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## An Application of Computational Fluid Dynamics to Optimize Municipal Sewage Networks; A Case of Tororo Municipality, Eastern Uganda.

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### Abstract

Two-phase pipe flow is a common occurrence in many industrial applications such as sewage, water, oil, and gas transportation. Accurate prediction of liquid velocity, holdup and pressure drop is of vast importance to ensure effective design and operation of fluid transport systems. This paper aimed at the simulation of a two-phase flow of air and sewage (water) using an open source software OpenFOAM. Numerical Simulations have been performed using varying dimensions of pipes as well as their inclinations. Specifically, a Standard k- $\epsilon$  turbulence model and the Volume of Fluid (VOF) free water surface model is used to solve the turbulent mixture flow of air and sewage (water). A two dimensional, 0.5m diameter pipe of 20m length is used for the CFD approach based on the Navier-Stokes equations. Results showed that the flow pattern behaviour is influenced by the pipe diameters as well as their inclination. It is concluded that the most effective way to optimize a sewer network system for Tororo Municipality conditions and other similar situations, is by adjusting sewer diameters and slope gradients and expanding the number of sewer network connections of household and industries from 535 (i.e., 31.2% of total) to at least 1,200 (70% of total).

**Keywords:** Computational Fluid Dynamics (CFD); Openfoam; Optimal Design Problem; Municipal Sewer Network.

### 1. Introduction

In urban development history, Municipal Sewage systems were built to collect rain runoff, wastewater, and sewage rapidly. These networks consist of pipes, pumping stations, force mains, manholes, and other facilities required to collect and transport wastewater [1]. Research on urban drainage pipelines focuses on hydraulics such as pipe slopes and flow rate so that sewage and faecal sludge are to be delivered efficiently [2]. The flows at or in the proximity of these structures are typically highly turbulent and often characterized by changes between the open channel (free surface) and pressurized conditions [3]. Such turbulent flows frequently involve complex interactions between air and water [4] as in the case of manholes with multiple in/out pipes, stepped spillways, and flow network structures. The latter structures are typically composed of an entrance manhole and inflow, overflow, and underflow conduits. The main design challenge of the network structure is the allowance of overflows only after underdrain capacity is exceeded while minimizing head losses that reduce the underdrain flow capacity. A second challenge is the prevention of significant backwater effects.

In hydraulic structures such as sewers and spillways, the air in the flow is important, perhaps an indispensable design factor [5]. The presence of air in wastewater: Increases the bulk of the flow thus influencing the height of the chute sidewalls; Prevents the damage of the chute caused by cavitation; Increases the momentum when the air within the boundary layer reduces the shear stress and Re-oxygenates the water flow which contributes to the downstream river quality and the preservation of aerobic species [5].

Sewer networks are considered as complex, large-scale systems since they are dependent on geographical distribution and decentralization with a hierarchical structure that is either on the surface or underground. Each sub-system is in itself composed of a large number of elements with time-varying behaviour, exhibiting



numerous operating modes and subject to changes due to external conditions (weather) and operational constraints [6]. The hydraulic regime within a sewer is a function of several design parameters including pipe material, sewer function, diameter and gradient. It also suggests that a minimum gradient should be achieved for a specific diameter. As a result, some old sewers do not meet the modern requirements for self-cleansing that result to high blockages [7].

Most cities around the world have sewage systems that combine sanitary and stormwater flows within the same network. This is why these networks are known as Combined Sewage Systems CSS [8]. This discharge to the environment, known as Combined Sewage Overflow (CSO), contains biological and chemical contaminants that create major environmental and public health hazards. Therefore, an urban catchment without an effective sewer system may easily encounter inundation and other consequential problems.

Tororo Municipal has combined sewer that experiences CSO with only two operational convectional sewerage treatment ponds that receive and treats wastewater from the domestic and industrial piped networks with many of the houses not connected to the urban sewer network [9]. It was constructed according to Bernoulli equation in the early 1970's for supply of clean water, and the collection, transportation and treatment of human excreta and greywater within the Municipality [10, 11]. This means that the flow conditions are based on the Bernoulli principle. The sewer network has not been developed and expanded to meet the capacity of rapidly growing population and industries in Tororo town and its surroundings'. It's common to find overflowing sewage spilling on the streets which poses a health risk to the residents [11, 12]. This is attributed to drainage system designs, rapid urbanisation and industrialisation.

