N are non-negative

$$\text{Also, } \left(\frac{S-S^*}{S} \right)^2 \geq 0, \left(\frac{C-C^*}{C} \right)^2 \geq 0, \left(\frac{R-R^*}{R} \right)^2 \geq 0, \left(\frac{I-I^*}{I} \right)^2 \geq 0 \quad (37)$$

Therefore

$$T' = -a_1 \left(\frac{\kappa+\nu}{N} \right) \frac{(S-S^*)(SC-S^*C^*)}{S} - \mu a_1 \frac{(S-S^*)^2}{S} - a_2 \frac{(C-C^*)^2(\rho+\mu+\mu_d)}{C} - a_3 \frac{(R-R^*)^2(\phi+\mu)}{R} + F(\omega) \quad (38)$$

$$\text{Where } F(\omega) = -a_1 \left(\frac{\kappa+\nu}{N} \right) \frac{(S-S^*)(SC-S^*C^*)}{S} \quad (39)$$

From La Salle 's invariant principle $F(\omega) \leq 0$ and hence $U' \leq 0$ as required.

Therefore, Endemic equilibrium is stable if $R_0 > 1$.

3.2.13 Basic Reproduction Number R_0 Sensitivity Analysis and its Analytical Interpretation

Sensitivity is the process conducted at the time when we want to check the influence of every parameter in the basic reproduction number (Diekmann *et al.*, 1990). This process assists when we want to choose the corruption control whereby the most sensitive parameters are considered seriously. In this study the normalized forward sensitivity index method used to determine the sensitivity of the model parameter included in the basic reproduction number. Therefore, if R_0 is differentiable with respect to its parameter z , then the partial derivatives of the basic reproduction number R_0 with respect to the parameters κ, ν, ρ, μ and μ_d of the model conducted by the formula shown below (Mapinda *et al.*, 2019).

$$T_z^{R_0} = \frac{\partial R_0}{\partial z} x \frac{z}{R_0}$$

Note that $R_0 = \frac{\kappa+\nu}{(\rho+\mu+\mu_d)}$ then

$$\frac{\partial R_0}{\partial \nu} = \frac{1}{(\rho+\mu+\mu_d)} > 1, \quad \frac{\partial R_0}{\partial \mu} = -\frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_0}{\partial \rho} = -\frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)^2} < 1, \quad (40)$$

$$\frac{\partial R_0}{\partial \mu_d} = -\frac{(\kappa+\nu)}{(\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_0}{\partial \kappa} = \frac{1}{(\rho+\mu+\mu_d)} > 1.$$

From the derivative of R_0 with respect to κ, ν, ρ, μ and μ_d we can see that some values of the derivative of R_0 are less than 1 while other values are greater 1. Analytically it shows that all values of R_0 which are less than 0 are the important factor for control of corruption in the country. Hence increasing more these parameters become the most control technique of corruption.

Table 4: Sensitivity Indices

Parameter	Sensitivity index
κ	0.3714
ν	0.6286
ρ	-0.1471
μ	-0.1176
μ_d	-0.7353

From Table 4 and Fig. 5 the parameters with the most negative sensitivity indices are rate of change of corruption due to natural recovery ρ and corruption induced death rate μ_d . From this case only ρ is considered because to increase corruption induced death rate μ_d is against human right and is unethical. Therefore, only natural recovery rate will be used to take into account as this change involves self-change of an individual behaviour without treatment. Natural recovery involves individuals making their own decision so that no external force applied to their decision. As more individual undergo self-change will lead into decline of corruption practices in the country (Robins, 1973 & National Institute on Alcohol Abuse and Alcoholism [NIAAA], 2012).

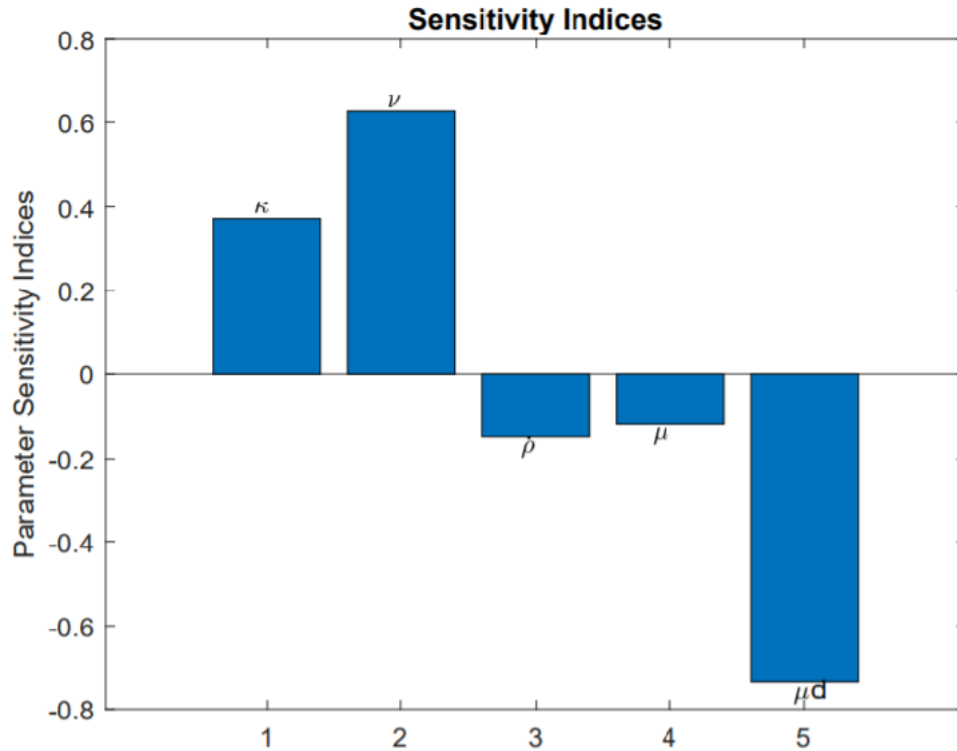


Figure 5: Sensitivity Indices for the Basic Model Parameters

3.3 The Model with Corruption Control Strategies

3.3.1 Model Formulation

This part the basic model is modified to include the control strategies. The corruption dynamics model is developed by extending the model which were formulated by Athithan *et al.* (2018) to include the immune compartment as well as the combination of mass education and religious teaching are included in the model as the control measure. The Corruption model with control measures divided the population into four compartments which are Susceptible individuals, Corrupt individuals, Reformed individuals and immune individuals. Also, the corruption dynamics model is formulated by letting the total population be $N(t)$, Susceptible class $S(t)$, Corrupt Class $C(t)$, Immune Class $I(t)$, and Reformed Class $R(t)$ as described in the following:

- (i) **Susceptible Class:** These are individuals who have never been involved in corruption but they are vulnerable for being influenced by the corrupt practice in the community. These individuals have never engaged in any practice that will have negative effect on the his/her country's economic development but he/she is prone to be corrupted.

- (ii) **Corrupt Class:** This class consists of individuals who always engage in corruption practices and are capable of influencing the susceptible individuals to become corrupt.
- (iii) **Reformed Class:** This class consists of people who are recovered and not able to spread corruption but can become susceptible or immune to corruption after a short period of time.
- (iv) **Immune Class:** These are individuals who can never involve in corruption irrespective of the circumstances around them.

This model assumed that the corrupt individuals will spread the corruption behaviour to the susceptible individuals in the community. Recruitment of individuals into the susceptible class originates from young individuals, who after coming of age seek government services on their own such as identity cards or business permits. During this process their good behaviour is vulnerable and can change to be corrupt. The immune class has those individuals who initially have good behaviour and can never become corrupt under any circumstances. Reformed class contains individuals who have been reformed due to mass education, religious teaching and natural recovery (self-change).

Susceptible individual increases due to birth rate and immigration at the rate of $\varepsilon\Lambda$ while the immune individual increases due to birth at the rate of $\Lambda(1-\varepsilon)$ and the immune rate of γ . Susceptible individual gets into corruption practice after contact and being convinced by the corrupt individual at the rate of $(k + v)$. The Susceptible, Corrupt, Reformed and Immune are all suffers at the death rate of $\mu S, (\mu + \mu_d), \mu R$ and μI respectively. The Corrupt individuals are capable of influencing the susceptible to engage in corruption practices. Corrupt individuals are recovered at the progression rate of $(\alpha + \beta + \rho)$. However, some corrupt individuals decrease at the induced death rate of μ_d while Immune individuals increases at the rate of $(\alpha + \beta)$ and γ . The model for this study formulated using ordinary differential equations. Numerical simulation and results visualization done by using MATLAB Software.

3.3.2 Corruption Dynamics Model Flow Diagram

Figure 6 is a diagrammatic representation of the model for this study showing the flow between the compartments: S-Susceptible, C- Corrupt, R- Reformed and I- Immune

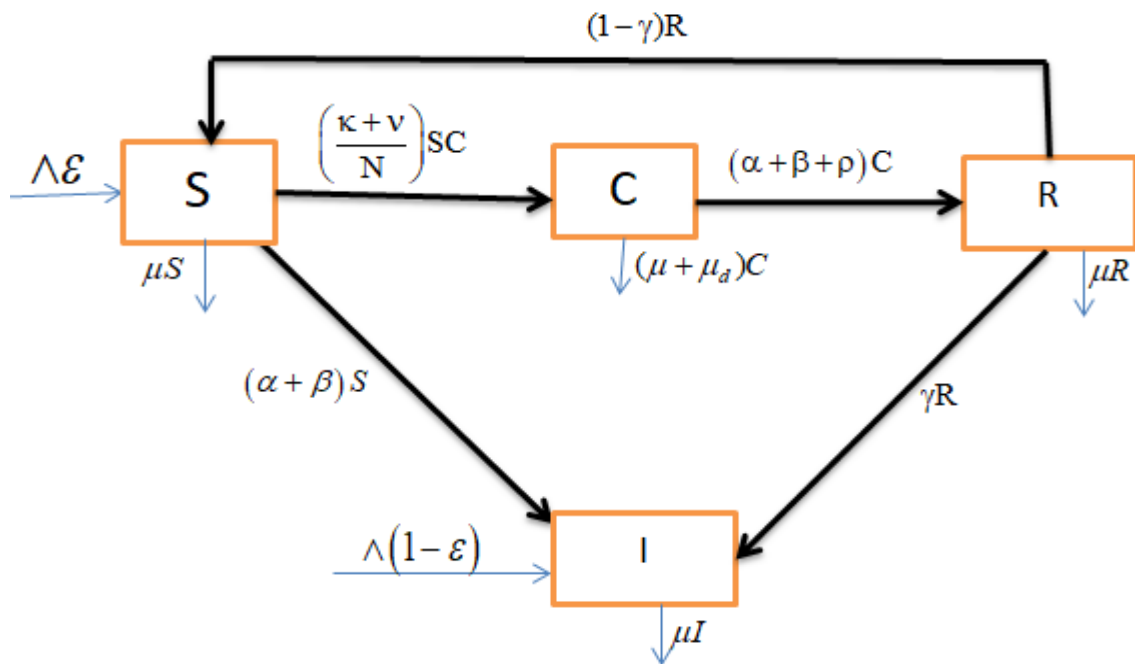


Figure 6: Model Flow Diagram in the Presence of Control Measures

Table 5: Definition of the Model Variables

Variable	Definition
$N(t)$	Total population at time t
$S(t)$	Number of Susceptible individuals at time t
$C(t)$	Number of Corrupt individuals at time t
$R(t)$	Number of Reformed individuals at time t
$I(t)$	Number of Immune individuals at time t

Table 6: Definition of the Model Parameters

Parameters	Definition	Values	Source
α	Rate of change of corruption due to Mass education	0.0095	Assumed
β	Rate of change of corruption due to Religious teaching	0.0125	Lemacha and Feyissa (2018)
ρ	Natural recovery rate	0.01	Binuyo (2019)
κ	Rate of change of corruption transmission due to desire (greed-driven)	0.13	Assumed
ν	Rate of change of corruption transmission due to poverty (need-driven)	0.22	TICPI (2017)
ε	Percentage of humans not born immune	0.7	Eguda <i>et al.</i> (2017)
Λ	Recruitment number	50	Assumed
μ	Natural removal rate	0.008	Assumed
ϕ	Rate at which recovered individuals become Susceptible	0.06	Binuyo (2019)
γ	Rate at which reformed individuals become Immune	0.06	Athithan <i>et al.</i> (2018)
μ_d	Corruption induced death rate	0.05	Assumed

3.3.3 Model Equations

From the diagram above the formulated system of differential Equations are as follows:

$$\text{Total population } N(t) = S(t) + C(t) + R(t) + I(t)$$

$$\frac{dS}{dt} = \varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha + \beta)S - \mu S \quad (41)$$

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C \quad (42)$$

$$\frac{dR}{dt} = (\alpha + \beta + \rho)C + (\mu + 1)R \quad (43)$$

$$\frac{dI}{dt} = \Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I \quad (44)$$

$$S(\mathbf{0}) > \mathbf{0} ; C(\mathbf{0}) > \mathbf{0} ; R(\mathbf{0}) > \mathbf{0} ; I(\mathbf{0}) > \mathbf{0}$$

3.3.4 Model Properties

To analyse the model is very crucial to test if the model is mathematically meaningful. The model is said to be mathematically meaningful if the solutions are positive and they are bounded. The following section will determine whether the solution is boundedness and positive.

(i) Positivity of the Solution

Here we are really going to prove if our model is mathematically meaningful and well posed. This conducted by looking for the positivity of the model solution so that there is no negative solution to the model variables. The Equation (41) - (45) above, need to show that $S(t)$ is always positive. First, we omit all terms from the equation which do not contain the variable S because our intention is to test for positivity of only $S(t)$ as follows:

$$\begin{aligned}
\frac{ds}{dt} &\geq -\left(\left(\frac{\kappa + \nu}{N}\right)C + \alpha + \beta + \mu\right)S \\
\int \frac{ds}{dt} &\geq -\int \left(\left(\frac{\kappa + \nu}{N}\right)C + \alpha + \beta + \mu\right)S \\
\int \frac{ds}{S(t)} &\geq -\int \left(\left(\frac{\kappa + \nu}{N}\right)C + \alpha + \beta + \mu\right) dt \\
\ln S(t)|_0^t &\geq -\int \left(\left(\frac{\kappa + \nu}{N}\right)C + \alpha + \beta + \mu\right) dt \\
\ln S(t) - \ln S(0) &\geq -\left[\int_0^t (\mu + \alpha + \beta) dt + \int_0^t \left(\frac{\kappa + \nu}{N}\right) C dt\right] \\
\frac{S(t)}{S(0)} &\geq -e^{-(\mu + \alpha + \beta)t} - \int_0^t \left(\frac{\kappa + \nu}{N}\right) C dt \\
S(t) &\geq S(0)e^{-(\mu + \alpha + \beta)t - \int_0^t \left(\frac{\kappa + \nu}{N}\right) C dt} \geq 0 \quad \text{for } \forall t \geq 0 \quad (45)
\end{aligned}$$

Consider Equation in (42) corrupt humans, therefore:

$$\begin{aligned}
\frac{dc}{dt} &\geq -[(\alpha + \beta + \rho)C + (\mu + \mu_d)C] \\
\frac{dC}{dt} &\geq -[(\alpha + \beta + \rho) + (\mu + \mu_d)]C \\
\int_0^t \frac{dC}{C(t)} &\geq -\int_0^t (\alpha + \beta + \rho + \mu + \mu_d) dt
\end{aligned}$$

$$\begin{aligned}
\ln C(t)|_0^t &\geq -[\alpha + \beta + \rho + \mu + \mu_d]t \\
\ln C(t) - \ln C(0) &\geq -[\alpha + \beta + \rho + \mu + \mu_d]t \\
\ln \left(\frac{C(t)}{C(0)} \right) &\geq -[\alpha + \beta + \rho + \mu + \mu_d]t \\
C(t) &\geq C(0)e^{-(\alpha+\beta+\rho+\mu+\mu_d)t} \geq 0 \quad \text{for } \forall t \geq 0
\end{aligned} \tag{46}$$

Again, considering Equation in (43) of reformed humans above we have to integrate by separating the variables, therefore:

$$\begin{aligned}
\frac{dR}{dt} &\geq -\mu R - (1 - \gamma)R - \gamma R \\
\frac{dR}{dt} &\geq -\mu R - R + \gamma R - \gamma R \\
\frac{dR}{dt} &\geq -(\mu + 1)R \\
\int_0^t \frac{dR}{R(t)} &\geq - \int_0^t (\mu + 1)dt \\
\ln R(t)|_0^t &\geq -(\mu + 1)t \\
\ln R(t) - \ln R(0) &\geq -(\mu + 1)t \\
\ln \left(\frac{R(t)}{R(0)} \right) &\geq -(\mu + 1)t \\
R(t) &\geq R(0)e^{-(\mu+1)t} \geq 0 \quad \text{for } \forall t \geq 0
\end{aligned} \tag{47}$$

Taking Equation (44) of immune humans, we have to look for positivity of the solution by omitting all variables without I and then we integrate by separating the variables as follows:

$$\text{From, } \frac{dI}{dt} = \Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I$$

$$\text{Thus, } \frac{dI}{dt} = -\mu I$$

$$\frac{dI}{dt} = -\mu I \Rightarrow \frac{dI}{I(t)} = -\mu$$

$$\int_0^t \frac{dI}{I(t)} \geq - \int_0^t \mu dt$$

$$\ln I(t)|_0^t \geq -(\mu)t$$

$$\ln \left(\frac{I(t)}{I(0)} \right) \geq -\mu t$$

$$\text{Therefore, } I(t) \geq I(0)e^{-\mu t} \geq 0, \quad \text{for } \forall t \geq 0 \tag{48}$$

(ii) Boundedness of Solution/Feasible Region

To check for boundedness of the solution of our model we need to add all derivatives of the model that means:

$$\begin{aligned}
 \frac{dS}{dt} + \frac{dC}{dt} + \frac{dR}{dt} + \frac{dI}{dt} &= \varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha + \beta)S - \mu S + \\
 &\quad \left(\frac{\kappa + \nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C + \\
 &\quad (\alpha + \beta + \rho)C + (\mu + 1)R + \\
 &\quad \Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I \\
 &= \Lambda - \mu S - \mu C - \mu R - \mu I - \mu_d C \\
 &= \Lambda - (S + C + R + I)\mu - \mu_d C \\
 \frac{d}{dt}(S + C + R + I) &= \Lambda - \mu N(t) - \mu_d C \tag{49}
 \end{aligned}$$

$\frac{d}{dt}(N(t)) = \Lambda - \mu N - \mu_d C$ we omit $\mu_d C$ hence what remained and hence check for integrating

factor (I.F)

$$\frac{dN}{dt} \leq \Lambda - \mu N$$

$$\frac{dN}{dt} + \mu N \leq \Lambda$$

I.F is given by $e^{\int \mu dt} = e^{\mu t}$ (50)

Now, from $\frac{dN}{dt} + \mu N \leq \Lambda$ and when we multiply both side by integrating factor, which is given by

$$e^{\mu t} \left(\frac{dN}{dt} + \mu N \right) \leq \Lambda e^{\mu t}$$

$$\frac{d}{dt} (Ne^{\mu t}) \leq \Lambda e^{\mu t}$$

$$\int_0^t \frac{d}{dt} (Ne^{\mu t}) \leq \int_0^t \Lambda e^{\mu t}$$

$$Ne^{\mu t} \leq \frac{\Lambda}{\mu} e^{\mu t} + A \text{ dividing by } e^{\mu t} \text{ both sides we have } N(t) \leq \frac{\Lambda}{\mu} + Ae^{-\mu t}$$

But $N(t) = S(t) + C(t) + R(t) + I(t)$

Now when we solve for A when $t = 0$ we obtain $N(0) \leq \frac{\hat{\Lambda}}{\mu} + Ae^0$ this implies $A \geq N(0) - \frac{\hat{\Lambda}}{\mu}$,

therefore $N(t) \leq \frac{\hat{\Lambda}}{\mu} + \left(N(0) - \frac{\hat{\Lambda}}{\mu} \right) e^{-\mu t}$

$N(t) \leq \max \left\{ \frac{\hat{\Lambda}}{\mu}, N(0) \right\}$ if $N(0) \leq \frac{\hat{\Lambda}}{\mu}$ means that $N(t) \leq \frac{\hat{\Lambda}}{\mu}$, otherwise $N(0)$ is the maximum

boundary of $N(t)$

Therefore $\omega = \left\{ (S, C, R, I) \in R_+^4 : N(t) \text{ if } N(0) \leq \frac{\hat{\Lambda}}{\mu} \right\}$ (51)

This shows that solution to model system (41) - (44) which start at the boundary of the region ω converge to the region and remain bounded. For this case our model is mathematically meaningful and hence now we can consider the model for analysis.

3.3.5 Model Analysis

In this section, we derive the basic reproduction number, equilibrium states and determine their stability.

3.3.6 The Effective Reproduction Number (R_e)

The effective reproduction number R_e is the average number of secondary cases that may happen as a consequence of introducing one corrupt individual in an entirely susceptible population when some controls are implemented (Whipple *et al.*, 1990). When $R_e > 1$ corruption exists and when $R_e < 1$ corruption dies in a given population. The effective reproduction number is computed by using the same approach as used in R_0 .

For our model we consider the Equation (42) of corrupt humans:

$$\frac{dC}{dt} = \left(\frac{\kappa + \nu}{N} \right) SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C$$

Now we take the first term and equate it to f and the remaining terms equal to ν , then multiplying by negative 1 for the function ν .

Therefore $\nu = (\mu + \mu_d)C + (\alpha + \beta + \rho)C$ but $N(t) = S(t) + C(t) + R(t) + I(t)$

$$f = \left(\frac{\kappa + \nu}{S+C+R+I} \right) SC \text{ and } \nu = (\mu + \mu_d)C + (\alpha + \beta + \rho)C \quad (52)$$

at the corruption free equilibrium (CFE) $S \neq 0, C = R = 0$,

If $F = \frac{\partial f}{\partial c} = \frac{(k+v)S^2}{(S)^2} = k+v$ and $V = \frac{\partial v}{\partial c} = (\alpha + \beta + p) + (\mu + \mu_d)$, then

$$V^{-1} = \frac{1}{(\alpha + \beta + p) + (\mu + \mu_d)}$$

Therefore $FV^{-1} = (k+v) * \frac{1}{(\alpha + \beta + p) + (\mu + \mu_d)} = \frac{k+v}{(\alpha + \beta + p) + (\mu + \mu_d)}$ for this case the

effective reproduction number is given by.

$$R_e = \frac{(\kappa+v)}{(\alpha+\beta+\rho+\mu+\mu_d)(\alpha+\beta+\mu)} \quad (53)$$

According to the results it indicates that the effective reproduction number (R_e) depends much on the infection rate between the rate of change of corruption due to poverty (v) and rate of change of corruption due to desire (k) such that when the sum of v and k that is $(\kappa + v)$ are greater than the sum of the rate of change of corruption due to mass education (α) and rate of change of corruption due to religious faith β then the effective reproduction number is greater than 1 hence corruption endemic. Note that α and β are the parameters representing effectiveness on the fight of corruption in the country by mass education and religious faith respectively. With a clear combination of these two parameters will probably mean an increase in these parameters. Therefore, according to our model, the increase of α and β is a successful strategy to control corruption ideology since it will reduce R_0 as a result the effective reproduction number will be less than 1.