The open-source CFD software OpenFOAM is equipped with a well-designed C++ library that allows the numerical simulation of various engineering applications [13]. Through its object-orientated structure and the open-source code concept, OpenFOAM is flexible and can be adjusted to very specific environmental fluid flow problems. This study presents a Mathematical model for the optimisation of urban sewer networks using the open source CFD software OpenFOAM based on interFoam solver.

## 2. Methods and Materials

The study combines CFD modeling and numerical methods. The urban sewage network simulations were done for the free-surface flow and for the sewer pipe systems. The free Open Source CFD package, OpenFOAM, based on the standard  $k-\epsilon$  model and the VOF free surface model was used to model the sewer flows, in order to predict maximum flow rate [14]. Structured/unstructured mesh were used with the optimum number of nodes.

In CFD modeling, Navier–Stokes equations of a one-dimensional incompressible flow for velocity  $U$  will be used as the Continuity Equation:

$$\nabla \cdot U = 0 \quad (1)$$

A transport relation for the volume fraction of each cell is solved to find the shape of the free surface (air/liquid interface) as the Conservation of linear momentum (Equation of motion):

$$\frac{\partial \alpha}{\partial t} + \nabla(\alpha U) = 0 \quad (2)$$

Where  $\alpha$  and  $U$  denote the volume fraction and the average flow velocity respectively.

For the given depth of open channel with a steady flow and the mean velocity  $U$ , Manning's equation will be applied to find hydraulic parameters and discharge  $Q$ .

$$U = \frac{1}{n} R^{2/3} S^{1/2}$$



$$Q = AU, \quad (3)$$

Where;  $R$  – Hydraulic radius, the  $A$  – wetted cross sectional area of the pipe,  $n$  – Manning’s roughness coefficient, and  $S$  – Slope of an energy gradient.

An important consideration for sewer performance is the increased volume flow in sewers resulting from increasing sewage efficient sewer connection practices. A large amount of sewage is reliant on the flow of sewers as a vehicle for its movement through the system. Therefore, a reduction in flow may impede the ability of a sewer to efficiently and effectively remove large amounts of sewage [1]. Thus, if the benefits of optimising sewer networks are to be fully realised, consideration must be given to how sewer design can be updated to accommodate increased volumes of sewer flows [15]. This can be achieved using equation (4):

$$\frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho U U) = -\nabla p - \nabla^2 (2\mu U) + F \quad (4)$$

where  $U$  is flow velocity,  $\rho$  is density,  $p$  is pressure,  $\mu$  is viscosity, and  $F$  is body force.

The solution of sewer network optimization problems requires determination of pipe diameters, flow rate and pipe slopes [1, 16]. Finding an optimal least-cost design of sewer networks is difficult because of numerous complex hydraulic and engineering constraints that lead to nonlinear equations [6, 7]. Moreover, the size of sewer networks is generally large, therefore, leading to large scale optimization problem which is generally difficult to solve [1].

## 2.1 CFD Models

Computational Fluid Dynamics (CFD) is most applied to gain insights into most fluid processes and associated phenomena and so presents potential to add value in the analysis of urban drainage systems. CFD presents itself as a useful tool for investigating domain space for physical system design and performance variables, and for diagnosing or troubleshooting system behaviour [17]. Below are various CFD models that have been used;

CFD and artificial neural network modelling of two-phase flow pressure drop using diameter and slopes as decision variables but 2cm and 6cm diameters were only used that limited the diameters and slope for the study [18].