3.3.7 Sensitivity Analysis of the Effective Reproduction Number and their Analytical Interpretation

By conducting partial derivatives of the effective reproduction number with respect to the parameters of this model is carried out and its interpretation is provided after the derivative. Therefore the partial derivatives of the effective reproduction number R_e with respect to the parameter of the model given $\alpha, \beta, \rho, \kappa, v, \mu$ and μ_d results into the following:

Considering $R_e = \frac{\kappa+v}{(\alpha+\beta+\rho+\mu+\mu_d)}$ then,

$$\frac{\partial R_e}{\partial \alpha} = -\frac{(\kappa+v)}{(\alpha+\beta+\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_e}{\partial v} = \frac{1}{(\alpha+\beta+\rho+\mu+\mu_d)} > 1$$

$$\frac{\partial R_e}{\partial \beta} = -\frac{(\kappa+\nu)}{(\alpha+\beta+\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_e}{\partial \mu} = -\frac{(\kappa+\nu)}{(\alpha+\beta+\rho+\mu+\mu_d)^2} < 1$$

$$\frac{\partial R_e}{\partial \rho} = -\frac{(\kappa+\nu)}{(\alpha+\beta+\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_e}{\partial \mu_d} = -\frac{(\kappa+\nu)}{(\alpha+\beta+\rho+\mu+\mu_d)^2} < 1, \quad \frac{\partial R_e}{\partial \kappa} = \frac{1}{(\alpha+\beta+\rho+\mu+\mu_d)} > 1 \quad (54)$$

From the derivative of R_e it has been observed that there are some values of the derivative of R_e which are less than 1 and other values are greater 1. Analytically it indicates that all values of R_e which are less than 1 are the important in the war of fighting corruption in the country. Therefore, increasing more these parameters become the most control strategy of corruption in the country. For example, α , β and μ_d representing mass education, religious teaching and corruption induced death rate respectively. Therefore, the more we increase mass education to citizens and religious teaching from religious leaders the more the control of corruption in the country. In Table 7 the most negative sensitive parameter is μ_d but we will not recommend as one of the control measure because increasing corruption induced death rate is against human right and it is unethical as well as not practical (Nyerere *et al.*, 2018). Table 7 and Fig. 7 below show the sensitive parameter values and its graph respectively.

Table 7: Sensitivity Indices

Parameter	Sensitivity Index
α	-0.4222
β	-0.5556
ρ	-0.1111
μ	0.6444
μ_d	-0.5556
κ	0.3714
ν	0.6286

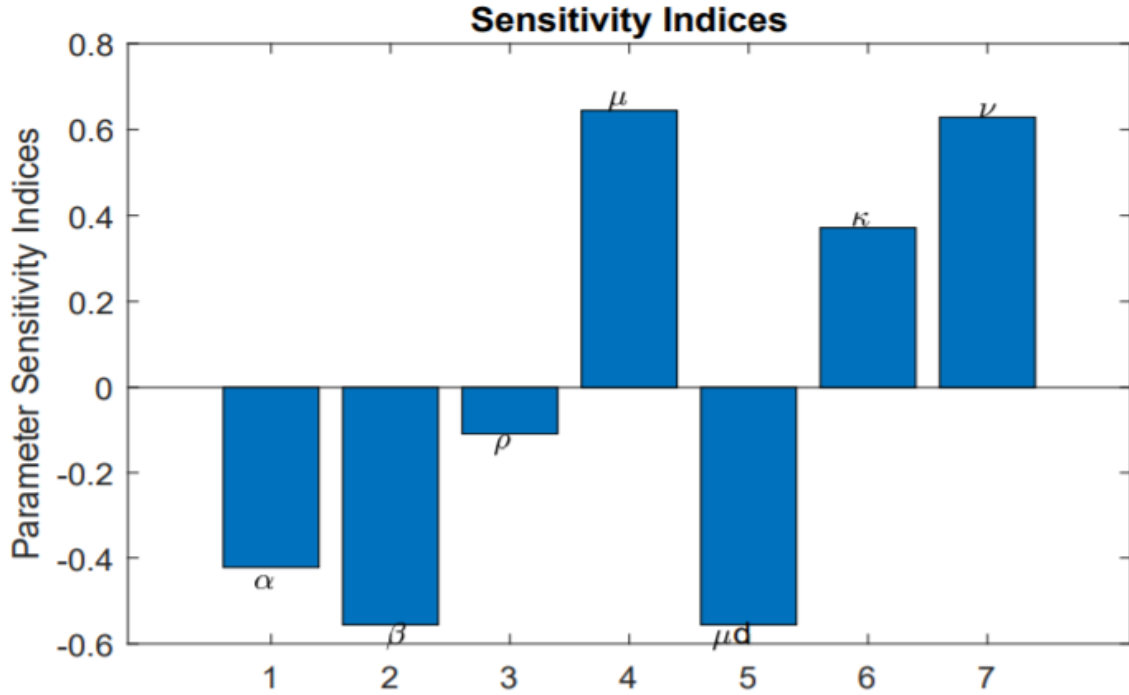


Figure 7: Sensitivity Analysis of R_e

3.4 Corruption Free Equilibrium (CFE)

Corruption free equilibrium (CFE) is the state when there is no corruption. In other words, corruption-free equilibrium is the state in which the population is free of corruption, so that there are only susceptible and immune individuals in the population, for this case the corruption free equilibrium is obtained when the RHS of the system is set to zero. At this point only susceptible and immune individuals exist in the population when equating system of the Equations (41) -(44) to zero the following obtained:

$$\varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa + \nu}{N}\right)SC - (\alpha + \beta)S - \mu S = 0$$

$$\left(\frac{\kappa + \nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C = 0$$

$$(\alpha + \beta + \rho)C + (\mu + 1)R = 0$$

$$\Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I = 0$$

Substituting $C = R = 0$ we have the following:

$$\varepsilon\Lambda - (\alpha + \beta)S - \mu S = 0$$

$$\varepsilon\Lambda = (\alpha + \beta + \mu)S$$

$$\therefore S = \frac{\varepsilon\Lambda}{(\alpha + \beta + \mu)} \quad (55)$$

Taking Equation $\Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I = 0$ solving for I when $R = C = 0$ we have

$$\begin{aligned} \Lambda(1 - \varepsilon) + (\alpha + \beta)S - \mu I &= 0 \\ I &= \frac{\Lambda(1-\varepsilon)+(\alpha+\beta)S}{\mu} = \frac{\Lambda(1-\varepsilon)+(\alpha+\beta)}{\mu} \frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)} \text{ since } S = \frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)} \\ I &= \frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)} \end{aligned} \quad (56)$$

Therefore, corruption free equilibrium (CFE) is given by:

$$CFE = (S, C, R, I) = \left(\frac{\varepsilon\Lambda}{(\alpha+\beta+\mu)}, 0, 0, \frac{\Lambda(1-\varepsilon)(\alpha+\beta+\mu)+(\alpha+\beta)\varepsilon\Lambda}{\mu(\alpha+\beta+\mu)} \right) \quad (57)$$

3.4.1 The Existence of Corruption Endemic Equilibrium Point

Corruption endemic equilibrium can be defined as the steady state solution of the corruption transmission model where corruption persists in the population and all compartments of the model are positive. Therefore, here needed to set the given Equations (41) -(44) equal to zero to solve for S, C, R and I , this means:

$$\begin{aligned} \varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa + \nu}{N}\right)SC - (\alpha + \beta)S - \mu S &= 0 \\ \left(\frac{\kappa + \nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C &= 0 \\ (\alpha + \beta + \rho)C + (\mu + 1)R &= 0 \\ \Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I &= 0 \end{aligned}$$

From Equation (3.43) the value of C can be found as follows:

$$\begin{aligned} (\alpha + \beta + \rho)C - (\mu + 1)R &= 0 \\ (\alpha + \beta + \rho)C &= \mu R + R \\ (\alpha + \beta + \rho)C &= (\mu + 1)R \\ \therefore C^* &= \frac{(\mu+1)R}{(\alpha+\beta+\rho)} \end{aligned} \quad (58)$$

Now substituting $C^* = \frac{(\mu+1)R}{(\alpha+\beta+\rho)}$ into Equation (42)

resulting to, $\left(\frac{\kappa+\nu}{N}\right)SC - (\alpha + \beta + \rho)C - (\mu + \mu_d)C = 0$

$$\left(\frac{\kappa + \nu}{N}\right)S * C - (\alpha + \beta + \rho) * C - (\mu + \mu_d) * C = 0$$

$$\left(\frac{\kappa + \nu}{N}\right)SC - (\alpha + \beta + \rho)C - (\mu + \mu_d)C = 0$$

But $C = \frac{(\mu+1)R}{(\alpha+\beta+\rho)}$ hence we have $\left(\frac{\kappa+\nu}{N}\right)S \frac{(\mu+1)R}{(\alpha+\beta+\rho)} - (\alpha + \beta + \rho) \frac{(\mu+1)R}{(\alpha+\beta+\rho)} - (\mu + \mu_d) \frac{(\mu+1)R}{(\alpha+\beta+\rho)} = 0$

Now, let $B = \left(\frac{\kappa+\nu}{N}\right)$

$$BS \frac{(\mu+1)R}{(\alpha+\beta+\rho)} - (\alpha + \beta + \rho) \frac{(\mu+1)R}{(\alpha+\beta+\rho)} - (\mu + \mu_d) \frac{(\mu+1)R}{(\alpha+\beta+\rho)} = 0 ,$$

multiplying each term by $(\alpha + \beta + \rho)$ results into

$BRS(1 + \mu) = (\alpha + \beta + \rho)(1 + \mu)R + (\mu + \mu_d)(1 + \mu)R$ divide by $BR(1 + \mu)$ each term remains

$$\therefore S^* = \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} \quad (59)$$

where $B = \left(\frac{\kappa+\nu}{N}\right)$

Now, from the values of S^* and C^* using Equation (41) to get R^* , therefore

$$\varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha + \beta)S - \mu S = 0 \text{ but } C^* = \frac{(\mu+1)R}{(\alpha+\beta+\rho)} \text{ and } S^* = \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B}$$

Hence,

$$\varepsilon\Lambda + (1 - \gamma)R - B \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} \frac{(\mu+1)R}{(\alpha+\beta+\rho)} - (\alpha + \beta) \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} - \mu * \frac{(\alpha + \beta + \rho) + (\mu + \mu_d)}{B} = 0$$

Let $(\alpha + \beta + \rho) = M, (\mu + \mu_d) = N$ simply we have

$$R^* = \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda \quad (60)$$

Now, to get I supposed to substitute S and R into Equation (44)

$$\Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I = 0 \text{ we get}$$

$$\Lambda(1 - \varepsilon) + (\alpha + \beta) \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} + \gamma \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda - \mu I = 0 \text{ and therefore:}$$

$$I^* = \frac{1}{\mu} (\Lambda(1 - \varepsilon) + (\alpha + \beta) \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} + \gamma \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda)$$

Generally, when $R_e > 1$ then the model system in (41) -(44) above will experience the corruption endemic equilibrium point at which $E^* = (S^*, C^*, R^*, I^*)$

$$S^* = \frac{(\alpha + \beta + \rho) + (\mu + \mu_d)}{B}$$

$$C^* = \frac{(\mu+1)R}{(\alpha+\beta+\rho)} = \frac{(\mu+1)*R}{(\alpha+\beta+\rho)} = \frac{(\mu+1)}{(\alpha+\beta+\rho)} \left(\frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda \right)$$

$$R^* = \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda$$

$$I^* = \frac{1}{\mu} (\Lambda(1 - \varepsilon) + (\alpha + \beta) \frac{(\alpha+\beta+\rho)+(\mu+\mu_d)}{B} + \gamma \frac{M((\alpha+\beta)(M+N)+\mu(M+N))}{B((M-M\gamma)-(M+N)(u+1))} - \varepsilon\Lambda) \quad (61)$$

Where by $(\alpha + \beta + \rho) = M$, $(\mu + \mu_d) = N$ and $B = \left(\frac{\kappa+v}{N} \right)$

3.4.2 Local Stability of Corruption Free Equilibrium Points

To check for local stability of equilibrium points we consider all model equations and find the Jacobian Matrix that will be used to evaluate whether the equilibrium point is stable or not depending on the sign of the eigenvalues. If all eigenvalues are negative, the equilibrium points are stable, otherwise it is unstable. Using linearization method by Jacobian matrix, local stability of corruption free equilibrium (CFE) obtained by letting the given four differential equations as function $f, g, h,$ and z as follows:

$$f(S, C, R, I) = \varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa+v}{N} \right) SC - (\alpha + \beta + \mu)S \quad (61)$$

$$g(S, C, R, I) = \left(\frac{\kappa+v}{N} \right) SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C \quad (62)$$

$$h(S, C, R, I) = (\alpha + \beta + \rho)C - (\mu + 1)R \quad (63)$$

$$z(S, C, R, I) = \Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I \quad (64)$$

Where $N(t) = S(t) + C(t) + R(t) + I(t)$, then its Jacobian Matrix (J) at corruption free equilibrium (CFE) is given by.

$$J = \begin{pmatrix} \frac{\partial f}{\partial S} & \frac{\partial f}{\partial C} & \frac{\partial f}{\partial R} & \frac{\partial f}{\partial I} \\ \frac{\partial g}{\partial S} & \frac{\partial g}{\partial C} & \frac{\partial g}{\partial R} & \frac{\partial g}{\partial I} \\ \frac{\partial h}{\partial S} & \frac{\partial h}{\partial C} & \frac{\partial h}{\partial R} & \frac{\partial h}{\partial I} \\ \frac{\partial z}{\partial S} & \frac{\partial z}{\partial C} & \frac{\partial z}{\partial R} & \frac{\partial z}{\partial I} \end{pmatrix} \quad (65)$$

$$J_{CFE} = \begin{pmatrix} -(\mu + \alpha + \beta) & -(k + v) & (1 - \gamma) & 0 \\ 0 & (k + v) - (\alpha + \beta + \rho + \mu + \mu_d) & 0 & 0 \\ 0 & (\alpha + \beta + \rho) & -(\mu + 1) & 0 \\ (\alpha + \beta) & 0 & \gamma & -\mu \end{pmatrix} \quad (66)$$

By the method of observation, the first eigenvalue is $\lambda_1 = -\mu < 0$. Therefore the reduced Jacobian matrix J^1 at CFE is given by:

$$J^1_{CFE} = \begin{pmatrix} -(\mu + \alpha + \beta) & -(\kappa + v) & (1 - \gamma) \\ 0 & (k + v) - (\alpha + \beta + \rho + \mu + \mu_d) & 0 \\ 0 & (\alpha + \beta + \rho) & -(\mu + 1) \end{pmatrix} \quad (67)$$

Also, through the method of observation the second eigenvalue is, $\lambda_2 = -(\mu + \alpha + \beta) < 0$, and hence the reduced Jacobian matrix is, J^2 at CFE.

$$J^2_{CFE} = \begin{pmatrix} (k + v) - (\alpha + \beta + \rho + \mu + \mu_d) & 0 \\ (\alpha + \beta + \rho) & -(\mu + 1) \end{pmatrix} \quad (68)$$

Again, through observation the third eigenvalue is $\lambda_3 = -(\mu + 1) < 0$ and hence the fourth eigenvalue is $\lambda_4 = (k + v) - (\alpha + \beta + \rho + \mu + \mu_d)$.

By considering λ_4 we find that the corruption free equilibrium (CFE) will be asymptotically stable only if $\lambda_4 < 0$, which means $(k + v) < (\alpha + \beta + \rho + \mu + \mu_d)$ must hold so as the condition for stability of CFE point.

3.4.3 The Global Stability of Corruption Free Equilibrium by Lyapunov Stability Theorem

Let $(x^*, y^*) = (0, 0)$ be the equilibrium point of $x' = f(x, y)$ and $V(x, y)$ be continuously differentiable positive definite function in the neighbourhood of the origin. The function $V(x, y)$ is Lyapunov function provided conditions below hold (Norelys & Manuel, 2014).

- (i) $V(0,0) = (0,0)$
- (ii) $V(x, y) > 0, \forall x, y \in \mu - \{0\}$
- (iii) $V'(x, y) \leq 0, \forall x, y \in \mu - \{0\}$ (69)
- (iv) $V'(x, y) < 0$, then $V(x, y)$ is strictly lyapunov.

There are different approaches to calculate the global stability using Lyapunov function but for the purpose of this study we shall use the LaSalle's invariance principle, (Massachusetts Institute of Technology [MIT], 2010).

Therefore,

By chain rule, let $V = \frac{1}{2}C^2$ and $\frac{dV}{dt} = \frac{\partial V}{\partial C} * \frac{\partial C}{\partial t}$

Then, $V = \frac{1}{2}C^2$ then $\frac{dV}{dC} = \frac{1}{2} * 2C = C$ and from Equation (42) above

$$\begin{aligned} \frac{dC}{dt} &= \left(\frac{\kappa+\nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C \\ \therefore \frac{dV}{dt} &= \frac{\partial V}{\partial C} * \frac{\partial C}{\partial t} = C\left(\frac{\kappa+\nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C \\ &= C^2 \left[\left(\frac{\kappa+\nu}{N}\right)S - (\mu + \mu_d) - (\alpha + \beta + \rho) \right] \\ \therefore \frac{dV}{dt} &= C^2 \left[(\kappa + \nu) \frac{S}{N} - (\mu + \mu_d) - (\alpha + \beta + \rho) \right] \end{aligned} \tag{70}$$

Considering Equation (3.53), $R_e = \frac{\kappa+\nu}{(\alpha+\beta+\rho+\mu+\mu_d)}$

making $(\kappa + \nu)$ the subject we have $\kappa + \nu = R_e[(\alpha + \beta + \rho) + (\mu + \mu_d)]$

when Substituting into the Equation results into,

$$\frac{dV}{dt} = C^2 \left[(\kappa + \nu) \frac{S}{N} - (\mu + \mu_d) - (\alpha + \beta + \rho) \right]$$

remain $\frac{dV}{dt} = C^2 \left[(\alpha + \beta + \rho + \mu + \mu_d) \frac{S}{N} R_e - (\alpha + \beta + \rho + \mu + \mu_d) \right]$

but $\frac{S}{N} \leq 1$,

Therefore,

$$\begin{aligned} \frac{dV}{dt} &\leq C^2 \left[(\alpha + \beta + \rho + \mu + \mu_d) \frac{S}{N} R_e - (\alpha + \beta + \rho + \mu + \mu_d) \right] \\ \frac{dV}{dt} &\leq C^2 [(\alpha + \beta + \rho + \mu + \mu_d) R_e - (\alpha + \beta + \rho + \mu + \mu_d)] \\ \frac{dV}{dt} &\leq C^2 ((\alpha + \beta + \rho + \mu + \mu_d) R_e - (\alpha + \beta + \rho + \mu + \mu_d)) \\ \frac{dV}{dt} &\leq C^2 (\alpha + \beta + \rho + \mu + \mu_d) [R_e - 1] \end{aligned} \quad (71)$$

$\therefore \frac{dV}{dt} \leq 0$ if and only if $R_e < 1$,

Therefore, CFE is globally asymptotically stable if $R_e < 1$

3.4.4 The Global Stability of Endemic Equilibrium point for model with control measures

If $R_e > 1$, thus the corruption-endemic equilibrium, E^* of the system of Equation (41)-(44) is globally asymptotically stable in ω when $\frac{S}{S^*} = \frac{C}{C^*}$ (Abdulrahman, 2014).

Proof, if we consider the Lyapunov function:

$$\begin{aligned} U &= a_1 \left(S - S^* - S^* \ln \frac{S}{S^*} \right) + a_2 \left(C - C^* - C^* \ln \frac{C}{C^*} \right) + a_3 \left(R - R^* - R^* \ln \frac{R}{R^*} \right) \\ &\quad + a_4 \left(I - I^* - I^* \ln \frac{I}{I^*} \right) \end{aligned} \quad (72)$$

taking the derivatives along the solutions of the model Equations as follows:

$$U' = a_1 \left(1 - \frac{S^*}{S} \right) S' + a_2 \left(1 - \frac{C^*}{C} \right) C' + a_3 \left(1 - \frac{R^*}{R} \right) R' + a_4 \left(1 - \frac{I^*}{I} \right) I' \quad (73)$$

But S', C', R' and I' are given below:

$$\begin{aligned} S' &= \varepsilon \Lambda + (1 - \gamma) R - \left(\frac{\kappa + \nu}{N} \right) SC - (\alpha + \beta) S - \mu S, C' \\ &= \left(\frac{\kappa + \nu}{N} \right) SC - (\mu + \mu_d) C - (\alpha + \beta + \rho) C \end{aligned}$$

$$R' = (\alpha + \beta + \rho) C + (\mu + 1) R, I' = \Lambda(1 - \varepsilon) + (\alpha + \beta) S + \gamma R - \mu I$$

Substituting these derivatives into U' we have the following:

$$\begin{aligned}
U' &= a_1 \left(1 - \frac{S^*}{S}\right) (\varepsilon\Lambda + (1 - \gamma)R - \left(\frac{\kappa + \nu}{N}\right)SC - (\alpha + \beta)S - \mu S) \\
&+ a_2 \left(1 - \frac{C^*}{C}\right) \left(\left(\frac{\kappa + \nu}{N}\right)SC - (\mu + \mu_d)C - (\alpha + \beta + \rho)C\right) \\
&+ a_3 \left(1 - \frac{R^*}{R}\right) ((\alpha + \beta + \rho)C + (\mu + 1)R) \\
&+ a_4 \left(1 - \frac{I^*}{I}\right) (\Lambda(1 - \varepsilon) + (\alpha + \beta)S + \gamma R - \mu I)
\end{aligned} \tag{74}$$

At equilibrium,

Taking into consideration of (61)-(64) at E^* and $a_i > 0$ for $i = 1, 2, 3, 4$ in ω and its constants a_i are continuous and differentiable in ω and $U(\omega^*) = 0$ for $\omega = (S^*, C^*, R^*, I^*)$. Global stability for endemic equilibrium holds if $U' \leq 0$, therefore at endemic equilibrium become:

$$\begin{aligned}
U' &= a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{\kappa + \nu}{N}\right)S^*C^* + \mu S^* + (\alpha + \beta)S^* - \left(\frac{\kappa + \nu}{N}\right)SC - \mu S - (\alpha + \beta)S\right] \\
&+ a_2 \left(1 - \frac{C^*}{C}\right) [(\alpha + \beta + \rho)C^* + (\mu + \mu_d)C^* - (\alpha + \beta + \rho)C - (\mu + \mu_d)C] \\
&+ a_3 \left(1 - \frac{R^*}{R}\right) [(\mu + 1)R^* - (\mu + 1)R] + a_4 \left(1 - \frac{I^*}{I}\right) [\mu I^* - \mu I]
\end{aligned} \tag{75}$$

by rearranging the terms leads to the following:

$$\begin{aligned}
U' &= a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{\kappa + \nu}{N}\right)(SC - S^*C^*) + (\alpha + \beta)(S - S^*) + \mu(S - S^*)\right] \\
&+ a_2 \left(1 - \frac{C^*}{C}\right) [(\alpha + \beta + \rho)(C - C^*) + (\mu + \mu_d)(C - C^*)] \\
&+ a_3 \left(1 - \frac{R^*}{R}\right) [(\mu + 1)(R - R^*)] - + a_4 \left(1 - \frac{I^*}{I}\right) [\mu(I - I^*)]
\end{aligned} \tag{76}$$