CFD analysis of the effect of elbow radius on pressure drop in multiphase flow was performed in four different 90-degree elbows with air-water two-phase flows [19]. The inside diameters of the elbows with the pressure drops at two different upstream and downstream locations were investigated using empirical, experimental, and computational methods. The mixture models solved the continuity, motion, and the volume fraction equations (5) and (6);

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m v_m) = 0 \text{ and } \rho_m = \sum \alpha \rho, \text{ where } \alpha \text{ is volume fraction.} \quad (5)$$

$$\frac{\partial \rho_m v_m}{\partial t} + \nabla \cdot (\rho_m v_m v_m) = -\nabla p + \nabla \cdot [\mu (\nabla v_m + (\nabla v_m)^T)] + F \quad (6)$$

Where  $F$  is body force,  $\rho_m$  is density of mixture,  $v_m$  is volume of mixture and  $\rho$  is density of water. The inside diameters of the elbows were restricted to 6.35mm and 12.7mm with radius to diameter ratios  $r/D$  of 1.5 to 3.0 but this could be extended to range of diameters and  $r/D$  to obtain various flow rates and pressure drop in sewer pipes. However, it was unable to handle different pipe lengths that are used in sewer networks.

Chen, *et al.*, 2013 [7], suggested a CFD Modelling approach for municipal sewer system design optimization to minimize emissions into receiving water body. He used the CFD model that assisted in design optimization of a municipal sewage system providing detailed flow fields inside of the system and CFD techniques of the

renormalized group (RNG) k- $\epsilon$  turbulence model and the VOF free surface model. For VOF and  $\alpha$  ranges from 0.0 for cell with no sewage to 1.0 that's full of sewage.

Where,  $\alpha$ , and  $u$  denote the volume fraction and the flow velocity respectively. The estimated inlet sewer velocities were specified as the inlet boundary conditions with turbulence intensity ( $I$ ) at the inlet boundary estimated based on the Reynolds number,  $Re$

$$I = 0.16Re^{-1/8} \quad (7)$$

In this study methods based on Computational Fluid Dynamics will be used to design gravity or pumped sewer networks with fixed layout in which diameters and pipe nodal cover depths will be considered as decision variables. The methods of [1,18, 20] will be adapted to accommodate the typical sewer constraints facing Tororo municipality and other locations of similar situations. These methods are chosen because of their robustness and an efficient optimization algorithm for optimal design of sewer networks. In particular, the method adaptation will ensure that all the sewer design constraints are systematically satisfied so that pressure drops and construction cost function are freely minimized. An algorithm for the method will be developed in this study, and its codes will be written and implemented on the free Open Source CFD package OpenFOAM [13].

## 2.2 Sewer networks Design Optimization

In this section, an optimal sewer network design that aim to find pipe diameter and slopes which minimizes pressure drops and sewer system cost is based on model proposed by [20]. Excavation and pipe installations costs are the main part of construction costs which are functions of the pipe slopes and pipe diameters [1].

Sewer network design is subject to the hydraulic constraints regarding full pipe condition, continuity equation, sewer flow velocity  $v$ , sewer pipe nodal cover depths (upstream and downstream)  $d$ , minimum sewer pipe slopes  $S$ , maximum and minimum relative flow depths  $h$ , ratio of flow depth to pipe diameter  $\beta$ , commercial pipe diameters  $D_c$  and progressive pipe diameters  $D_l$  described in equations (8) and (9) as follows.

$$v_{min} \leq v_i \leq v_{max} \quad i = 1 \dots N \quad (8)$$

$$d_{min} \leq d_i \leq d_{max} \quad i = 1 \dots N \quad (9)$$

This constraint can be rewritten in terms of the nodal elevation as:

$$h_{min} \leq h_i \leq h_{max} \quad i = 1 \dots N \quad (10)$$

Where  $h_i$  the nodal elevation of the  $i^{th}$  node,  $N$  is the number of nodes in the network, and  $h_{min}$  and  $h_{max}$  are the allowable minimum and maximum nodal elevations, respectively.

$$S_{min} \leq S_i \leq S_{max} \quad i = 1 \dots N \quad (11)$$

$$\beta_{min} \leq \beta_i \leq \beta_{max}; \beta = h/D \quad i = 1 \dots N \quad (12)$$

$D \in D_c$  and,  $D \leq D_l$ ;  $D$  = diameter of sewer pipe,  $v_0$  = Self - Cleansing velocity.

Due to the advancements in computer technology in recent years, the CFD technique has been a powerful and effective tool to model the real life behaviour of fluids [18]. The free the open source CFD packages, OpenFOAM, based on the standard k- $\epsilon$  model and the Volume of Fluid (VOF) free surface model was used to model the sewer fluid flows, in order to predict maximum flow rate [13, 14]. In meshing, hexahedral mesh was used due to its capabilities in providing high-quality solution, with a fewer number of cells than comparable tetrahedral mesh for simple geometry.



### 2.2.1 Mathematical Model Equations

The mass and momentum equations are solved for isothermal, incompressible and immiscible two-phase flows, written in their conservative form:

$$\text{Continuity Equation: } \nabla \cdot U = 0 \quad (13)$$

A transport relation for the volume fraction of each cell is solved to find the shape of the free surface (air/liquid interface) as:

$$\text{Equation of Motion: } \frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho U U) = -\nabla P + \nabla (2\mu\tau) + \rho g + F \quad (14)$$

where,  $F$  is the continuum surface force (CSF) vector and  $\tau$  is the deformation tensor given as follow.

$$\tau = \frac{1}{2}(\nabla U + [\nabla U]^T) = \frac{1}{2}[\nabla \cdot (\nabla U) + (\nabla U) \cdot \nabla] \quad (15)$$

By substituting Equation (15) and replacing  $\mu = \rho\nu$ , Equation (14) becomes;

$$\frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho U U) - \nabla \cdot (\rho\nu(\nabla U)) = -\nabla p + (\nabla U) \cdot \nabla \cdot (\rho\nu) + \rho g + F \quad (16)$$

The modified pressure  $p^*$  ("p\_rgh" in OpenFOAM code) is adopted in interFoam removing the hydrostatic pressure ( $\rho g \cdot x$ ) from the pressure  $P$ . This is advantageous for the specification of pressure at the boundaries of the space domain. The gradient of the modified pressure is defined as:

$$\nabla p^* = \nabla P - \nabla(\rho g \cdot x) \Leftrightarrow \nabla P = \nabla p^* + \rho g + g \cdot x \nabla \rho \quad (17)$$

The mixture density  $\rho$  and viscosity  $\mu$  of sewage depend on the volume fraction of the flow and they are calculated from the equations;

$$\rho = \alpha\rho_{water} + (1 - \alpha)\rho_{air} \quad \text{and} \quad \mu = \alpha\mu_{water} + (1 - \alpha)\mu_{air} \quad (18)$$

where,  $\alpha$  is the air volume fraction in the cell. The interface between two phases will be tracked by the volume fraction. Conservation of  $\alpha$  can be represented by the interface mass balance using the following equation:

$$\frac{\partial \alpha}{\partial t} + \nabla(\alpha U) = 0 \quad (19)$$

$$U = \alpha U_{water} + (1 - \alpha)U_{air} \quad (20)$$

Also, the compression velocity,

$$U_r = U_{liquid} - U_{air} \quad (21)$$

Where,  $\alpha$  and  $U$  denote the volume fraction, the average flow velocity respectively. The volume fraction  $\alpha$  ranges from 0.0 for a cell with no sewage to 1.0 for a cell full of sewage.

The transport/advection equation is given by Equation (22).

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha U) + \nabla \cdot (\alpha(1 - \alpha)U_r) = 0 \quad (22)$$

where  $U_r$  is compression velocity. This Equation uses an interfacial compression term included to mitigate the effects of numerical diffusion on the gas-liquid interface, rather than using interface reconstruction schemes.

The volumetric surface force function is explicitly estimated by the Continuum Surface Force (CSF) model, where  $\sigma$  is the surface tension and  $\kappa$  is the surface curvature calculated as

$$\kappa = \nabla \cdot (\nabla \alpha / |\nabla \alpha|) \tag{23}$$

Then

$$F = \sigma \kappa \frac{2\rho}{\rho_{air} + \rho_{liquid}} \nabla \alpha \approx \sigma \kappa \nabla \alpha \tag{24}$$

### 2.2.2 Computational Geometry

The computational geometry that was used for this study is described in Figure 1. A pipe of 20m and 0.5m in length and diameter respectively, was considered for the evaluation of the model.