By simplifying the system above leads into the following equations:

$$\begin{aligned}
&= a_1 \left(1 - \frac{S^*}{S}\right) \left[\left(\frac{\kappa + \nu}{N}\right)SC \left(1 - \frac{S^*}{S}\right) + (\alpha + \beta)S \left(1 - \frac{S^*}{S}\right) + \mu S \left(1 - \frac{S^*}{S}\right)\right] \\
&+ a_2 \left(1 - \frac{C^*}{C}\right) ((\alpha + \beta + \rho)C \left(1 - \frac{C^*}{C}\right) + (\mu + \mu_d)C \left(1 - \frac{C^*}{C}\right)) + \\
&+ a_3 \left(1 - \frac{R^*}{R}\right) \left[(\mu + 1)R \left(1 - \frac{R^*}{R}\right)\right] + \\
&+ a_4 \left(1 - \frac{I^*}{I}\right) \left(\mu I \left(1 - \frac{I^*}{I}\right)\right) \\
&= a_1 \left(\frac{S - S^*}{S}\right) \left[\left(\frac{\kappa + \nu}{N}\right) (S^*C^* - SC) + (\alpha + \beta)(S^* - S) + \mu(S^* - S)\right]
\end{aligned}$$

$$\begin{aligned}
& +a_2 \left(\frac{C - C^*}{C} \right) [(\mu + \mu_d)(C - C^*) + (\alpha + \beta + \rho)(C - C^*)] \\
& +a_3 \left(\frac{R - R^*}{R} \right) [(\mu + 1)(R^* - R)] + a_4 \left(\frac{I - I^*}{I} \right) [\mu(I^* - I)] \tag{77}
\end{aligned}$$

$$\begin{aligned}
U' &= a_1 \left(\frac{S - S^*}{S} \right) \left[- \left(\frac{\kappa + \nu}{N} \right) SC \left(1 - \frac{S^* C^*}{SC} \right) - (\alpha + \beta) S \left(1 - \frac{S^*}{S} \right) - \mu S \left(1 - \frac{S^*}{S} \right) \right] \\
& +a_2 \left(\frac{C - C^*}{C} \right) \left[-(\mu + \mu_d) C \left(1 - \frac{C^*}{C} \right) + (\alpha + \beta + \rho) C \left(1 - \frac{C^*}{C} \right) \right] \\
& +a_3 \left(\frac{R - R^*}{R} \right) \left[-(\mu + 1) R \left(1 - \frac{R^*}{R} \right) \right] + a_4 \left(\frac{I - I^*}{I} \right) \left[-\mu I \left(1 - \frac{I^*}{I} \right) \right] \\
& = a_1 \left(\frac{S - S^*}{S} \right) \left[- \left(\frac{\kappa + \nu}{N} \right) SC \left(\frac{SC - S^* C^*}{SC} \right) - (\alpha + \beta) S \left(\frac{S - S^*}{S} \right) - \mu S \left(\frac{S - S^*}{S} \right) \right] \\
& +a_2 \left(\frac{C - C^*}{C} \right) \left[-(\mu + \mu_d) C \left(\frac{C - C^*}{C} \right) - (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right) \right] \\
& +a_3 \left(\frac{R - R^*}{R} \right) \left[-(\mu + 1) R \left(\frac{R - R^*}{R} \right) \right] + a_4 \left(\frac{I - I^*}{I} \right) \left[-\mu I \left(\frac{I - I^*}{I} \right) \right] \tag{78}
\end{aligned}$$

when we open the brackets, we obtain the following:

$$\begin{aligned}
U' &= -a_1 (\alpha + \beta) S \left(\frac{S - S^*}{S} \right)^2 - a_1 \mu S \left(\frac{S - S^*}{S} \right)^2 - a_2 (\mu + \mu_d) C \left(\frac{C - C^*}{C} \right)^2 \\
& -a_2 (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right)^2 - a_2 (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right)^2 \\
& -a_3 (\mu + 1) R \left(\frac{R - R^*}{R} \right)^2 - a_4 \mu I \left(\frac{I - I^*}{I} \right)^2 + F(\omega) \\
U' &= -a_1 \left(\frac{\kappa + \nu}{N} \right) \frac{(S - S^*)(SC - S^* C^*)}{S} - a_1 (\alpha + \beta) S \left(\frac{S - S^*}{S} \right)^2 - a_1 \mu S \left(\frac{S - S^*}{S} \right)^2 \\
& -a_2 (\mu + \mu_d) C \left(\frac{C - C^*}{C} \right)^2 - a_2 (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right)^2 \\
& -a_3 (\mu + 1) R \left(\frac{R - R^*}{R} \right)^2 - a_4 \mu I \left(\frac{I - I^*}{I} \right)^2 \tag{79}
\end{aligned}$$

From the assumption we note that all parameters are greater than zero and all state variables S, C, R, I, N are non-negative.

$$\text{Also, } \left(\frac{S - S^*}{S} \right)^2 \geq 0, \left(\frac{C - C^*}{C} \right)^2 \geq 0, \left(\frac{R - R^*}{R} \right)^2 \geq 0, \left(\frac{I - I^*}{I} \right)^2 \geq 0 \tag{80}$$

Therefore,

$$\begin{aligned}
U' &= -a_1 (\alpha + \beta) S \left(\frac{S - S^*}{S} \right)^2 - a_1 \mu S \left(\frac{S - S^*}{S} \right)^2 - a_2 (\mu + \mu_d) C \left(\frac{C - C^*}{C} \right)^2 \\
& -a_2 (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right)^2 - a_2 (\alpha + \beta + \rho) C \left(\frac{C - C^*}{C} \right)^2
\end{aligned}$$

$$-a_3(\mu + 1)R\left(\frac{R-R^*}{R}\right)^2 - a_4\mu I\left(\frac{I-I^*}{I}\right)^2 + F(\omega) \quad (81)$$

$$\text{Where } F(\omega) = -a_1\left(\frac{\kappa+\nu}{N}\right)\frac{(S-S^*)(SC-S^*C^*)}{S} \quad (82)$$

From La Salle's Invariant principle $F(\omega) \leq 0$ and hence $U' \leq 0$ as required

Therefore, Endemic equilibrium is stable if $R_e > 1$.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Numerical Simulation

In this part we give numerical simulation of the model systems in (1) -(3) and (41) -(44). The simulation started by considering the basic model and concluded by including control strategies

4.1.1 Numerical Simulation of the Basic Model

This section explains how the basic model simulated to study the corruption dynamics under the absence of control strategies. Therefore, in this part we showed the dynamics of corruption without control measures using the proposed model in Fig. 4. The model is simulated using the parameters value shown in Table 6. The results are shown in Fig. 8.

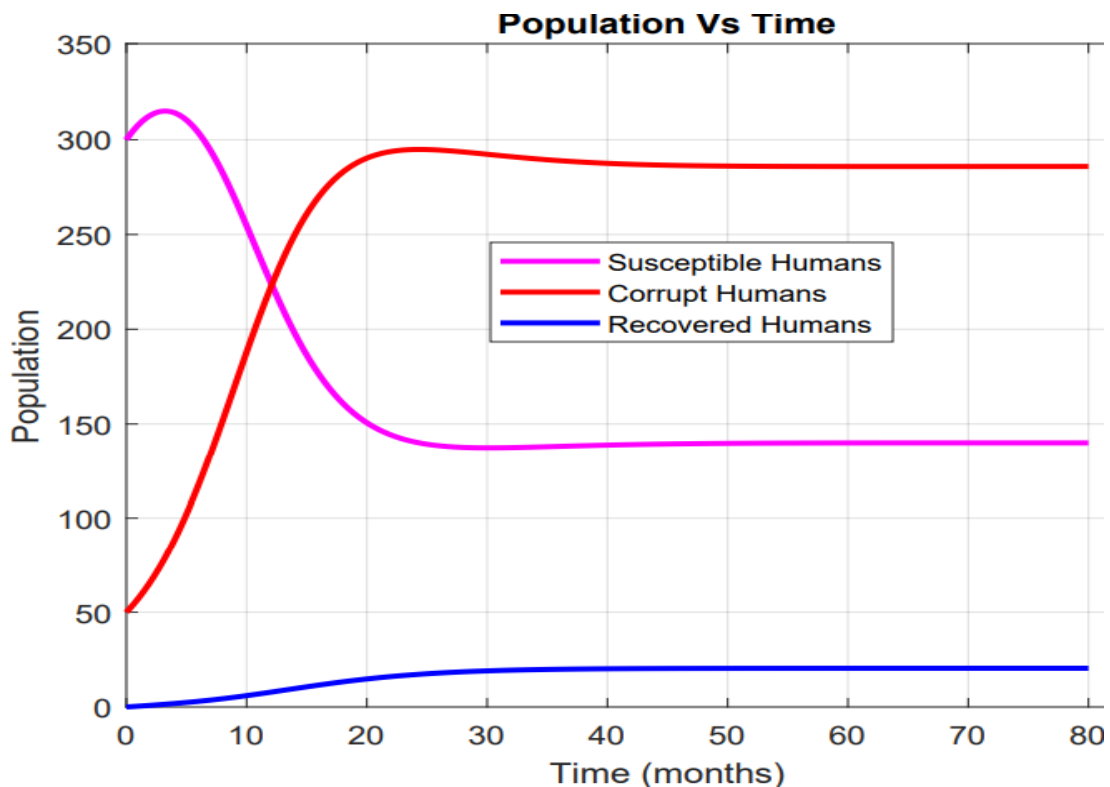


Figure 8: General Corruption Dynamics of the Basic Model

From Fig. 8 recovered humans increases from $t = 0$ to $t = 56$ due to increase in the number of corrupt humans who undergo natural recovery rate caused by self-change hence lead into increase of recovered humans, from $t = 56$ it remains constant throughout. It should be noted that the increase of recovered humans leads also into increase of susceptible humans since at

the recovery compartment there is a temporally stay meaning that after a short period of time of recovery individuals leave the compartment to either susceptible for the rate of ϕ and others undergo natural removal at the rate of μR .

According to (Robbins *et al.*, 1973; Tucker *et al.*, 2020) natural recovery or self-change involve to stop engaging in something with your own decision (without treatment or support group), hence under this situation to stop in engaging in corruption due to self-change will help to decline the corruption practices at a certain degree in the community. In addition, some people choose natural recovery because they feel strong enough to stop engaging in corruption without the assistance from anyone else (quitting in their own decision), this occurs with people who are strong motivated but it may not be something that can keep up indefinitely (Robbins *et al.*, 1973).

4.2 Numerical Simulation of the Model with Control Measures

4.2.1 Numerical Simulation when Control Measures are Set to Zero

Therefore, in this section it has been shown the dynamics of corruption with control strategies using the numerical simulation of the proposed model system in (41) - (44). The model is simulated using the parameters value shown in Table 6 when the control strategies are set to zero that is $\alpha = 0$ and $\beta = 0$ while other values of parameters are as stated in Table 6. The results are shown in Fig. 9.

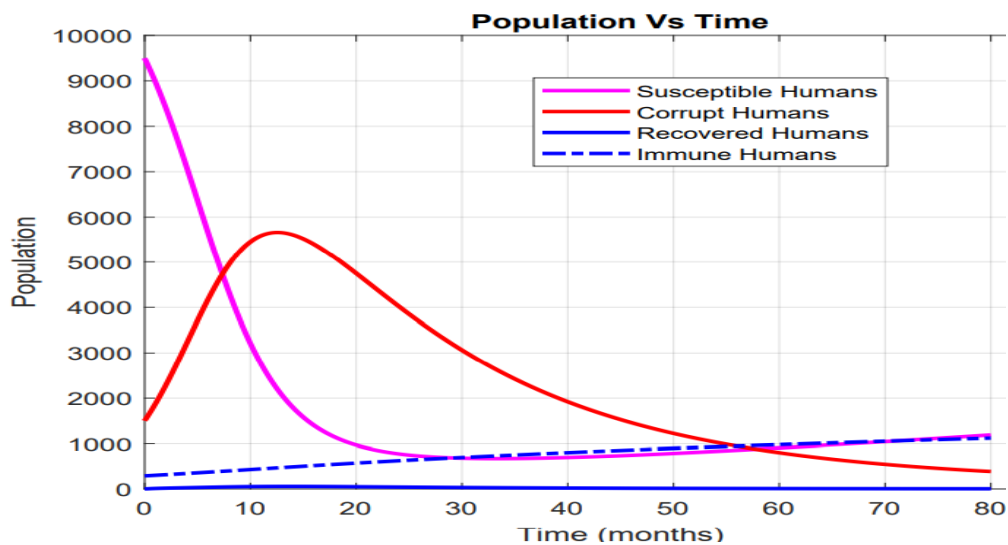


Figure 9: Corruption Dynamics with control when $\alpha = 0$ and $\beta = 0$

From Fig. 9 it indicates that susceptible humans decrease asymptotically due to high corruption contact rate of $(\kappa + \nu)$ and the remove rate of μ . Also, from the graph the number of corrupt humans increases at the very beginning from $t = 0$ up to $t = 12$ due to increase rate of desire (greed-driven) and poverty (need-driven) in the society which lead into most of humans to engage in corruption practices. After $t = 12$ the number of corrupt individuals decreases due to the fact that some individuals undergo corruption induced death rate and others are recovered from corruption due to natural recovery (self-change) of individuals.

Also Fig. 9 shows that, immune individuals at the beginning increases due to birth rate of $(1 - \varepsilon)$ and after a long period of time the number of immune individuals remain constant that is without change. Recovered individuals from $t = 0$ up to $t = 20$ increases due to natural recovery rate and thereafter decreases drastically to zero throughout due to fact that some individuals goes to immune class at the rate of γ and others goes to susceptible class at the rate of $(1 - \gamma)$ as a result the recovered class remain without individuals as from assumption that at the recovered class there is a temporally immunity. Corruption-free equilibrium state is given by:

$$CFE = (S, C, R, I) = \left(\frac{\varepsilon\Lambda}{(\alpha + \beta + \mu)}, 0, 0, \frac{\Lambda(1 - \varepsilon)(\alpha + \beta + \mu) + (\alpha + \beta)\varepsilon\Lambda}{\mu(\alpha + \beta + \mu)} \right)$$

and the effective reproduction number $R_e = \frac{(\kappa + \nu)}{(\alpha + \beta + \rho + \mu + \mu_d)(\alpha + \beta + \mu)}$

4.2.2 The effects of Mass Education against Corruption

In this part the model simulated to check the effect of mass education with the parameter value shown in Table 1.

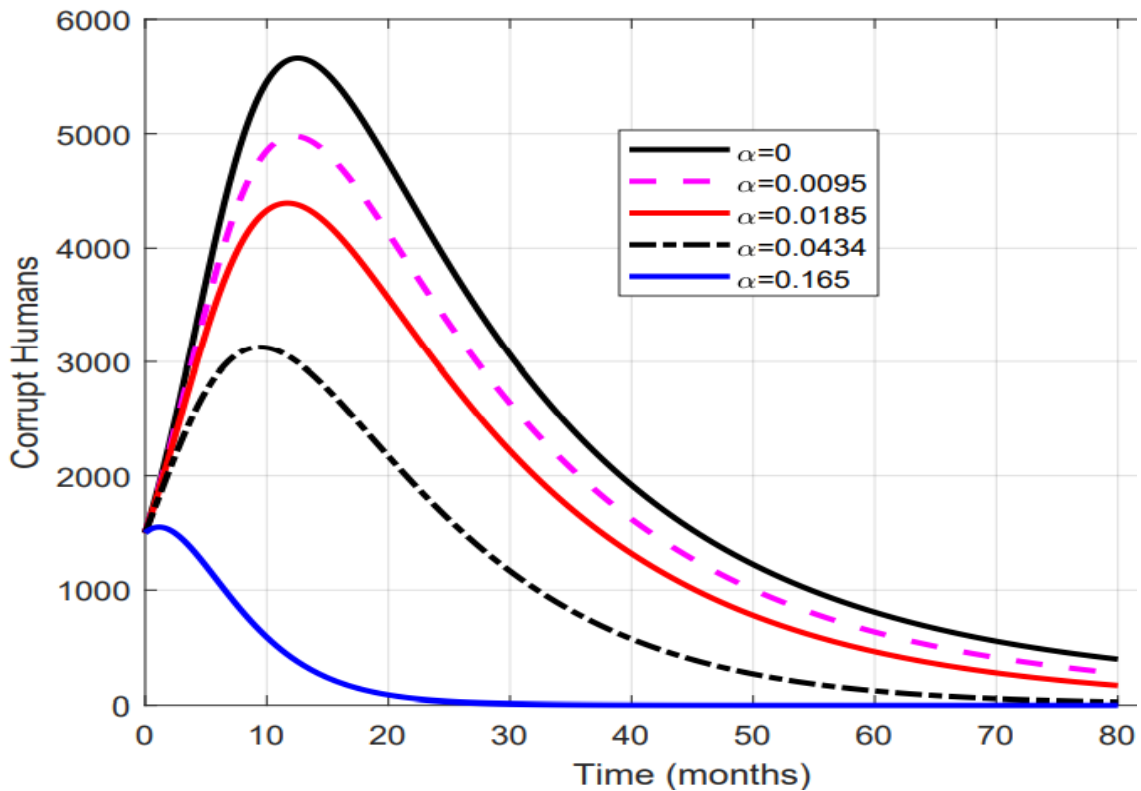


Figure 10: Number of Corrupt Humans with Variation Rate of Mass Education

From Fig. 10 it shows that the higher the rate of provision of mass education to the community concerning the effect of corruption to the development, the lower the number of corrupts humans in the community, this inform the government to invest more in providing education to the citizens about the negative impacts of corruption. The curriculum developers can be required to include corruption studies in the curriculum of all level of education in the country so that everyone will be aware of it. As it can be seen at $t=38$ corruption decline which is almost after 3 years since its implementation. Therefore, investing in provision of mass education to the majority of the community will help to reduce the rate of corruption in the community.

4.2.3 The effects of Religious Teaching against Corruption

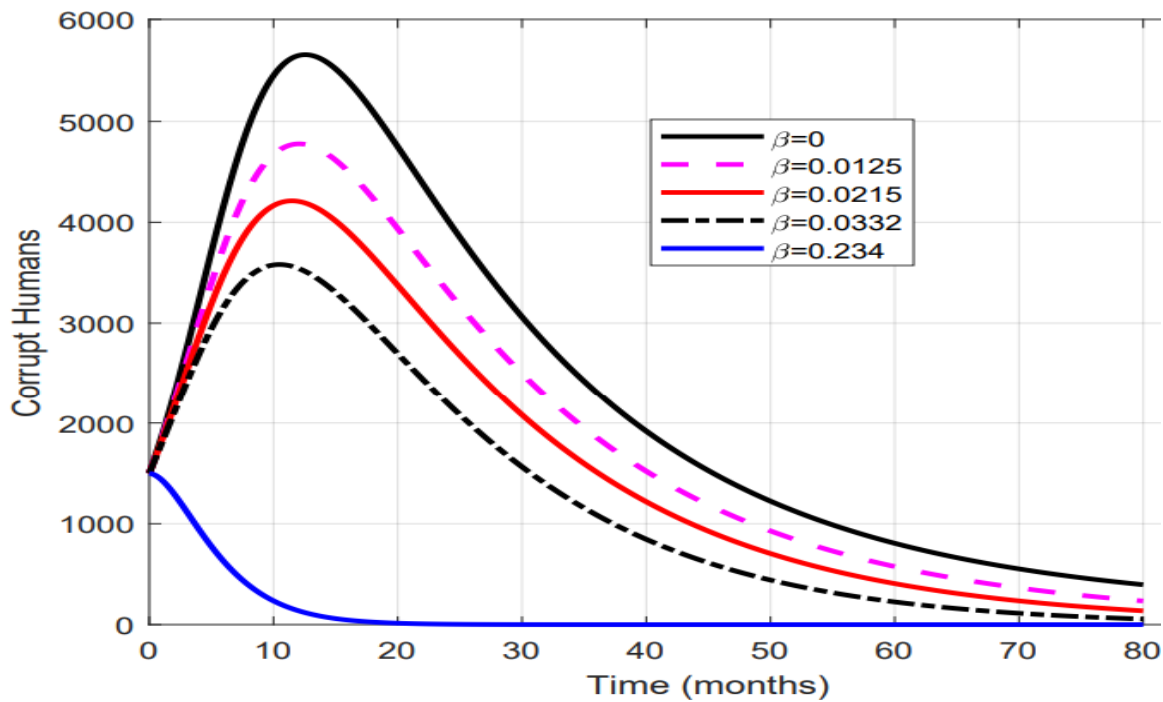


Figure 11: Number of corrupt humans with varying rate of religious teaching, in absence of mass education

By considering Fig. 11 it shows that the higher the rate of provision of education through religious leaders to their followers about corruption as a vice, the number of corrupt humans in the community decline. The model assumed that most followers trust their religious leaders for this case there is a great need to involve more religious leaders in the war of fighting corruption in the country. From the graph it indicates that corruption decline from $t = 26$ which is approximately after 2 years after the commencement of the strategy.

4.2.4 Effects of Combining Mass Education and Religious Teaching against Corruption

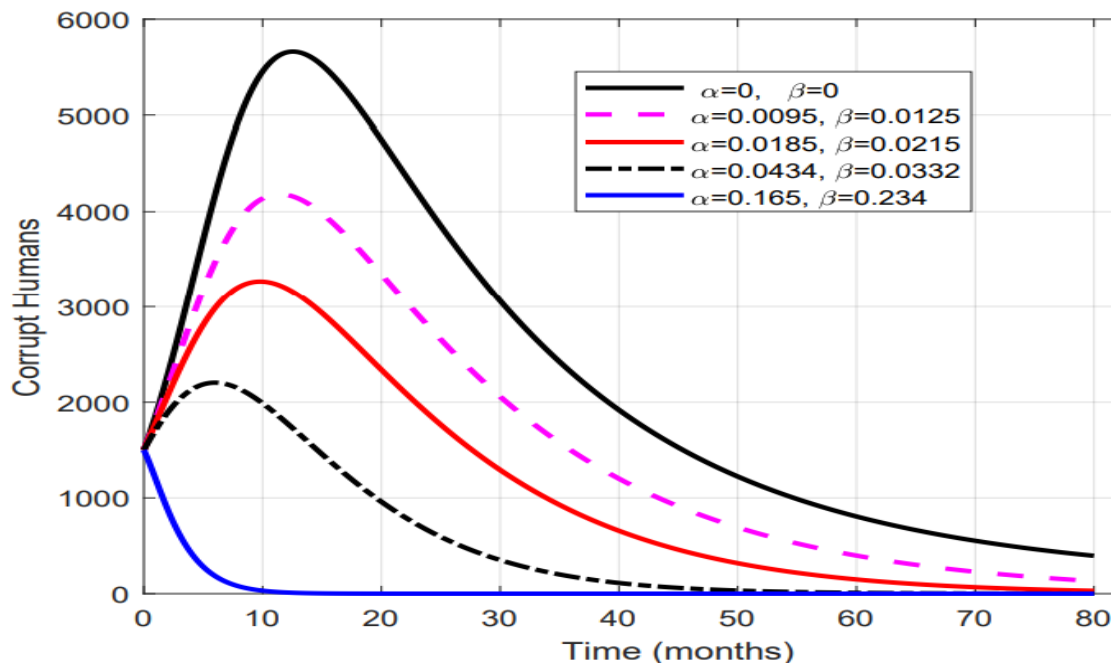


Figure 12: Number of Corrupt Humans by Varying both Mass Education and Religious Teaching

Figure 12 shows that the combination of mass education and religious teaching have significant impact on the community since when you increase their rates collectively, the corruption tend to decrease rapidly compared to when mass education and religious teaching has been used separately as can be seen from Fig. 10 and Fig. 11 respectively. It should be noted that when mass education used as a single control measure the corruption decline at $t = 38$ which is almost 3 years whereas when religious teaching used as a single control measure the corruption decline at $t = 26$ which is almost after two years, this indicates that religious teaching is more effective. It should be noted from Fig. 12 that when mass education and religious teaching are combined together as control measure, they tend to reduce corruption immediately from $t = 17$ which is almost after one and a half years and hence bring more success compared to when each strategy used as a single control. From these results it shows that the government should invest more in giving both mass education to the citizen as well as to emphasize more the religious leaders to teach their followers not to engage in corruption through conducting of special seminars about corruption as it is a vice to engage in corruption and it is against their faith and doctrine. For this case the education curriculum developers should incorporate corruption studies in the education system from pre-school to Universities as serious study to every Tanzanian in order to create awareness to everyone in the country.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The deterministic mathematical model in this study for corruption dynamics is formulated and analysed. The purpose was to get an understanding of transmission dynamics of corruption under the absence and presence of control measures. The main activities in this work includes (a) to formulate and analyse the basic model for corruption dynamics under the absence of control measure (b) to formulate and analyse the mathematical model taking into consideration the combination of mass education and religious teaching as control measure.

From the analytical analysis of basic reproduction number R_0 it shows that the basic reproduction number R_0 is directly affected by the parameters κ , ν , ρ , μ and μ_d . The parameters κ and ν measures the force of infection of transmission rate from susceptible individuals to corrupt individuals, this means that the higher the corruption contact rate the higher the prevalence of corruption in the country since the basic reproduction number become greater than 1. The analytical analysis of R_e shows that R_e is directly affected by the parameters α , β , κ , ν , ρ , μ and μ_d . The parameters κ and ν measures the force of infection of transmission rate from susceptible individuals to corrupt individuals, this means that the higher the corruption contact rate the higher the prevalence of corruption in the country since the effective reproduction number become greater than 1.

Whereas, if parameters represent mass education and religious teaching, respectively, and as these parameters increase, the effective reproduction number becomes smaller than 1, the community's corruption is controlled. These findings suggest that more effort should be put into public education for citizens, as well as a greater emphasis on religious leaders to successfully teach their followers about the detrimental effects of corruption, as it is contrary to their faith and teaching.