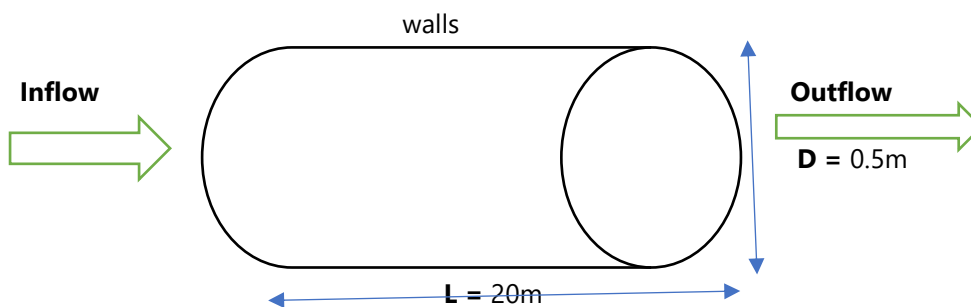


Figure 1: Computational Geometry of the Sewer.

## 3 Results and Discussion

In the market, the commercially available sewer pipe sizes are in the range from DN100mm to DN600mm and DN650mm to DN1200mm on request [21, 22]. These pipe sizes and slopes are important factors in the design of the sewer network. The recommended minimum slopes (S) for different pipe sizes (D) for sewer systems are shown in Table 1.

Table 1: The minimum slopes and pipe sizes for sewer pipes

D (mm)	150	200	250	300	375	450	525	600
S	0.00430	0.00330	0.00250	0.00190	0.00140	0.00110	0.00092	0.00077

### 3.1 Flow pattern visualisation

The general combined standard k- $\epsilon$  and VOF model visualisations generated in the analysis for optimal sewer flow rates are presented in Figures 2 to 10.

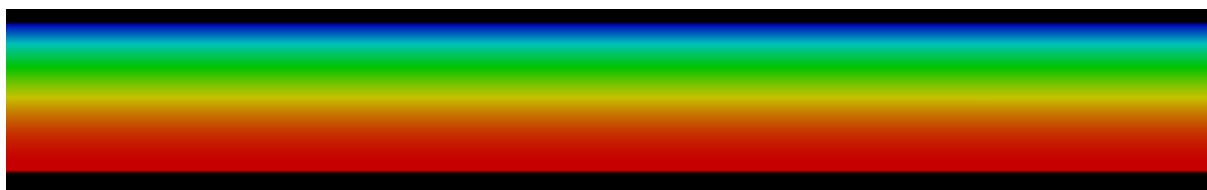


Figure 2: The Velocity Field Flow inside a Pipe (ms<sup>-1</sup>)

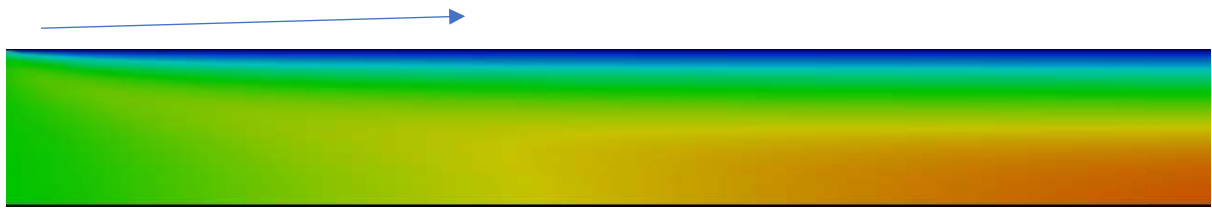


Figure 3: The Pressure Field Flow inside a Pipe ( $m^2s^{-2}$ )