The effective reproduction number R_e was obtained and the analysis indicates that if $R_e < 1$ the corruption-free equilibrium is globally asymptotically stable. Due to the nature of corruption it is a bit difficult to eradicate it completely but can be reduced at a reasonable level which does not have harmful impact to the growth of the economy of the country. Also, it should be noted that for whatever circumstances if $R_e > 1$ the corruption-free equilibrium point

is unstable and the endemic equilibrium persist. Also, it is well known that to reduce corruption in the country it is possible if and only if the government is strictly implementing the laws for the corruption victims as well as reducing the level of poverty in the country; to increase the public sector wages when necessary to do so in order to make the public servants not get into temptation of involving in corruption during provision of services to the citizens.

Despite the great effort made by Prevention and Combating of Corruption Bureau (PCCB) to combat corruption in the country but the efforts not bring rapid changes in corruption practices due to insufficient transparency; failure to involve fully the community as well as religious leaders; political interference to PCCB. To solve this the government should show zero tolerance for whoever involve in corruption practices; the government leaders must be honest and display full and visible commitment in dealing with this threat; increasing more effort in provision of mass education to all citizens and incorporate more religious leaders in the fight of corruption since most of Tanzanian are religious, so by effective use of religious leaders will add value to this war as most of the citizens respect very well their religious leaders. Mathematical model has proved this through analytical and numerical simulation both agree with the formulated model. Hence, this study will positively add efforts to the current efforts brought forward by the government of Tanzania to address the issue of corruption and way forward to control the prevalence of it in the country.

5.2 Recommendations

In order to control corruption in the country, the government of the United Republic of Tanzania should insist integrity and request each individual to sustain pressure from lower level of leadership to the top level of leadership as well as to expand and enhance transparency and accountability for leaders. Mass education and religious teaching should be highly emphasized as large population follows on these control measures, but to attain success in this war the government must be very strict to those involving in corruption as well as using strong transformational leader in all sphere of landscape in the government. The study recommends the following:

- (i) Inclusion of corruption studies as independent topic in the education curriculum from pre-school to Universities so that majority of the citizen will be well informed on the meaning, sources, types, background and effect of corruption to social, political and

economic development of the country. Meanwhile corruption is taught as a small part within other topics which do not dig deeper the dynamics of corruption in the country.

- (ii) Anticorruption (PCCB) must be granted prosecution power

5.2.1 Future Work

This study not exhausted everything concerning corruption dynamics, therefore further studies are needed to dig more for understanding of corruption dynamics in the country in the following areas:

- (i) To check the optimal control and cost effectiveness analysis.
- (ii) To check the social factors hindering citizens not to report the corruption practices they see to higher authority to allow immediate action to be taken.
- (iii) To check the time analysis during the implementation of control strategies.

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APPENDICES

Appendix 1: Timeline of corruption in Tanganyika/Tanzania from (1958-2018)

A.1 Timeline of corruption in Tanganyika/Tanzania from 1958-2004

Table 3.1 presents a summary timeline of anti-corruption in Tanganyika/Tanzania to date.

Table 3.1: PCB-PCCB Timeline

1958	Prevention of Corruption Ordinance.
1971	Prevention of Corruption Act (amended 2002).
1974	Anti-Corruption Squad under Ministry of Home Affairs.
1983	Economic Sabotage Act
1984	Economic and Organised Crime Control Act
1990	Prevention of Corruption Bureau. Oversight moved from Home Affairs to President's Office.
1995	November. Benjamin Mkapa elected President, publicly declares his assets.
1995	Public Leadership Code of Ethics Act.
1996	<i>Presidential Commission against Corruption</i> produces the ' Warioba report ' on corruption.
2000	National Anti-Corruption Strategy and Action Plan (NACSAP) launched.
2003	'... while grand corruption is eating up the country's efforts to develop her economy, the fight against corruption [is] waged against low-level government officers.' Mateo Qaresi, RC, Mbeya Region.
2004	Public Procurement Act.

³⁰ Policy Forums Tanzania Governance Reviews document the relative openness of Kikwete's first term and the subsequent tightening up. The revised draft of the proposed new constitution that was launched during Kikwete's second term, known as the 'Chenge draft', made no reference to PCCB. See Mwassa Jingi 2017. 'Why PCCB 'not issue' in Katiba?', Citizen, 2 November.

A.2 Timeline of corruption in Tanzania from 2005-2015

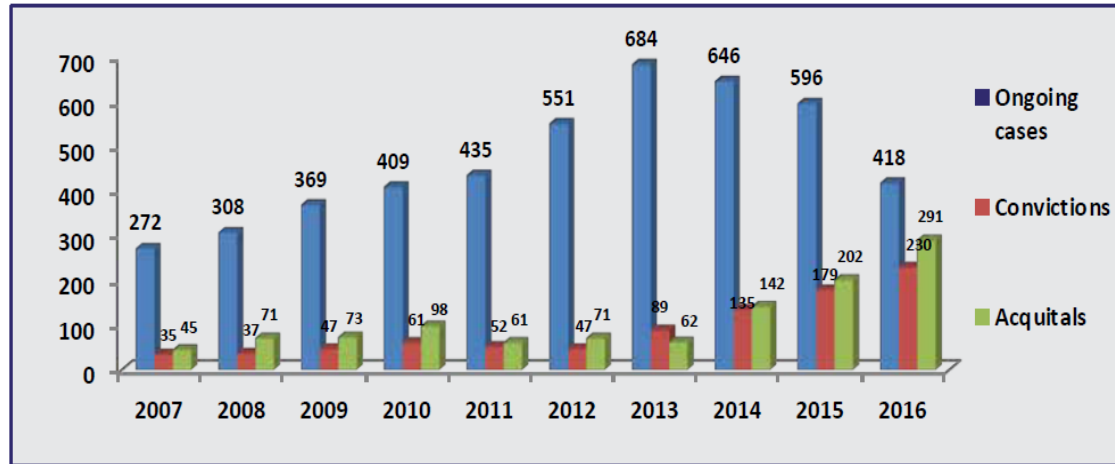
2005	Former Minister of Finance Edwin Mtei considers that "the PCB are only mice; and mice cannot bell free-roaming cats." Stanley Kamana, 2005. 'Mkapa's Problem: Everyone is Totally Corrupt', <i>The East African</i> , 24 Jan
2005	November: Jakaya Kikwete elected President.
2006	Anti-Money Laundering Act, Economic and Organised Crimes Control Act.
2006	President Kikwete appoints Dr Edward Hoseah Director General of PCB.
2007	Prevention and Combating of Corruption Act. PCCB replaces PCB.
2007	Prime Minister Edward Lowassa resigns over the Richmond affair.
2008	Public Audit Act.
2008	March: Edward Hoseah elected chairman of the East African Association of Anti-Corruption Authorities.
2008	Collaboration between PCCB and Britain's Serious Fraud Office (SFO) leads to resignation of former Attorney General, Andrew Chenge , after a transfer of US\$1m to an offshore account in his name is discovered.
2008	Basil Mramba and Daniel Yona arraigned over the Alex Stewart gold assaying scandal.
2008	NACSAP II (2008-11).
2010	Election Expenses Act.
2015	September Whistleblowers Protection Act.
2015	Basil Mramba and Daniel Yona sentenced to three years jail and payment of Shs5m each.
2015	November. President Magufuli pledges law to expedite anti-corruption and economic sabotage cases. Faustine Kapama 2018. 'Probes fail 500bn/- corruption cases', Daily News, 29 January.
2015	16 th December. President Magufuli replaces PCCB Director General Edward Hoseah with Valentino Mlowola. www.google.com/search?rls=aso&client=gmail&q=Hosea%20sacked&authuser=0

A.3 Timeline of corruption in Tanzania from 2016-2018

2016	Anti-Corruption Court law passed by parliament.
2016	NACSAP III (2016-22).
2017	14 th May. Land, houses and other property worth Shs3.6bn belonging to Godfrey Gugai , PCCB's chief accountant frozen pending investigations into how they were acquired. Faustine Kapama 2017. 'PCCB accountant risks losing 3.6bn/- worth of houses, vehicles', Daily News, 15 May
2017	19 th June. PCCB arraign James Rugemalira and Harbinder Singh Sethi over the Escrow affair: https://ippmedia.com/en/news/jpti-barons-nabbed
2017	24 th August. Brigadier General John Julius Mbungu appointed PCCB's deputy Director General.
2017	13 th November. PCCB's former Chief Accountant Godfrey Gugai arrested and assets frozen.
2018	6 th September. President Magufuli replaces PCCB DG Valentino Mlowola with Diwani Athumani , former Director of Criminal Investigations (DCI). Reporter 2018. 'PCCB debate hots up as JPM removes anti-graft czar', Citizen, 7 September.

Appendix 2: Number of Ongoing Cases, Convictions and Acquittals from 2007 -2016

Chart 4.4 Number of ongoing cases, convictions and acquittals, PCCB 2007-16



Source: PCCB data⁹³

Appendix 3: MATLAB Codes for Basic Model

C1 MATLAB codes for Figure 8

```
%Numerical simulation of the basic model, %General corruption dynamics of the basic mode
clc; clear; close all; %h=step size, %T(t)=Total population, %H=gamma model parameter=phi
%g=recruitment number, %Parameter value are as shown below
p=0.01; g=50; H=0.06; k=0.13; v=0.22; D=0.05; u=0.08; t(1)=0;
S(1)=300; C(1)=50; R(1)=0;
%Runge Kutta fourth order method
T(1)=S(1)+C(1)+R(1);
fS=@(t,S,C,R) g+H*R-((k+v)/(T))*S*C-u*S;
fC=@(t,S,C,R) ((k+v)/(T))*S*C-(p+u+D)*C;
fR=@(t,S,C,R) p*C-(H+u)*R;
    tfinal=80; h=0.001; N=ceil(tfinal/h);
for i=1:N
    t(i+1)=t(i)+h;
    k1S=fS( t(i),    S(i),    C(i),    R(i));
    k1C=fC( t(i),    S(i),    C(i),    R(i));
    k1R=fR( t(i),    S(i),    C(i),    R(i));
    k2S=fS( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R);
    k2C=fC( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R);
    k2R=fR( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R);
    k3S=fS( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R);
    k3C=fC( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R);
    k3R=fR( t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R);
    S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S);
    C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C);
    R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R);
end
%Plotting
plot(t,S,'m','linewidth',2)
hold on
plot(t,C,'r','linewidth',2)
hold on
```

```
plot(t,R, 'b','linewidth',2)
hold on
legend ('Susceptible Humans', 'Corrupt Humans', 'Recovered Humans')
title ('Population Vs Time')
xlabel ('Time (months)')
ylabel ('Population')
grid on
```


Appendix 4: MATLAB Codes for Model with Control Measures

D1 MATLAB codes for Figure 9

```
clc;clear;close all;%h=step size%T(t)=Total population%H=gamma model
parameter%g=recruitment number
p=0.01;a=0;b=0;e=0.7;g=50;H=0.06;k=0.13;v=0.22;D=0.05;u=0.008;t(1)=0;
S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;
T(1)=S(1)+C(1)+R(1)+I(1);
fS=@(t,S,C,R,I) e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;
fC=@(t,S,C,R,I) ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;
fR=@(t,S,C,R,I) (a+b+p)*C-(u+1)*R;
fI=@(t,S,C,R,I) g*(1-e)+(a+b)*S+H*R-u*I;
tfinal=80;h=0.001;N=ceil(tfinal/h);
for i=1:N
    t(i+1)=t(i)+h;
    k1S=fS( t(i), S(i), C(i), R(i), I(i));
    k1C=fC( t(i), S(i), C(i), R(i), I(i));
    k1R=fR( t(i), S(i), C(i), R(i), I(i));
    k1I=fI( t(i), S(i), C(i), R(i), I(i));

    k2S= fS( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
    k2C= fC( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
    k2R= fR( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
    k2I= fI( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
    k3S= fS( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
    k3C= fC( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
    k3R= fR( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
    k3I= fI( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);

    k4S=fS( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
    k4C=fC( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
    k4R=fR( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
    k4I=fI( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
```

```

S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S+k4S);C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C+k4C)
; R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R+k4R) I(i+1)=I(i)+(h/6)*(k1I+2*k2I+2*k3I+k4I);
end
plot(t,S,'m','linewidth',2)
hold on
plot(t,C,'r','linewidth',2)
hold on
plot(t,R,'b','linewidth',2)
hold on
plot(t,I,'b-','linewidth',2)
grid on
legend('Susceptible Humans','Corrupt Humans','Recovered Humans','Immune Humans')
title('Population Vs Time')
xlabel('Time (months)')
ylabel('Population')