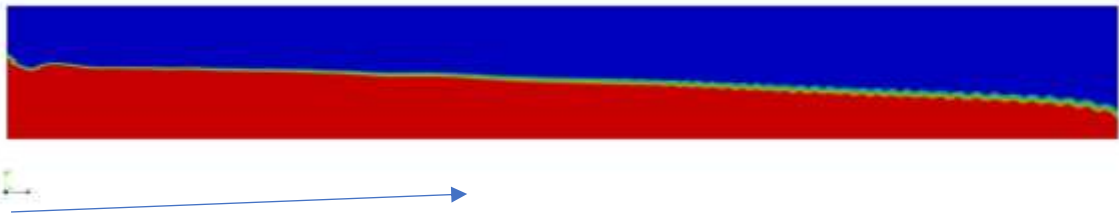


Figure 4: Liquid and Air Flows inside a Pipe for 1.0m Diameter at 0 degrees

Flow direction



Figure 5: Liquid and Air Flows inside a Pipe for 1.0m Diameter at 3 degrees



Figure 6: The Pressure Field Flow inside a Pipe inclined at 3 Degrees ( $m^2s^{-2}$ )

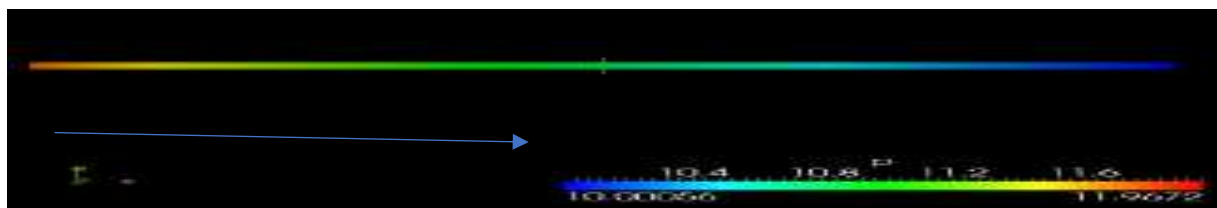
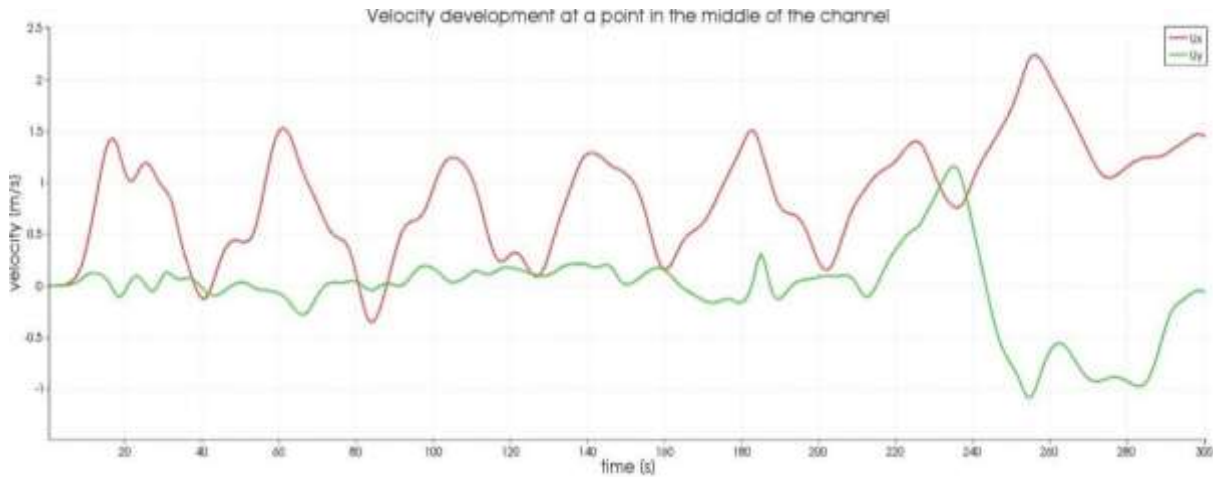


Figure 7: The Pressure Field Flow inside a Pipe ( $m^2s^{-2}$ )