```

D2 MATLAB codes for Figure 10 &11&12

```

clc;clear;close all;p=0.01;a=0;b=0;e=0.7;g=50;H=0.06;k=0.13;v=0.22;

```

```

D=0.05;u=0.008;t(1)=0;S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;

```

```

T(1)=S(1)+C(1)+R(1)+I(1);

```

```

fS=@(t,S,C,R,I) e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;

```

```

fC=@(t,S,C,R,I) ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;

```

```

fR=@(t,S,C,R,I) (a+b+p)*C-(u+1)*R;

```

```

fI=@(t,S,C,R,I) g*(1-e)+(a+b)*S+H*R-u*I;

```

```

tfinal=80;h=0.001;N=ceil(tfinal/h);

```

```

for i=1:N

```

```

    t(i+1)=t(i)+h;

```

```

    k1S=fS( t(i), S(i), C(i), R(i), I(i));

```

```

    k1C=fC( t(i), S(i), C(i), R(i), I(i));

```

```

    k1R=fR( t(i), S(i), C(i), R(i), I(i));

```

```

    k1I=fI( t(i), S(i), C(i), R(i), I(i));

```

```

k2S= fS( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C, R(i)+ ( h/2)*k1R,I(i)+(h/2)*k1I);

```

```

k2C= fC( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C, R(i)+ ( h/2)*k1R,I(i)+(h/2)*k1I);

```

$k2R = fR(t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);$
 $k2I = fI(t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);$

$k3S = fS(t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R, I(i)+(h/2)*k2I);$
 $k3C = fC(t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R, I(i)+(h/2)*k2I);$
 $k3R = fR(t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R, I(i)+(h/2)*k2I);$
 $k3I = fI(t(i)+h/2, S(i)+(h/2)*k2S, C(i)+(h/2)*k2C, R(i)+(h/2)*k2R, I(i)+(h/2)*k2I);$

$k4S = fS(t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R, I(i)+(h)*k3I);$
 $k4C = fC(t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R, I(i)+(h)*k3I);$
 $k4R = fR(t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R, I(i)+(h)*k3I);$
 $k4I = fI(t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R, I(i)+(h)*k3I);$

$S(i+1) = S(i) + (h/6) * (k1S + 2*k2S + 2*k3S + k4S);$
 $C(i+1) = C(i) + (h/6) * (k1C + 2*k2C + 2*k3C + k4C);$

$R(i+1) = R(i) + (h/6) * (k1R + 2*k2R + 2*k3R + k4R);$
 $I(i+1) = I(i) + (h/6) * (k1I + 2*k2I + 2*k3I + k4I);$

end

plot(t,C,'k','linewidth',2)

hold on

p=0.01;a=0.0095;b=0.0125;e=0.7;g=50;H=0.06;k=0.13;v=0.22;D=0.05;u=0.008;

t(1)=0;S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;

T(1)=S(1)+C(1)+R(1)+I(1);

fS=@(t,S,C,R,I) e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;

fC=@(t,S,C,R,I) ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;

fR=@(t,S,C,R,I) (a+b+p)*C-(u+1)*R;

fI=@(t,S,C,R,I) g*(1-e)+(a+b)*S+H*R-u*I;

tfinal=80;

h=0.001;N=ceil(tfinal/h);

for i=1:N

t(i+1)=t(i)+h;

k1S=fS(t(i), S(i), C(i), R(i), I(i));

k1C=fC(t(i), S(i), C(i), R(i), I(i));

k1R=fR(t(i), S(i), C(i), R(i), I(i));

k1I=fI(t(i), S(i), C(i), R(i), I(i));

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k2S=fS( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C, R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);
k2C=fC( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);
k2R=fR( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);
k2I=fI( t(i)+h/2, S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R, I(i)+(h/2)*k1I);

k3S=fS( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3C=fC( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3R=fR( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R, I(i)+(h/2)*k2I);
k3I=fI( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);

k4S=fS( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4C=fC( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4R=fR( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4I=fI( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);

S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S+k4S);C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C+k4C)
;
R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R+k4R);I(i+1)=I(i)+(h/6)*(k1I+2*k2I+2*k3I+k4I);
end
plot(t,C, 'm--', 'linewidth',2)
p=0.01;a=0.0185;b=0.0215;e=0.7;g=50;H=0.06;k=0.13;v=0.22;D=0.05;u=0.008;
t(1)=0;S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;T(1)=S(1)+C(1)+R(1)+I(1);
fS=@(t,S,C,R,I) e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;
fC=@(t,S,C,R,I) ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;
fR=@(t,S,C,R,I) (a+b+p)*C-(u+1)*R;
fI=@(t,S,C,R,I) g*(1-e)+(a+b)*S+H*R-u*I;
tfinal=80;h=0.001;N=ceil(tfinal/h);
for i=1:N
t(i+1)=t(i)+h;
k1S=fS( t(i), S(i), C(i), R(i), I(i));
k1C=fC( t(i), S(i), C(i), R(i), I(i));
k1R=fR( t(i), S(i), C(i), R(i), I(i));
k1I=fI( t(i), S(i), C(i), R(i), I(i));

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k2S=fS( t(i)+h/2,S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2C=fC( t(i)+h/2,S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2R=fR( t(i)+h/2,S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2I=fI( t(i)+h/2,S(i)+(h/2)*k1S, C(i)+(h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);

k3S=fS( t(i)+h/2,S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3C=fC( t(i)+h/2,S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3R=fR( t(i)+h/2,S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3I=fI( t(i)+h/2,S(i)+(h/2)*k2S, C(i)+(h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);

k4S=fS( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4C=fC( t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4R=fR( t(i)+h,S(i)+(h)*k3S, C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4I=fI( t(i)+h,S(i)+(h)*k3S, C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);

S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S+k4S);C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C+k4C)
;
R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R+k4R);I(i+1)=I(i)+(h/6)*(k1I+2*k2I+2*k3I+k4I);
end
plot(t,C,'r','linewidth',2)
hold on
p=0.01;a=0.0434;b=0.0332;e=0.7;g=50;H=0.06;k=0.13;v=0.22;D=0.05;u=0.008;
t(1)=0;S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;T(1)=S(1)+C(1)+R(1)+I(1);
fS=@(t,S,C,R,I) e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;
fC=@(t,S,C,R,I) ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;
fR=@(t,S,C,R,I) (a+b+p)*C-(u+1)*R;
fI=@(t,S,C,R,I) g*(1-e)+(a+b)*S+H*R-u*I;
tfinal=80;h=0.001;N=ceil(tfinal/h);
for i=1:N
    t(i+1)=t(i)+h;
    k1S=fS( t(i),    S(i),        C(i),        R(i),    I(i));
    k1C=fC( t(i),    S(i),        C(i),        R(i),    I(i));
    k1R=fR( t(i),    S(i),        C(i),        R(i),    I(i));

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k1I=fI( t(i),    S(i),    C(i),    R(i),    I(i));

k2S=fS( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2C=fC( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2R=fR( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2I=fI( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+ (h/2)*k1C,R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);

k3S=fS( t(i)+h/2,S(i)+(h/2)*k2S,C(i)+ (h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3C=fC( t(i)+h/2,S(i)+(h/2)*k2S,C(i)+ (h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3R=fR( t(i)+h/2,S(i)+(h/2)*k2S,C(i)+ (h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3I=fI( t(i)+h/2,S(i)+(h/2)*k2S,C(i)+ (h/2)*k2C,R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);

k4S=fS( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R, I(i)+(h)*k3I);
k4C=fC( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4R=fR( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
k4I=fI( t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C,R(i)+(h)*k3R,I(i)+(h)*k3I);
S(i+1)=S(i)+(h/6)*(k1S+2*k2S+2*k3S+k4S);C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C+k4C)
;
R(i+1)=R(i)+(h/6)*(k1R+2*k2R+2*k3R+k4R);I(i+1)=I(i)+(h/6)*(k1I+2*k2I+2*k3I+k4I);
end
plot(t,C,'k-', 'linewidth',2)
hold on
p=0.01;a=0.165;b=0.234;e=0.7;g=50;H=0.06;k=0.13;v=0.22;D=0.05;u=0.008;t(1)=0;
S(1)=9500;C(1)=1500;I(1)=300;R(1)=0;T(1)=S(1)+C(1)+R(1)+I(1);
fS=@(t,S,C,R,I)  e*g+(1-H)*R-((k+v)/(T))*S*C-(a+b)*S-u*S;
fC=@(t,S,C,R,I)  ((k+v)/(T))*S*C-(u+D)*C-(a+b+p)*C;
fR=@(t,S,C,R,I)  (a+b+p)*C-(u+1)*R;
fI=@(t,S,C,R,I)  g*(1-e)+(a+b)*S+H*R-u*I;
tfinal=80;h=0.001;N=ceil(tfinal/h);
for i=1:N
    t(i+1)=t(i)+h;
    k1S=fS( t(i),    S(i),    C(i),    R(i),    I(i));
    k1C=fC( t(i),    S(i),    C(i),    R(i),    I(i));
    k1R=fR( t(i),    S(i),    C(i),    R(i),    I(i));

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k1I=fI( t(i),      S(i),      C(i),      R(i),      I(i));

k2S=fS( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+(h/2)*k1C, R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2C=fC( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+(h/2)*k1C, R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2R=fR( t(i)+h/2, S(i)+(h/2)*k1S,C(i)+(h/2)*k1C, R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);
k2I=fI( t(i)+h/2,S(i)+(h/2)*k1S,C(i)+(h/2)*k1C, R(i)+(h/2)*k1R,I(i)+(h/2)*k1I);

k3S=fS( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C, R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3C=fC( t(i)+h/2,S(i)+(h/2)*k2S,C(i)+(h/2)*k2C, R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3R=fR(t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C, R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);
k3I=fI( t(i)+h/2, S(i)+(h/2)*k2S,C(i)+(h/2)*k2C, R(i)+(h/2)*k2R,I(i)+(h/2)*k2I);

k4S=fS(t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
k4C=fC(t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
k4R=fR(t(i)+h, S(i)+(h)*k3S,C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);
k4I=fI(t(i)+h, S(i)+(h)*k3S, C(i)+(h)*k3C, R(i)+(h)*k3R,I(i)+(h)*k3I);

S(i+1)
=S(i)+(h/6)*(k1S+2*k2S+2*k3S+k4S);C(i+1)=C(i)+(h/6)*(k1C+2*k2C+2*k3C+k4C);
R(i+1) =R(i)+(h/6)*(k1R+2*k2R+2*k3R+k4R);I(i+1)=I(i)+(h/6)*(k1I+2*k2I+2*k3I+k4I);
end
plot(t,C,'b','linewidth',2)
grid on
%legend('\alpha=0','\alpha=0.0095','\alpha=0.0185','\alpha=0.0434','\alpha=0.165')%Figure 9
%title('Corrupt Humans by Varying Mass Education')
legend('\beta=0','\beta=0.0125','\beta=0.0215','\beta=0.0332','\beta=0.234')%Figure 10
%title('Corrupt Humans by Varying religious teaching')
grid on
legend('\beta=0','\beta=0.0125','\beta=0.0215','\beta=0.0332','\beta=0.234') %Figure 11
% title('Corrupt Humans by Combining Mass Education and Religious teaching')
grid on
legend(' \alpha=0, \beta=0','\alpha=0.0095, \beta=0.0125','\alpha=0.0185,
\beta=0.0215','\alpha=0.0434, \beta=0.0332','\alpha=0.165, \beta=0.234')%Figure 12
xlabel('Time (months)')
ylabel('Corrupt Humans')

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RESEARCH OUTPUTS

Publication Paper

Danford, O., Kimathi, M., & Mirau, S. (2020). Mathematical Modelling and analysis of corruption dynamics with control measures in Tanzania. *Journal of Mathematics and Informatics*, 19, 57-79. <http://dx.doi.org/10.22457/jmi.v19a07179>

Poster Presentation

Corruption Dynamics in Tanzania: Modelling the effects of control strategies.