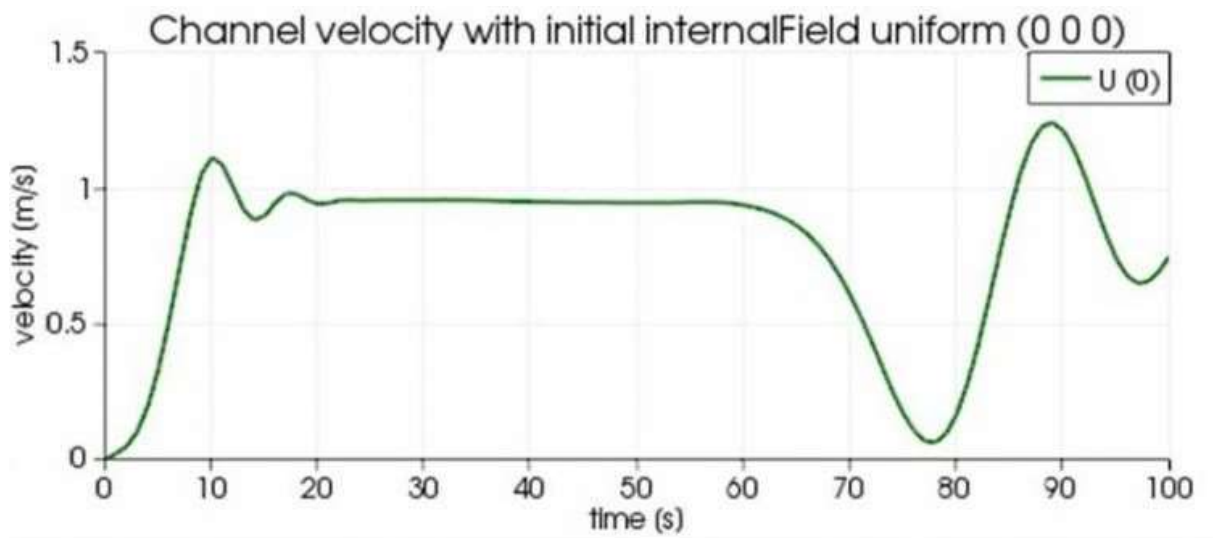




**Figure 8: The Pressure Field Flow inside a Pipe ( $m^2s^{-2}$ )**



**Figure 9: A Graph showing Velocity Development of Fluid in the Channel**



**Figure 10: A Graph of Velocity Field against Time in the Channel**

## 4 Conclusions

### 4.1 Improvement of the sewer system infrastructure

The most effective way to optimize a sewer network system based on fluid pressure is by adjusting sewer diameters and slope gradients in order to achieve optimal flow rate to transport and deliver sewage to the treatment plant. It is also one of the easiest ways to control flow rate. However, specifying pipe sizes accurately may help to control those factors that can change over time. For example, in old metallic and reinforced concrete piping systems, as it is observed in the municipal records, the piping diameters should have been wider in design work than was initially necessary in order to account for material friction and some corrosion and/or scaling.



Currently, there are 535 sewer network connections of households and industries in Tororo Municipality, accounting for 31.2% of the total number. Therefore, they should be expanded from 31.2% to at least 70% of the total number by increasing the number of connections from the current 535 to at least 1200 connections to drastically increase the amount of collection and transportation of sewage to treatment plant by 80% of the current amount. This will reduce operational costs of the municipal authorities and improve the hygiene and health in the town.

Records from both NWSC and municipal office show that UPVC (Unplasticised Polyvinyl Chloride) and HDPE (High Density Polyethylene) pipes are the most preferred and used as they are most effective for the transportation and delivery of sewage and wastewater to treatment plant when compared to metallic and reinforced concrete pipes. UPVC and HDPE pipes are smoother and are manufactured in various pipe sizes for sewage flow and can last for at least 30 years under normal temperatures from 0 - 80°C while metallic ones are prone to corrosion.

#### 4.2 Conclusions for improvement of the model simulations

Running mesh refinements for the pipe/sewer geometries should be conducted properly through continuous refinements and use of well-defined boundary and initial conditions in order to achieve correct and accurate velocity and pressure profiles. The study therefore concludes that: InterFoam solver in Open Foam is suitable to be used for turbulent fluid flow simulation; The velocity profile in the straight pipe agrees well with the analytical solution of the classical formula. For analysing turbulent fluid flow in the complex geometry pipe, the unstructured mesh can be used.

Finally, to optimize a sewer network system for effective fluid flow, the recommended sewer diameters and their corresponding slopes (see Table 1) should be adhered to during network design and construction in order to achieve optimal flow rate to transport and deliver sewage to the treatment plant.

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